PHY211 WAVES & OPTICS LAB

Laboratory Manual

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Recommended Reading

1. Waves (Berkeley Physics Course Volume 3), by F. S. Crawford Jr, McGraw-Hill (1968).
2. Vibrations and Waves, by A. P. French, CBS Publishers (2003).
3. The Physics of Vibrations and Waves 6th Edition, by H. J. Pain, Wiley (2005).
4. Optics 4th Edition, by E. Hecht, Addison Wesley (2001).
5. Fundamentals of Optics 4th Edition, by F. Jenkins and H. White, McGraw-Hill (2001).

Contents

Chapter 1

Sonometer: The Resonant String

1.1 Aim

To observe standing wave modes in a stretched string. To vary tension, length and linear density of the sonometer wire and observe changes in frequency of vibrated string. To find the linear density of an unknown wire.

1.2 Experimental Setup

Sonometer box with non-magnetic/magnetic wire. Slotted weights and pulley. Screw gauge. Set of tuning forks of different frequencies. Set of different sonometer wires with attached leads to hang from pulley. Magnetic coil vibrator at a fixed frequency.

1.3 Physics Concepts

A sonometer is a "bare-bones" apparatus to study the transverse vibrations of strings. The sonometer is a hollow box with two fixed bridges near the ends and a pulley fixed at one end. A string (metal wire) is fixed at one end, run over the bridges and the pulley and attached to a weight holder hanging below the pulley. Weights can be added to the hanger to vary the tension in the wire and a third movable bridge can be placed to change the length of the vibrating string. The sonometer demonstrates the relationship between the frequency of sound produced by a plucked string, the tension, length and mass per unit length of the string. Many musical instruments such as guitars, pianos and violins operate by creating standing waves. The frequency of mode n of a perfect string fixed at both ends is given by

$$f_n = n\frac{\nu}{2L} \tag{1.1}$$

where L is the string length and f_n the frequencies at which the string will resonate. The lowest or fundamental node has $n = 1$. When transverse waves are excited in the stretched wire of the sonometer, the bridges act as rigid reflectors of these waves. The length of the wire between two bridges becomes a bound medium with waves reflected at both ends and stationary waves are formed with bridges as nodes. In the fundamental mode, when the wire

Figure 1.1: Experimental setup for sonometer.

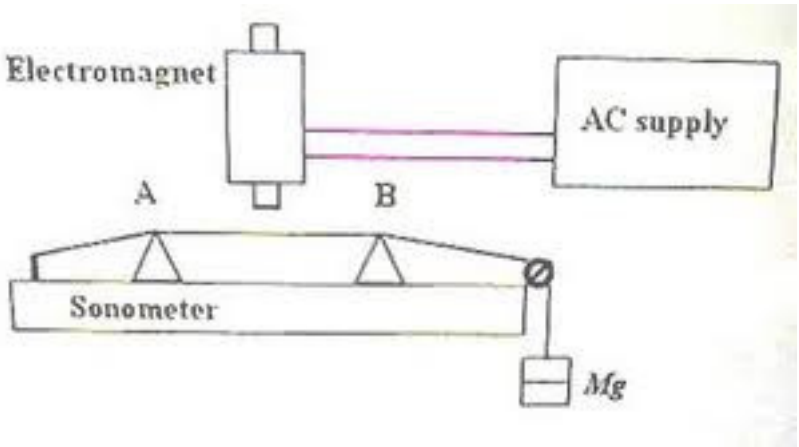

Figure 1.2: Block diagram of sonometer setup.

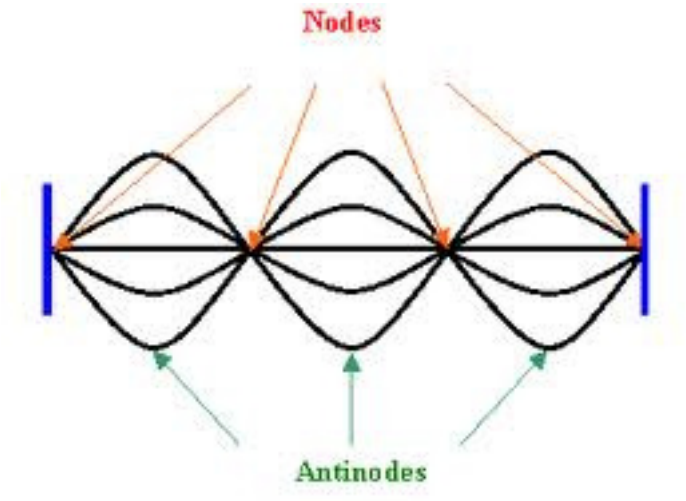

Figure 1.3: Nodes and antinodes in standing waves.

vibrates in one loop, the wavelength λ of the transverse waves excited in the string is given by $\lambda/2 = l$ where l is the distance between the bridges. If elastic forces are negligible compared to the tension, the velocity of transverse waves in the string is given by

$$v = \sqrt{\frac{T}{m}} \tag{1.2}$$

where T is the tension and m is the mass per unit length of the wire. The fundamental mode of the wire is given by

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}} \tag{1.3}$$

The frequency of the wire can be varied by varying T or l. If you know the linear density (mass per unit length) of the string and the tension on the string you can calculate the frequency at which the string will vibrate for a given length, like the length of a guitar neck or a piano backboard. You can vary the tension of the sonometer string and see how it changes the frequency at which the string will resonate. When a current carrying wire is placed in a magnetic field perpendicular to its length, the wire experiences a magnetic force whose direction is perpendicular to both the wire and the magnetic field. The wire thus experiences a force in the vertical direction with its sense being given by Fleming's left hand rule. If alternating current is being passed through the wire, it will experience an upward force in one half cycle and a downward force in the next half cycle. The wire gets impulses alternately in opposite directions at the frequency of the current and it begins to execute forced transverse vibrations with the frequency f_{AC} of the source. If the distance between the bridges is adjusted so that the natural frequency of vibrations f of the wire becomes equal to the frequency of the source, resonance occurs and the wire will begin to vibrate with a large amplitude. In this case

$$f_{AC} = \frac{1}{2l}\sqrt{\frac{T}{m}} \tag{1.4}$$

where $T = Mg$ is the tension in the sonometer wire, M is the hanging mass, g is the acceleration due to gravity, $m = \pi r^2 \rho$ is the mass per unit length of the wire, l is the length of the sonometer wire between the bridges at resonance, ρ is the density of the sonometer wire, and r is the radius of the sonometer wire.

The sonometer thus allows us to "drive" the string into vibration (without plucking the string) by using a magnetic field. This permits us to test which harmonics can be made to ring within a certain length/type of string. The sonometer can also be used to analyze those parts of the string that are "louder" than others. The string can be vibrated at a frequency set by the function generator. The "sensor" senses the motion of the string and turns it into a voltage that can be seen on the oscilloscope or can be turned into numbers using the microphone input to the computer.

The frequency of mode n for a perfect string that is fixed at both ends is given by

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}} \tag{1.5}$$

where L is the length of the string, f_n the frequencies at which the string will resonate, n the mode number, T the tension in the wire and m the mass per unit length of the string. This formula predicts that each harmonic is an exact integer multiple of the fundamental - it is a linear relationship. While this is a good approximation, it does not predict the exact frequencies of the overtones particularly for thick strings (when the string is said to be non-linear). The wave equation describing motion on the string contains non-linear terms. In this experiment you will try to measure as many overtones as you can and see if their frequencies deviate from those predicted in the linear equation. String non-linearity is important for instruments with heavy base strings. For example, the overtones on a piano are systematically sharp compared to integer multiples of the fundamental. In this case f_n is proportional to n plus another term. We could write for example $f_n = A_n + Bn^2$ with A and B constants. For the piano the constant B is positive and so the high overtones have a higher frequency than predicted with B=0.

After a string is plucked many modes of oscillation are excited. However each mode or overtone may decay at a different rate leading to a change in the timbre of the sound. We can try to measure this effect by comparing the strength of one of the higher overtones to the fundamental at different times after plucking. A string that has strong higher overtones at longer times after plucking is said to have a bright sound. A strong with higher overtones that quickly decay after plucking is said to have a soft sound. The decay rate of overtones is affected by string composition, structure and tension. For example bronze strings are considered softer or less bright than steel strings. Nickel/steel strings are often chosen for electric guitars. Heavier strings wind up under higher tension for the same pitch. A string under higher tension is harder to play (you need more force to push the string down). There may also be differences in the time the note is sustained after plucking (how long the note is loud).

1.4 Experimental Procedure

- Vary the tension and the length of the wire and obtain the fundamental mode of frequency of the sonometer wire for these different values. Devise your own strategy for error analysis and make plots of relevant quantities and from the slope of the resultant curve, make your own inferences.
- Repeat the experiment using different strings.
- Increase the tension of the string and see what happens to the pitch (does it increase or decrease). Measure the pitch of two different strings under the same tension.
- Find the linear density of an unknown wire, using the method of resonance on a sonometer.
- Use resonance matching and the set of tuning forks to find an unknown frequency (of the magnetic vibrator).
- Measure the diameter of the sonometer wire using a screw gauge and find the mean radius of the wire. Plot a graph between $\sqrt{M}$ and the length l. Estimate the frequency from the slope of the graph.
- **Optional:** Do an experiment to test the non-linearity of the sonometer string. Connect the output of the sonometer to the computer and record its sound on the computer. Pluck the string and record the output. Measure the frequency of as many harmonics as you can. Use a heavy string to measure more overtones as its fundamental frequency is lower. Plot n versus the frequency of each harmonic. If the linear model for wave propagation on a string does not hold good then your wave equation fails to perfectly predict the frequencies of all the modes and you have found that the waves on the string are non-linear.

1.5 Safety & Precautions

- The sonometer wire should be uniformly stretched and free of kinks.
- There should be no friction at the pulley.
- The magnet resonator should be placed at the middle of the bridges.
- Make the paper rider as light as possible so that it jumps off easily at resonance.

Chapter 2

Normal modes in a two-dimensional pendulum

2.1 Aim

To study the normal modes and symmetry breaking in a two-dimensional coupled pendulum apparatus.

2.2 Introduction

This experiment deals with an experiment to demonstrate normal modes and symmetry breaking in a two-dimensional pendulum. A simple pendulum hanging from a point suspension is actually a two-dimensional pendulum. The oscillation can be begun in any plane and the pendulum will oscillate in the same plane with a frequency dictated by its length and of course the acceleration due to gravity. The system Hamiltonian has a complete cylindrical symmetry and therefore oscillations in all planes are completely identical. In practice, if one sets up a pendulum, the plane typically moves in a certain interval of time and the pendulum acquires an elliptical character to its motion.These effects are usually ignored in most experiments as errors in initial conditions.

Initial Observation :
A careful study of such a system reveals that these effects have a certain systematic character and that the plane of oscillation of the pendulum moves and after some time comes back to its original direction in a somewhat mysterious way.

Why does it happen?
It turns out that this is due to the fact that suspensions are not completely cylindrically symmetric and that the symmetry breaking leads to non-degenerate normal modes of oscillation whose interplay gives rise to these effects. This is also a major problem while building a Foucault's pendulum.

How it is realized :
We explicitly break the symmetry of the pendulum by attaching a small weak spring to the suspension along one direction. We show how this leads to a very interesting phenomenon of the pendulum becoming non-planar, being set into elliptical motion, moving into an entirely different plane and coming back to its original plane with a certain time period. We demonstrate that this time period is associated with the amount of symmetry breaking in the system by using springs of varying spring constants.

You will learn through this experiment :

The following three concepts are introduced:

1. Realization of normal modes using a single pendulum.
2. The concept of symmetry breaking.
3. The difficulties associated with building a Foucault's pendulum.

2.3 Theory

Consider a two-dimensional linear oscillator with complete cylindrical symmetry. This implies that force is proportional to $\vec{r}$ in the x-y place. The Hamiltonian of such an oscillator is written as

$$H_(p_x, p_y, x, y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}k(x^2 + y^2) \tag{2.1}$$

where p_i is the momentum in the ith direction and m and k are the mass and the force constant respectively. This Hamiltonian is symmetric under rotations in the plane. If we displace such an oscillator to a distance d in any direction and let it oscillate, the frequency of oscillation is independent of the direction in which we pull it and is given by $w_0 = \sqrt{\frac{k}{m}}$. The oscillator moves back and forth in the same direction forever, without the direction of oscillation changing in time. Therefore there are an infinite number of linear polarization modes that we can begin this oscillatory motion in. We can also start the oscillator in circular or elliptical motion and in that case the motion pattern will again remain the same as time passes, and there is only one frequency of oscillation in the system. Another way to think about this system is to think of two independent modes of the oscillator, one along x and one along y, which are independent and have the same frequency.

A modification where we break the symmetry and make the force constant in one direction different from the other, leads to the Hamiltonian

$$H(p_x, p_y, x, y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}(k_1x^2 + k_2y^2) \tag{2.2}$$

where k_1, k_2 are the force constants in two different directions. This is easy to conceive and we can think of the mass m attached to two springs, one along x and other along y, and each spring is of a different spring constant. The cylindrical symmetry is now broken and this will reflect in the motion.

What is the motion pattern of such a system? Let us write the equations of motion:

$$\begin{aligned} m\ddot{x} &= -k_1 x \\ m\ddot{y} &= -k_2 x \end{aligned} \tag{2.3}$$

This means that if we pull the oscillator in the x direction it oscillates with the frequency $w_1 = \sqrt{\frac{k_1}{m}}$ and if we pull it along the y direction it oscillates with the frequency $w_2 = \sqrt{\frac{k_2}{m}}$. There are two fundamental frequencies in the system namely ω_1 and ω_2. Since these motions are decoupled, the motion pattern does not change and the direction of oscillation remains the same with the passage of time.

Consider starting the oscillator in a direction which is not along either x or y. What is the motion in this case? Let us imagine that the oscillator is pulled by an amount d along some general direction making an angle θ with respect to the x axis. This means that the initial condition of the oscillator is $x_0 = d\cos\theta$ and $y_0 = d\sin\theta$. The equations of motion given in equation (2.3) immediately tell us what the solution is going to be. The general solution of the equations (2.3) is

$$\begin{aligned} x(t) &= A\cos(\omega_1 t + \phi_1) \\ y(t) &= B\cos(\omega_2 t + \phi_2) \end{aligned} \tag{2.4}$$

For the initial conditions that we have started the oscillator with, the solution reduces to

$$\begin{aligned} x(t) &= x_0\cos(\omega_1 t) \\ y(t) &= y_0\cos(\omega_2 t) \end{aligned} \tag{2.5}$$

Clearly if $\omega_1 = \omega_2$, the motion will be oscillatory along a straight line. However for the case when symmetry is broken and $\omega_1 \neq \omega_2$ what is the motion pattern? It turns out that the motion pattern in this case is is very interesting. Consider the case when the difference in ω_1 and ω_2 is small. Which means that the amount symmetry breaking is small. In this case, if we start the oscillator in a straight line oscillatory motion at an angle θ with the x axis, the motion reveals a very interesting pattern with time. After sometime the motion acquires a somewhat elliptical character with its semi-major axis not along θ. The semi-major axis slowly moves toward x and the semi-minor axis slowly grows. After the semi-major axis crosses the x axis, the semi-minor axis begins to decrease and the motion settles into straight line path again on the opposite side at an angle $-\theta$. From this position the pattern repeats in the reverse direction and so on. We have shown an example of such a motion in Figure (2.1).

The motion described above is in fact a demonstration of normal modes. There are two normal frequencies in the system which are ω_1 and ω_2 and the normal mode motion corresponds to starting the system in either the x or the y direction. For such a motion, the trajectory remains the same over time. However if we start the oscillator in a different direction which is not one of the normal modes, there is transfer of energy from one motion pattern to another over a certain time scale which depends upon the coupling strength of the modes represented, via the difference in frequency of the two normal modes.

We now consider the notion of symmetry breaking in this system. A natural question to ask is what if we attach a large number of springs in different directions in the plane to the

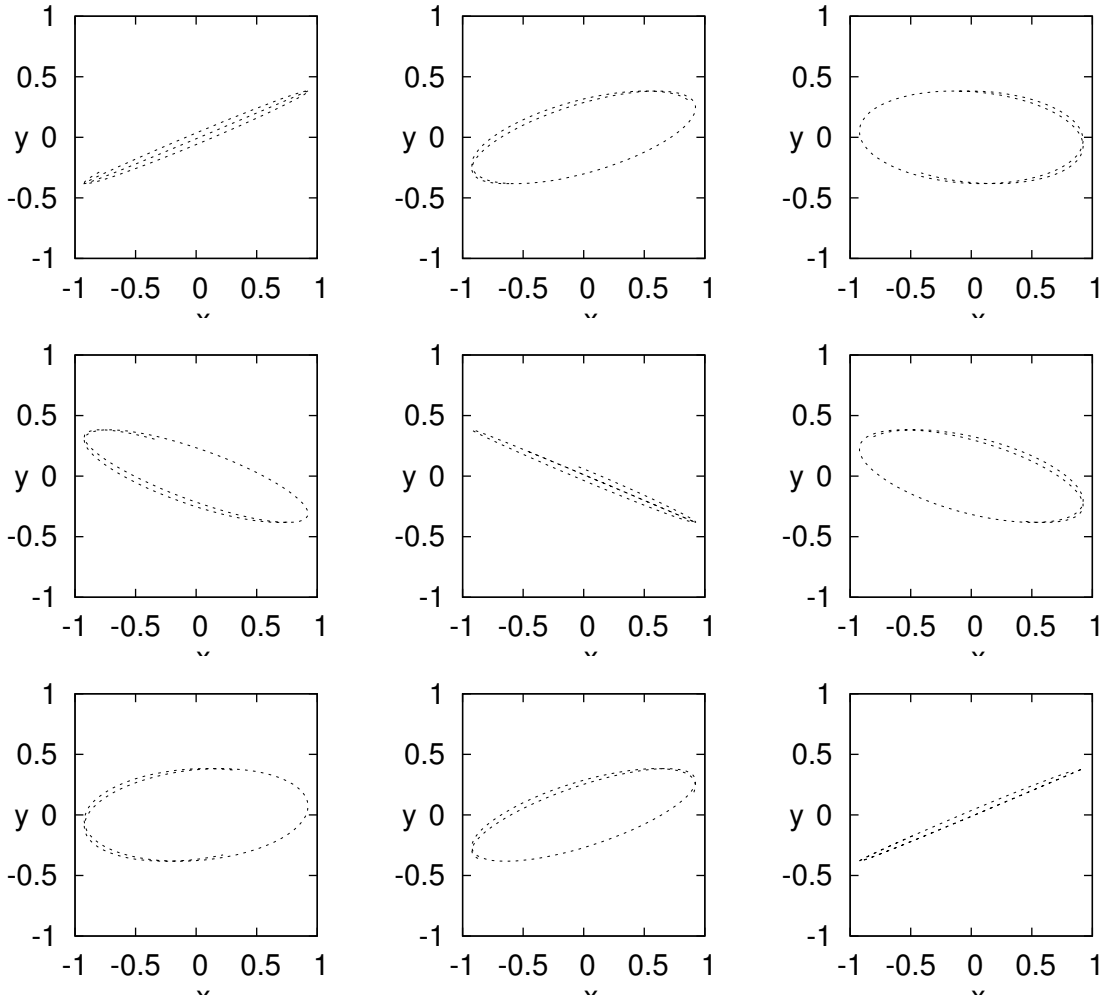

Figure 2.1: Simulation of the motion of a two-dimensional oscillator where $\omega_1 \neq \omega_2$. The difference in frequencies is very small. The oscillation is started at an angle of $\pi/8$ with respect to the x axis. Several snapshots of the motion are displayed for progressive values of time. In the beginning the oscillator oscillates in a straight line, with the passage of time the oscillations become elliptical and, the direction of oscillation gradually rotates and the motion again becomes straight line motion at a certain time along $-\pi/8$. From this the oscillation slowly comes back to the original motion pattern.

mass? It turns out that regardless of the method we choose to break the symmetry, as long as we restrict ourselves to linear springs, there are always two normal modes of the system.

The time scale over which the direction of oscillation comes back to its original direction is inversely proportional to the frequency difference between the modes. The amount by which the direction turns accumulates over the oscillation process. Therefore however small the turning per oscillation may be, its additive effect over many oscillations is appreciable. This then connects with the Foucault pendulum experiment where we would like the oscillator (in the absence of the Coriolis force) to keep oscillating in the same direction forever(or at least for one whole day)!

We realized this two-dimensional oscillator as a two-dimensional pendulum hanging from a point suspension. The symmetry breaking possibility is realized by attaching a spring to the suspension and the angular amplitudes are kept small so that the linear approximation is valid. The construction details of the setup and the experiments are described in the following sections.

2.4 Design of the apparatus

The two-dimensional oscillator system described above can be realized by a two dimensional pendulum oscillating under the effect of gravity. The frequency can be chosen by adjusting the length of the pendulum. The symmetry is broken by attaching a spring toward one side of the suspension so that if the pendulum is pulled in that direction, apart from gravity, the spring also provides an additional restoring force. The linearity is achieved by restricting to small amplitudes.

A photograph of the actual oscillator setup is depicted in Figure (2.2) in two parts. The upper photograph shows the suspension which is a combination of a cylindrically symmetric suspension and a spring arrangement aimed at breaking the symmetry. The bob is a cylinder of mass m, copper wire was used for the suspension. Springs of different strengths can be attached to change the strength of symmetry breaking. We can start the oscillator by giving it a push in any desired direction.

The system is designed such that the size and the time periods lead to a visual observation of the motion. The time period is about 3 seconds, which means that we can easily observe the system and its motion. A balance has to be maintained between the time for observation and the system dynamics. On the one hand we want the system to be slow enough so that we can see the motion, on the other hand the system must perform a large number of oscillations to achieve a cumulative effect of symmetry breaking before dissipation becomes dominant. Spring constants of springs have to chosen such that the perturbation is small. A longer length also allows one to remain in the linear domain even for visually large bob amplitudes.

2.5 Experimental Procedure

2.5.1 Motion pattern

Start the oscillations in different directions and carefully observe the qualitative features of the motion of the bob. Write down your observations in words.

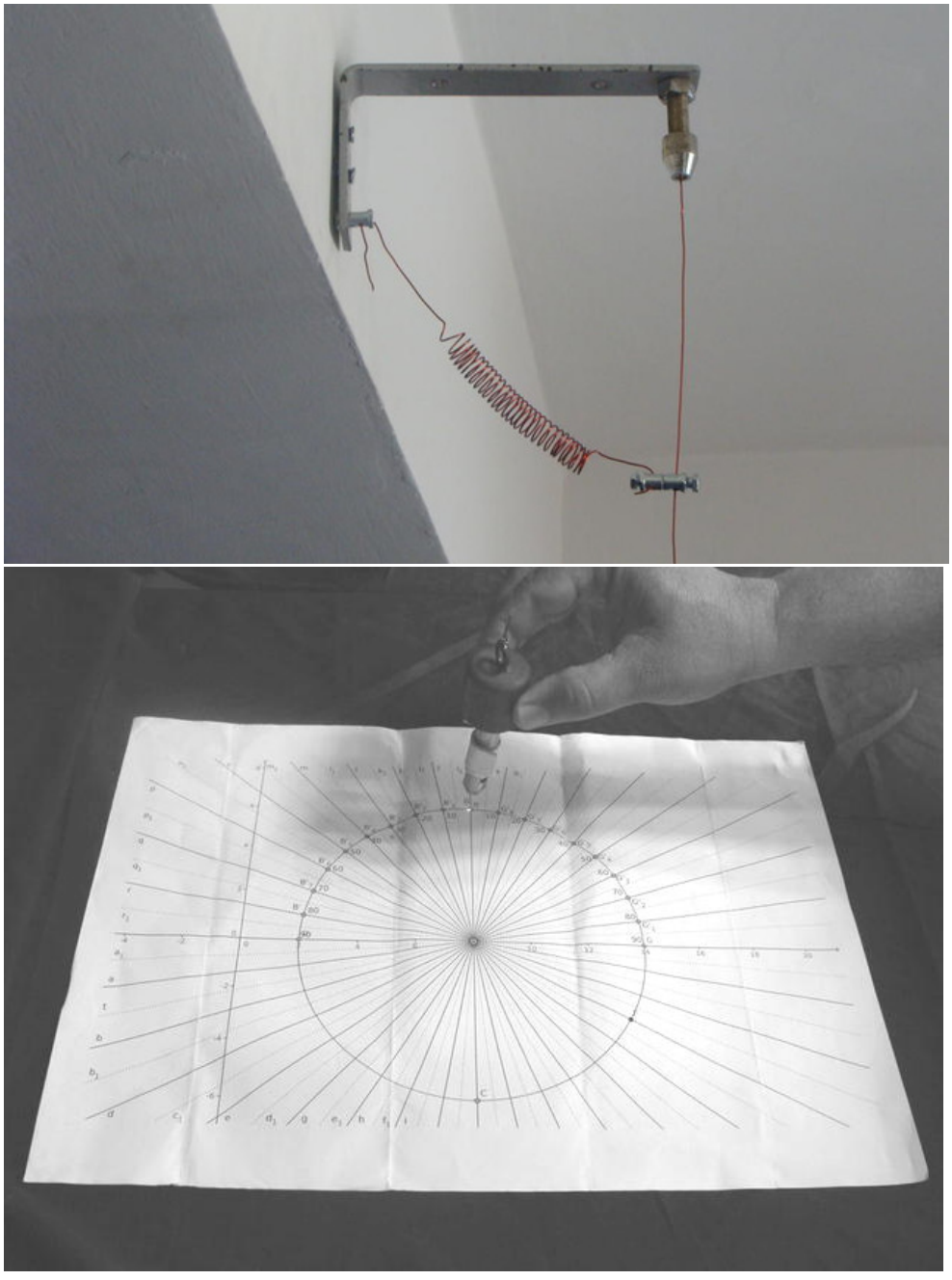

Figure 2.2: The top photograph shows the suspension of the pendulum with spring attached to break symmetry. The pendulum bob being released at a particular angle is shown in the photograph below.

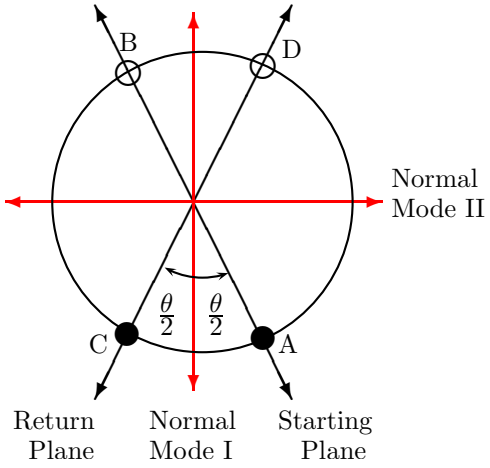

Figure 2.3: Experimental determination of normal modes. If the pendulum is started in a planar motion in the plane AB it oscillates, changes its motion to an elliptical one and again settles into a planar motion along CD at a later time. If we mark these planes on a diagram then we can compute the normal mode by computing the bisector of the angle between these planes. The second normal mode is orthogonal to this direction.

2.5.2 Determination of normal modes

Start the pendulum in some direction and note the direction which is marked on the sheet. The pendulum oscillates and changes its motion pattern and finally again settles into linear motion in a different direction, which is again marked on the sheet. This situation is shown in Figure 2.3, where we indicate that if the initial plane of motion is along A-B and the final plane of motion is C-D then the pendulum turns by a total angle of θ during its motion. The direction that bisects these two lines is one of the normal modes of motion. The second normal mode is along the direction perpendicular to this direction. The normal mode directions thus computed are indicated in the figure. Having determined the normal directions, you can verify that indeed you have found the normal modes correctly by starting the pendulum along any of the normal modes and observing the motion which then remains planar for a very long time, without turning at all.

2.5.3 Return time measurement

Having determined the normal modes, you know the basic characteristics of the motion. If the pendulum is started along AB, over time the motion becomes elliptical, settles along CD and then returns to the original direction AB. Return time is defined as the time the pendulum takes to go from AB to CD and back to AB. Measure the return time for several different angles of the pendulum and tabulate your results. Return time turns out to be independent of the angle with respect to the normal mode from which we release the bob. The return time in fact is a measure of the amount by which the symmetry is broken.

2.5.4 Determination of normal mode frequencies

Next start the pendulum in each of the normal modes and measure the time periods of motion for the two normal modes. The time periods and the associated frequencies will be different. This difference has developed because of symmetry breaking and can be related to the extent to which symmetry is broken. This difference in frequency of the two normal modes is directly related to the return time. The relationships are same for different starting angles and depend only upon the amount of symmetry breaking.

2.5.5 Repeating with a different spring

If you wish you can repeat the whole set of observations for different springs, i.e. for different amounts of the symmetry breaking. With the qualitative observations remaining the same the quantitative results change indicating a different amount of broken symmetry. You can also repeat the experiments with several different springs attached and figure out for yourself what will be the consequences of having more than one springs. It turns out that since spring forces add like vectors you do not see any difference in the motion pattern and several springs act in the same way as one resultant spring. Another way to put it will be that a symmetric potential matrix in two dimensions has only one off-diagonal term!

2.6 Discussion

One observes that the symmetry breaking leads to rotation of the plane of the oscillator over a time scale related to the strength of symmetry breaking. What if we want to build a Foucault's pendulum? In that case the oscillator plane should not shift (due to this effect) over one day. Let us do an order of magnitude calculation regarding the required precision for the system. The time period of shift is related to $\Delta\omega$ and demanding stability over a period of 24 hours implies that the suspension should be such that the frequencies should be the same up to parts per million, in order to observe the effects of Coriolis force!

2.6.1 Quadratic Hamiltonians

An interesting point to note here is regarding quadratic Hamiltonians. A general quadratic Hamiltonian for a two-dimensional oscillator can be written as

$$H(p_x, p_y, x, y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}(k_x x^2 + k_y y^2 + 2k_{xy} xy) \tag{2.6}$$

This Hamiltonian has a simple structure where the kinetic energy part is invariant under rotation and the potential energy part can be diagonalized by going to an appropriate frame which is related to the original frame by a rotation. Let us look at the potential energy term in quadratic form

$$k_x x^2 + k_y y^2 + 2k_{xy} xy = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} k_x & k_{xy} \\ k_{xy} & ky \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.7}$$

Since the coupling matrix is symmetric, we can always find a frame in which the matrix is diagonal by performing a rotation namely

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} k_x & k_{xy} \\ k_{xy} & ky \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} k'_x & 0 \\ 0 & k'_y \end{pmatrix} \tag{2.8}$$

The rotation matrix given above corresponds to a rotation of the coordinate system such that the new coordinates are given by

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.9}$$

The novel aspect of our experiment is that this rotation matrix and the normal modes can found experimentally! We start the oscillator in any arbitrary direction, observe its motion and mark the point of return. The normal mode direction lies half-way between the starting direction and final direction from which the oscillator returns. This allows us to find the angle θ of the rotation which diagonalizes the potential energy term and takes us to the normal coordinates.

Finally it turns out that if we place a large number springs in arbitrary directions, the sum total of their effect is represented by a quadratic Hamiltonian of the form given in equation (2.6). As long as the springs are linear, there is no further complexity to the problem, and we can always find the normal modes of the system by following the same experimental procedure.

Chapter 3

Melde's Experiment

3.1 Aim

To find the frequency of the wave and the frequency of the tuning fork by Meldes experiment.

3.2 Experimental Setup

Thread, weights, pan, Melde's arrangement (battery, electrically maintained tuning fork, stand with clamp and pulley)

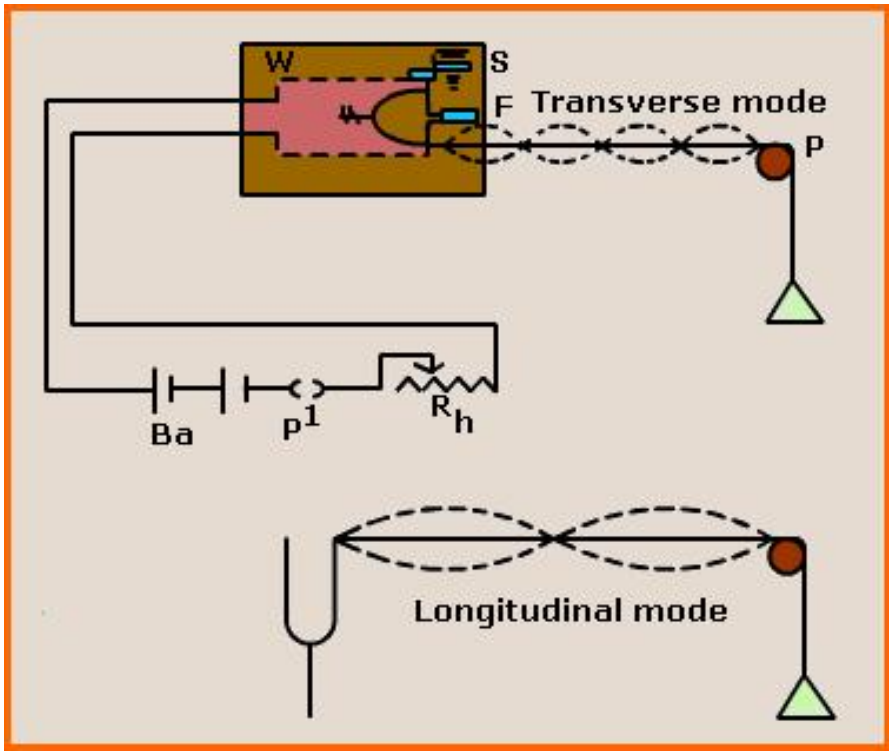

Figure 3.1: Melde's arrangement

3.3 Physics Concepts

This is an easy and efficient method by which nodes and antinodes formed on a string can be demonstrated. A tuning fork is fitted in a wooden block. A long string or thread of cotton is taken. One end of this string is tied to one of the prongs of the tuning fork and the second end of this string passes over a smooth pulley in such a way that a small pan can be hang on it. Small weights are put in the pan to create tension in the string. This arrangement is shown in the diagram.

The fork is set into vibrations by connecting an electric supply as shown in the diagram. Thus the transverse vibrations are produced in the string and by adjusting the length and/or weights in the pan, these vibrations are reflected from the point of contact between the string and the pulley and a number of loops appear on the string, as shown in the diagram.

When the prongs of the tuning fork vibrate in a plane, parallel to the direction of propagation of the waves, it is called A position or longitudinal position. If the tuning fork, along with the wooden block is rotated through 90°, so that the plane of vibration of the tuning fork is normal (i.e. at right angles) to the direction of propagation, it is called B position or transverse position.

For the same length and same weight in the pan (i.e. for the same tension) the number of loops in B position is double than in A position. Thus, either in A position or in B position, standing transverse waves are generated in the string with a series of nodes and antinodes at equal distances.

The equation connecting number of loops (n or n), length of the string (l), tension in the string (T), mass per unit length of the string (m) and frequency of the string (f) is as follows.

For longitudinal position (A)

$$f = \frac{n}{2l}\sqrt{\frac{T}{m}} \tag{3.1}$$

For transverse position (B)

$$f' = \frac{n'}{2l}\sqrt{\frac{T}{m}} \tag{3.2}$$

If the frequency of tuning fork is F, then for A position f = $\frac{F}{2}$ and for 'B' position f = F.

In the above equations, if all the quantities on the R.H.S. are known, the frequency of the tuning fork F can be calculated.

3.4 Experimental Procedure

Set Meldes experiment apparatus as shown in the diagram in longitudinal position and switch ON the power supply, put some weight in scale pan and adjust the length of the string in such a way that you find some loops on the string (e.g. 2 loops). Now rotate the apparatus by 90° so that it is set for B position, for the same values of weight in pan and for the same length of the string, the number of loops created in B position are double (e.g. 4 loops) than in A position. Note down these readings in observation table. Repeat the experiment for other values of weight in scale pan. Calculate the tension in the string (T=wt in the pan+wt of

the pan). Measure one meter length of the thread and find its mass and obtain the mass per unit length.

3.5 Safety & Precautions

- The thread should be uniform and inextensible.
- Well defined loops should be obtained by finely adjusting the weights in the pan.

Chapter 4

Resonance Column and Speed of Sound

4.1 Aim

To measure the speed of sound in air at room temperature, using the resonance of an air column in a resonant column.

4.2 Basics of the Setup

The resonant column apparatus consists of a long metal pipe/column mounted on a vertical board (with a marked metre scale alongside) and a rubber tubing connecting the column to a beaker containing water, which is the "reservoir". The water level in the column can be adjusted by lowering/raising the beaker using the attached clamp. The resonance tube can be leveled using the leveling screws provided at the bottom. Tuning forks of different frequencies are provided to achieve different resonant frequencies.

4.3 Physics Concepts

When a system is driven at a natural frequency, we say that it is set into "resonance" and the particular frequency at which it occurs is called the resonance frequency. From the relationship between frequency ν, wavelength λ and speed v, which is $\lambda\nu = v$, it can be inferred that if the frequency is known and the wavelength is experimentally determined, the speed of the wave can be determined. Sound propagates as longitudinal waves in air. When a tuning fork of a particular frequency is made to vibrate over the open end of the partially filled (with water) resonant column, sound waves propagate in the column which are reflected back from the surface of the water and form standing waves due to interference.

Air columns in a tube of a fixed length L have particular resonant frequencies - longitudinal standing waves are set up in the tube due to the interference between waves traveling down and up the tube. These standing waves must have a node at the closed end of the tube and an antinode at the open end of the tube. In a standing wave points of no displacement are called

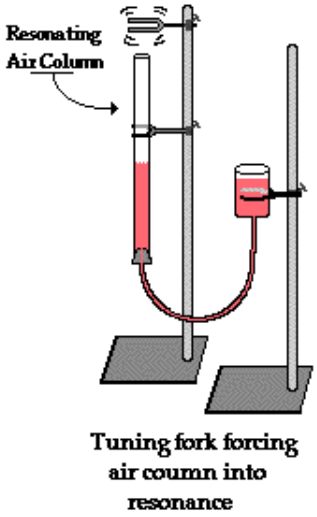

Figure 4.1: The resonant column experimental setup.

nodes, while points of maximum displacement are called antinodes. The distance between two consecutive nodes or antinodes is $\lambda/2$ while the distance between a node and the antinode next to it is $\lambda/4$. With these node-antinode requirements, hence only a certain number of wavelengths can be "fitted" into the tube length. Resonance occurs when the length of the tube is almost equal to an odd number of quarter wavelengths i.e $L = \frac{n\lambda}{4}$ where $n = 1, 3, 5, ...$ Plugging this back into the relation between frequency, wavelength and speed, it can be easily seen that

$$\nu_n = \frac{nv}{4L}, \ n = 1, 3, 5... \tag{4.1}$$

Hence an air column of length L has particular resonant frequencies which are in resonance with the corresponding odd-harmonic driving frequencies. In actual fact, it was observed by Rayleigh that the position of the antinode at the top of the air column is slightly above the top of the tube, about 0.6 times the radius of the tube (called the end-correction).

In a sound wave the general method to find the speed is by using the relation

$$v = \sqrt{\frac{\partial p}{\partial \rho}} \tag{4.2}$$

where p is the pressure and ρ is the mass density. For liquids this relationship is recast in terms of the bulk modulus and for solids it is written in terms of the Young's modulus. For gases (like air) the bulk modulus is not easy to deal with since in general it will vary with temperature, pressure and density. So the speed of sound equation is recast for ideal gases to be:

$$v = \sqrt{\frac{\gamma kT}{m}} \tag{4.3}$$

where γ is the adiabatic index (a constant, for dry air it is 1.4) which is equivalent to the ratio of the specific heat at constant pressure to the specific heat at constant volume; k is

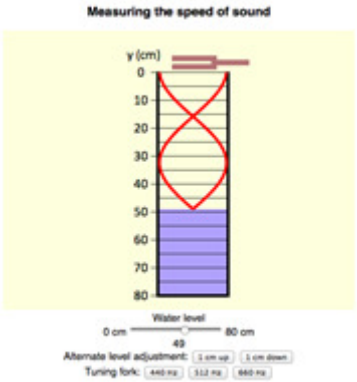

Figure 4.2: Standing waves in a resonating air column.

the Boltzmann's constant, T is the temperature and m is the molecular mass. Air can be considered to be a nearly ideal gas. The speed of sound in air v (in m/s) is hence temperature dependent and is given by

$$v = 331.5\text{m/s} + 0.6\text{m/s/C} \times T \tag{4.4}$$

where T is the air temperature in degrees Celsius. The speed of sound increases by 0.6 m/s for each degree of temperature increase. Equation 4.4 retains the first two terms of the Taylor expansion of the actual equation

$$v = 331.5\sqrt{1 + \frac{T}{273.15}}\text{m/s} \tag{4.5}$$

and the value of 331.5 m/s represents the speed of sound in air at 0°C (in actuality, the speed of sound in air at 0° may vary from 331.2 to 331.6 depending on the various assumptions made about air pressure and moisture present).

4.4 Experimental Procedure

- Strike a tuning fork on the rubber pad provided, hold it over the open end of the column and adjust the water level in the column till you obtain a resonance. Note down the corresponding meter scale reading of the length of the column above the water level. Once you have found the approximate postion of the resonance point, make several trials by running the water surface up and down and make an estimate of the error involved in locating the point of resonance. This is the first resonance position (say L_1). Confirm this resonance position by taking four readings: two for when level of water is falling and two for when water level is rising. There might not be a pinchcock provided with the apparatus, so pinch the rubber tube yourself, while your partner holds the tuning fork steady.

- Lower the water level position so that is around three times the length L_1. Repeat the above procedure to get a second resonance position (say L_2). Again obtain the mean of four readings (two for water rising and two for water falling).

- Repeat the experiment and obtain the two mean resonance positions for a different tuning fork frequency.
- Use a set square to measure the lower meniscus of the water level.
- Use a thermometer to measure the temperature of air.
- Calculate the velocity of sound in air at room temperature v_T by plugging in the values of the tuning fork frequency ν and the resonance position lengths L_1, L_2 into the formula:

$$v = 2\nu(L_2 - L_1) \tag{4.6}$$

- Infer the velocity of sound at 0°C from your data. Compare the value of the speed of sound at 0° that you infer from your experimental data, with the actual value of 331.5 m s^{-1} and find the % deviation. Discuss the possible sources of error in your experiment.

4.5 Safety & Precautions

- Check that the resonant tube is vertical, if not make it vertical using the leveling screws. See to it that the prongs of the tuning fork do not touch the edge of the tube.
- If the lab is very noisy, you may not be able to hear the faint resonance - in that case, shift the apparatus to a quieter place in the lab.

Chapter 5

Spectrometer Experiments

5.1 Aim

To find the prism angle and the angle of minimum deviation of the prism. To determine the refractive indices of a glass prism as a function of wavelengths of mercury light, by refraction of light through prism at minimum deviation. To plot the dispersion curve for the glass prism and calculate the dispersive power of the prism. To obtain the coefficients in Cauchy's equation from the graph of n versus $(1/\lambda^2)$. If a sodium vapor lamp is used, the aim is to determine the wavelengths of sodium light.

5.2 Experimental Setup

Prism. Spectrometer. Mercury vapor lamp. Sodium vapor lamp. Spirit level. Vernier calipers, screw gauge, magnifying glass.

5.3 Physics Concepts

An optical instrument used to measure the wavelengths or frequencies of light emitted by various light sources is known as a spectrometer. All spectrometers depend on an optical element, usually a prism or a grating, to separate the light into its individual colors or wavelengths. Once separated, the different wavelengths can be measured. The refractive index of the prism material depends on the wavelength of light and can be written in terms of the Cauchy equation as

$$n = A + B/\lambda^2 + C/\lambda^4... \tag{5.1}$$

where the constant A is the coefficient of refraction and B is the coefficient of dispersion. Cauchy's equation is an approximation that applies quite well to many non-absorbing materials in the optical region. A plot of the first two terms in the Cauchy equation gives a straight line with slope B and intercept A(the value of n at very large wavelengths). The prism spectrometer consists of a collimator, a prism table and a telescope. Light enters through a slit at one end of the collimator and emerges parallel to the axis of the collimator at the other end. The collimated light then strikes a prism that has been positioned on the spectrometer

Figure 5.1: The prism spectrometer apparatus.

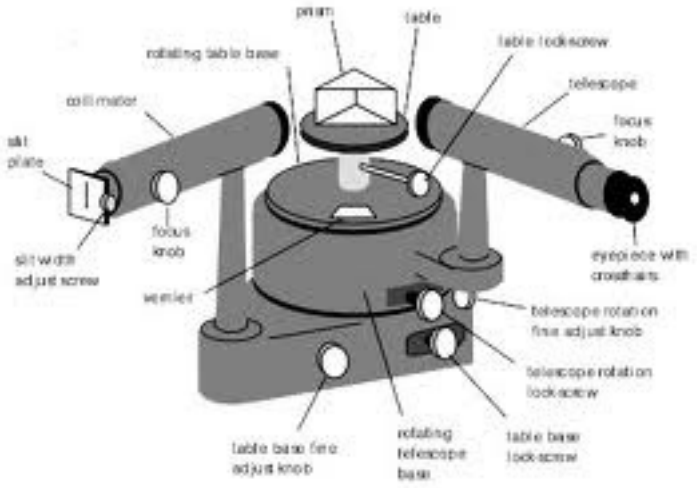

Figure 5.2: The prism spectrometer block diagram.

table. The prism deviates and disperses the collimated light according to Snell's law and the different wavelengths of light present in the light source. The sets of parallel rays (one set for each wavelength) then enter the telescope and form separate distinct images of the collimator slit when viewed through the telescope. Cross hairs in the telescope may be positioned over each spectral line, thereby fixing the angular position of the telescope for each wavelength. The angular position of the telescope is determined with a circular scale, which is graduated in fractions of a degree. A distinct relationship exists between the position of the telescope and a given wavelength of light passing through the prism. This relationship permits the prism spectrometer to be used to measure the wavelength of any given spectral light component.

When a ray of monochromatic light passes through a prism, it is refracted twice, once as it enters and again as it leaves the prism. The angle A between the two prism surfaces or planes where refraction takes place is called the prism angle. The intersection of the two refracting planes is called the refracting edge. The base of the prism is that side of the prism opposite the prism angle. The normals to the prism faces are N and N$'$. The incident and refracted rays are in different directions and the angle D between the incident and emergent rays is defined as the angle of deviation. If a beam of collimated light containing several wavelengths is incident on the prism face, different wavelengths will be refracted by different amounts and will have different angles of deviation. The particular angle of deviation depends on the prism angle, the index of refraction of the prism at that wavelength and the angle of incidence. The angle of deviation D is a minimum if the angle of emergence is equal to the angle of incidence. When the prism spectrometer is set at minimum deviation for a given wavelength, the index of refraction n (at that wavelength) is given by

$$n = \frac{\sin{(A + D_{min})/2}}{\sin{A/2}} \tag{5.2}$$

where A is the prism angle, D_{min} is the measured angle of minimum deviation for the particular wavelength. By determining the angle of minimum deviation for each wavelength, one can calculate the appropriate refractive index. The graph of index of refraction versus wavelength is called a dispersion curve.

From the dispersion curves for different optical materials it can be seen that the index of refraction varies with the wavelength λ. In fact, the index of refraction decreases as the wavelength increases, and the rate of decrease is less at longer wavelengths. For example, for borosilicate crown glass, the index of refraction decreases rapidly between 200 and 300 nm, while it decreases only gradually between 500 and 600 nm. This rate of change of index of refraction with wavelength is called dispersion and can be written as $\Delta n/\Delta\lambda$. The relative dispersion or dispersive power, DP, of a prism in the visible region is defined as

$$DP = \frac{(n_v - n_y)}{(n_{avg} - 1)} \tag{5.3}$$

where n_v, n_y are refractive indices of the material of the prism for violet color and yellow color respectively. $n_{avg} = (n_v + n_y)/2$ is the average refractive index. Typical values for dispersive power are 0.02 for crown glass (low dispersion) and 0.033 for flint glass (high dispersion).

The resolving power of a prism is defined as the ratio of the wavelength λ to the minimum

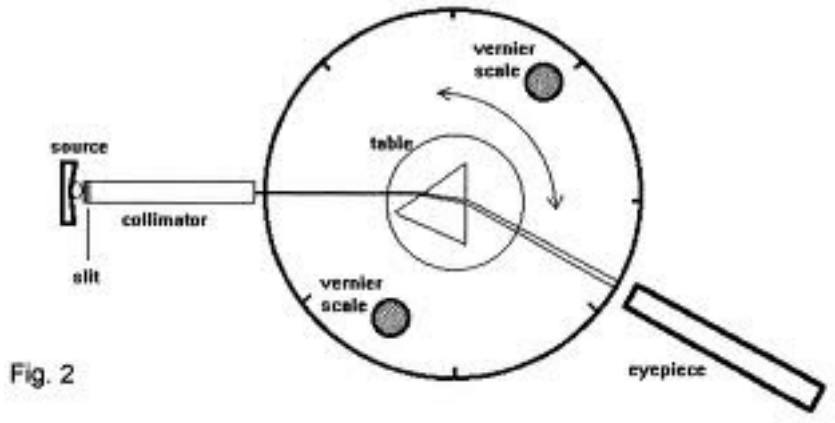

Figure 5.3: Light path in the prism spectrometer.

wavelength difference that can be distinguished at that wavelength.

$$RP = \frac{\lambda}{\Delta\lambda} \tag{5.4}$$

For a prism

$$RP = B\frac{\Delta n}{\Delta\lambda} \tag{5.5}$$

where B is the length of the base of the prism.

5.4 Experimental Procedure

5.4.1 Spectrometer Adjustments

Normal operation of the spectrometer requires parallel light emerging from the collimator, passing through the prism and entering the telescope. Therefore, the collimator must be adjusted to produce parallel light and the telescope adjusted to receive parallel light. **Note:- All adjustments once made should not be disturbed. If these settings are disturbed, the entire procedure has to be repeated step by step.**

1. Level the prism table if necessary using a spirit level and leveling screws.

2. Before placing the prism on the table, aim the telescope at a distant object through a window. Rotate the knurled ring close to the eyepiece so that the eyepiece moves in and out. (You may have to slide the eyepiece in and out). Bring the image of the distant object into clear focus in the plane of the cross hairs (focal plane of the telescope). The telescope now is adjusted for parallel light; do not further change the focus of the telescope after this adjustment.

3. Next, adjust the collimator for parallel light. With the proper clamp loosened, bring the telescope into a straight line with the collimator. By turning the knurled ring at

the end of the collimator, slightly open the adjustable slit. Place an incandescent light source in front of the slit and look through the telescope. Slide the tube that holds the slit in and out of the barrel of the collimator and, if necessary, move the telescope sideways a little until a sharp image of the slit is seen in the center of the focal plane of the telescope. Set the slit exactly vertical. The collimator now is set to produce parallel light and should not be further disturbed.

4. Find the least count of the vernier scale $LC = \theta/n$ where θ is the smallest main scale division and n is the number of vernier divisions. The least count represents the limit of error in reading the scale.

5.4.2 Adjustment of prism table and Schuster's method

1. Using the spirit level and adjusting screws, the prism table should be adjusted in horizontal plane. The prism table is adjusted to be at the same height of the collimator and telescope.

2. Now the prism is placed on the prism table in such a position that one of the refracting surfaces faces the collimator. The light emerging from the other refracting surface of the prism is viewed through the telescope.

3. The prism table and the telescope both are then rotated slowly and simultaneously so that the spectral lines always remain at the cross-wire. A state is reached when on rotating the prism table further, the direction of rotation of the spectral lines is reversed. This position is the position of minimum deviation for that particular wavelength (where the angle of incidence will be equal to the angle of emergence). Determine this position of minimum deviation very carefully by using the fine adjustment screws on the telescope. Perform this procedure several times and take the average of your values for the minimum deviation position.

4. Now keeping the telescope fixed, the prism table is rotated through a small angle. The spectral lines are seen to be blurred. The telescope is adjusted so that spectral lines, become distinct and clear.

5. Now the prism table is rotated through a small angle in the reverse direction. Again the spectral lines are seen blurred. The collimator is adjusted so that spectral lines become distinct and clear.

6. The methods described in steps (4) and (5) are repeated again and again till the spectral lines become distinct and clear throughout the entire rotation of the telescope. In this position the spectrometer will be set for parallel rays coming out of the collimator, focusing parallel rays by telescope at its cross-wire and the prism in the position of angle of minimum deviation.

5.4.3 Measurement of the prism angle

1. Place the prism on the table of the spectrometer so that its refracting edge is vertical and faces the collimator.

2. Move the telescope until the collimator slit image, reflected off one of the prism faces, is visible in the telescope. Carefully position the telescope until the slit image is in the center of the focal plane of the telescope and the cross hair is aligned with one edge of the slit image. Read and record the angular position of the telescope on the graduated scale, making use of the vernier to obtain maximum accuracy.

3. Swing the telescope around to receive the light reflected off the other face of the prism. (When rotating the telescope, push on the supporting arm rather than on the telescope itself.) Align the cross hair with the same edge of the slit used in position in the point above. Read and record the angular position of the telescope. The difference in readings of the two angular positions is equal to twice the prism angle (2A). Divide this reading in half to obtain the desired prism angle A.

5.4.4 Measurement of the angle of minimum deviation

1. Turn on a mercury lamp source. After the mercury lamp has brightened to its highest intensity (several minutes), set the lamp near the slit end of the collimator. With clamps loosened, swing the telescope around in line with the collimator. With the prism removed, center the slit image in the focal plane of the telescope. Make the slit fairly narrow, but not so narrow that a single-slit diffraction pattern occurs.

2. Read and record the angular position of the telescope.

3. Place the prism on the spectrometer table so that one face makes an angle of about 45 with the light coming from the collimator. At the same time, ensure that the collimator light fills the face of the prism. Look through the telescope while moving it around slowly, until the colored images of the collimator slit are seen. Align the cross hair of the telescope on one of the spectral lines, say the very bright green line (546.1 nm).

4. To find the position of minimum deviation, the prism table is rotated slowly (loosen the table clamp!) in the direction that causes the image of the green line to move toward the first position of the telescope, that is, toward the undeviated direction. Follow this image with the telescope until a position is found where the image just reverses its motion. Repeat this several times until the turn-around point is clearly determined. This is the direction of minimum deviation for the green line. Carefully align the cross hair on one of the spectral lines and read the angular position of the telescope. The difference between this reading and the reading for the undeviated direction is equal to the angle D_{min}. Now with information on angle A and angle D_{min}, the index of refraction of the glass prism, at the wavelength corresponding to the spectral line viewed above, can be calculated. The procedure outlined above can be repeated for each line visible in the spectrum, thereby obtaining a value of n for each wavelength.

Color	Wavelength(nm)	Rel. Intensity
violet	404.7	15
violet	407.8	10
blue-violet	433.9	08
blue-violet	434.8	08
blue-violet	435.8	75
blue	491.6	20
blue-green	496.2*	13
green	536.3*	3
green	546.1	100
yellow-green	567.7	3
yellow	576.9	75
yellow	579.1	75
yellow	589.0	3
orange	607.3*	3
orange	612.4*	4
red	623.2*	10
red	670.8*	3
red	690.7*	10

Table 5.1: Spectrum of mercury lamp.

5.4.5 Experiments to Do

1. Setup and align the prism spectrometer for use in the measurement of refractive index and wavelength.

2. Determine the prism angle A by moving the prism several times. Take the average of the readings.

3. Determine the angle of minimum deviation for each of ten of the more intense spectral lines. Plot n versus λ and $n-1$ versus $1/\lambda^2$.

4. The table lists eighteen of the more intense spectral lines from a mercury lamp. Most lines are due to mercury atoms but some (marked with an asterisk) are due to commonly found impurities. Use the colors and relative intensities (given on an arbitrary scale of 0-100 for weak-to-intense) as a guide to identify the spectral lines you see. Bear in mind that different people see colors and intensities differently.

5. Measure the index of refraction of a glass prism at several different wavelengths and plot the dispersion curve for this prism. Using this dispersion curve, determine the resolving power and the dispersive power of the prism and estimate values for the refractive index.

6. Plot n versus $1/\lambda^2$ and obtain the Cauchy constants A and B for the prism from the least squares fit to this straight line (estimate the goodness of fit).

7. Find the resolving power of the prism. First find $\frac{dn}{d\lambda}$ using the Cauchy formula (keep the first two terms in the approximation).

8. **Measuring the wavelength of sodium light using the prism spectrometer:** Replace the mercury lamp with a sodium vapor lamp. Determine the angle of minimum deviation for each of the clearly visible spectral lines and calculate the corresponding value of the n. Find the corresponding wavelength for each of the refractive indices you determined from your calibration curves. Compare these wavelengths to the actual wavelengths of sodium light and see if you are able to separate the two sodium-D lines. In case you are not able to resolve the two yellow lines, then explain why you are not able to do so, by calculating theoretically what the resolving power R should be in order to resolve the lines and what your resolving power of this particular spectrometer in the lab turns out to be.

9. Perform a rigorous error analysis and discuss your results.

5.5 Safety & Precautions

- The spectrometer is a very delicate instrument and must be handled with care. All movable parts such as the prism table move freely if the clamps are properly loosened. NEVER FORCE ANYTHING.

- The slit should be as narrow as possible but the knife edges of the slit should not touch each other. The telescope and the collimator should be separately set for parallel rays.

- The height of the prism table should be so adjusted that the maximum light must fall on the entire surface of the prism.

- While taking observations the telescope and the prism table must be clamped.

- Use a magnifying lens for taking readings on both the verniers.

Chapter 6

Polarimeter

6.1 Aim

To study the optical rotation caused by chiral molecules using a polarimeter setup and to find the specific rotation of a solution of sugar using the polarimeter. To find the unknown concentration of a solution from its optical activity.

6.2 Experimental Setup

Polarimeter, polaritube, physical balance, sodium vapor lamp, measuring cylinder, beaker, filter paper, dropper, sugar, distilled water.

6.3 Physics Concepts

Usually waves associated with light produced from a source such as a candle or a light bulb are unpolarized i.e. the electric field oscillates in random directions (though always at right angles to the direction of wave propagation) and there is no favored direction for polarization.

A polarization filter is a device with a specified direction called the polarization axis. The filter absorbs all the incoming waves electric field component perpendicular to the polarization axis and transmits the electric field parallel to the polarization axis (linearly polarized light). An unpolarized wave just before passing through the polarizer has its electric field pointing randomly in all directions perpendicular to the direction of wave propagation. After passing through the polarizer, the wave has only half the energy, with the electric field oscillating solely along the polarization axis. The most general form of polarized light is elliptically polarized light, with circularly polarized and linearly polarized light being special cases. Certain optically active substances such as quartz, turpentine and sugar solution can rotate the plane of polarization of a plane polarized light wave passing through the substance. Dextro-rotatory (right-handed) substances rotate the plane of polarization toward the right (in the clockwise direction on looking toward the source) while laevo-rotatory (left-handed) substances rotate the plane of polarization toward the left (in the anti-clockwise direction on looking toward the source). The angle by which the plane of polarization is rotated depends upon the concentra-

Figure 6.1: Experimental setup for the polarimeter.

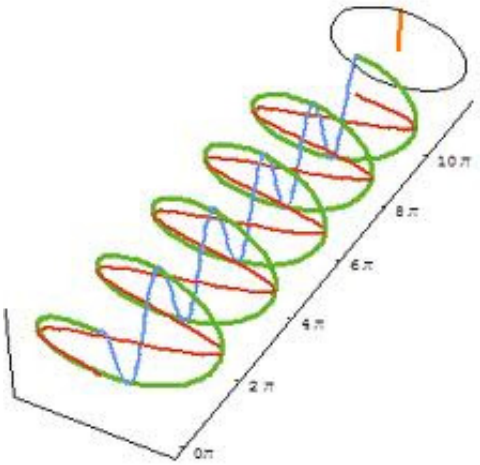

Figure 6.2: Elliptically polarized light.

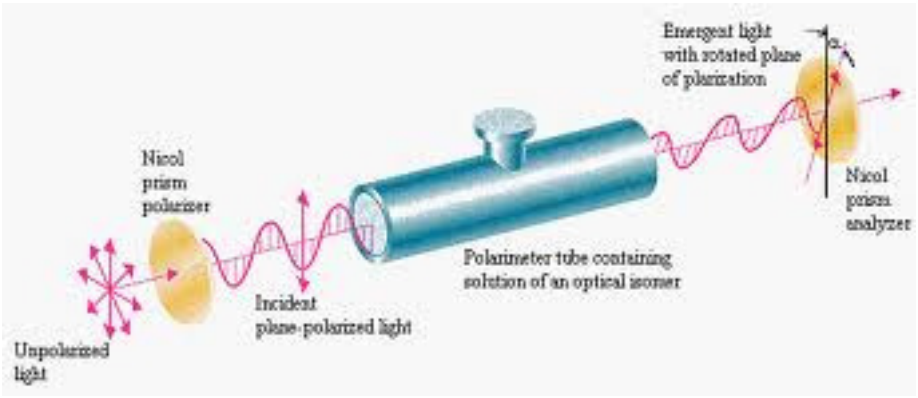

Figure 6.3: Polarimeter block diagram.

tion of the solution (or density of substance), the wavelength of light and the temperature. The specific rotation S at a given temperature and wavelength of light is defined as the optical rotation produced by one decimetre length of solution of unit concentration.

$$S = \frac{\theta \times V}{l \times m} = \frac{\theta}{lc} \tag{6.1}$$

where θ is the optical rotation produced in degrees, l is the length of the polaritube (in decimetre), m is the mass dissolved in water (gm), V is the volume (cm^3) and c is the concentration of the solution (gm/cm^3). A nicol prism is an optical device made from calcite and can be used to produce and analyze plane polarized light. When light is passed through a doubly refracting crystal it splits into an ordinary and an extraordinary part, both plane polarized. In the nicol prism, one of these rays is cut off by total internal reflection. Laurent's half-shade polarimeter is an instrument used to find the optical rotation of optically active solutions. The light from a source (sodium vapor lamp) passes through a narrow slit and is made parallel by a lens. The light is plane polarized by a nicol prism (called the polarizer) and then passes through a half-shade device and then a glass tube containing the optically active solution. The light then falls on another nicol prism (called the analyzer) and the emergent light is viewed through a telescope. The analyzer can be rotated about the tube axis and its rotation measured on a graduated circular scale. The position of the analyzer is adjusted such that the field of view is completely dark. The tube is refilled with the optically active solution and inserted back in position. The field of view now becomes illuminated and darkness can be achieved by rotating the analyzing nicol prism through a certain angle depending on the optical rotation of the solution. When the nicol prism is rotated the total darkness of the field of view is achieved gradually and hence the exact position at which it occurs is difficult to determine. Laurent's half-shade device circumvents this problem. The half-shade device consists of a semi-circular glass plate glued to a semi-circular quartz plate, cut with its optic axis parallel to the line of separation of the two plates. The thickness of the quartz is such that it introduces a phase difference of π between the ordinary and extraordinary light rays (a half-wave plate). The thickness of the glass plate is such that it absorbs the same amount of light as the quartz plate. When the principal plane of the analyzer nicol is parallel to the

quartz optic axis, the two halves (glass and quartz) will appear equally bright. When the principal plane of the analyzer nicol is perpendicular to the quartz optic axis, the two halves will appear equally dark. Since the eye can better detect a slight change when the two halves are equally dark, all readings are taken for this position.

6.4 Experimental Procedure

- Clean the polarimeter tube and place the polarimeter in front of a sodium vapor lamp. Look through the telescope and adjust the eyepiece so that the two halves of the half-shade circle are clearly in focus.
- Fill the glass tube with distilled water, position it inside the polarimeter and cover it. Rotate the analyzer by rotating the circular scale till the two halves of the half-shade device are equally dark. In this position, there will be a drastic change in intensity of the two halves for a slight rotation on either side. Note the reading on the scale.
- Rotate the analyzer by approximately 180^0 and record the other equally dark position reading. Keep rotating by 180^0 and note down all the other readings for equally dark halves.
- Note down the ambient(room) temperature.
- Find the least count of the analyzer (as the value of one division of the main circular scale in degrees divided by the total number of divisions of the vernier scale).
- Fill the tube with sugar solution of a known concentration and again note down the scale readings when the two halves are in an equally dark position. Repeat for different concentrations of the sugar solution.
- Plot a graph between θ and the concentration c. Do a least squares fit and obtain the slope and calculate the specific rotation at ambient temperature.
- Now prepare an unknown concentration solution and repeat your observations. From the graph plotted between θ and c, obtain the unknown concentration.

6.5 Safety & Precautions

- The glass tube should be cleaned.
- There should be no air bubble in the glass tube when filled with solution. Before filling the tube with a solution, rinse it out with the same solution.
- Screw the caps of the glass tube tightly so that there is no leakage.
- Take the readings in the "equally dark" position.

Chapter 7

Fresnel Biprism

7.1 Aim

To setup the interference fringes of the double slit variety using a single source and Fresnel's biprism. To determine the wavelength of sodium light using the Fresnel's biprism.

7.2 Experimental Setup

Optical bench with lens mounts, sodium vapor lamp, Fresnel biprism,convex lens, slit,eyepiece with micrometer.

7.3 Physics Concepts

When two coherent beams of light of equal amplitude meet, an interference pattern consisting of a series of bright and dark fringes results (known as constructive and destructive interference, respectively). Fresnel's biprism is an example of an interferometric experiment depending on the division of wavefront to achieve interference of light from two coherent sources. The biprism consists of a glass prism with two of its faces making an angle of nearly 180^0 with each other, and both equally inclined to the third face, making the other two angles each equal to around $30^{''}$.

The Fresnel biprism is placed with its refracting edge parallel to the slit; this edge divides the incident light into two parts. When light from the source falls on the lower portion of the biprism, it is bent upwards and appears to come from the first "virtual" source. When light from the source falls on the upper portion of the biprism, it is bent downwards and appears to come from the second "virtual" source. The two virtual sources act as two coherent sources. Interference fringes of equal width (alternate bright and dark fringes) can be seen in the field of view of the eyepiece. The fringe width β is given by

$$\beta = \frac{D}{d}\lambda \tag{7.1}$$

where $d = \sqrt{d_1 d_2}$ is the distance between the two virtual sources, D is the distance between the slit and the eyepiece and λ is the wavelength of the source.

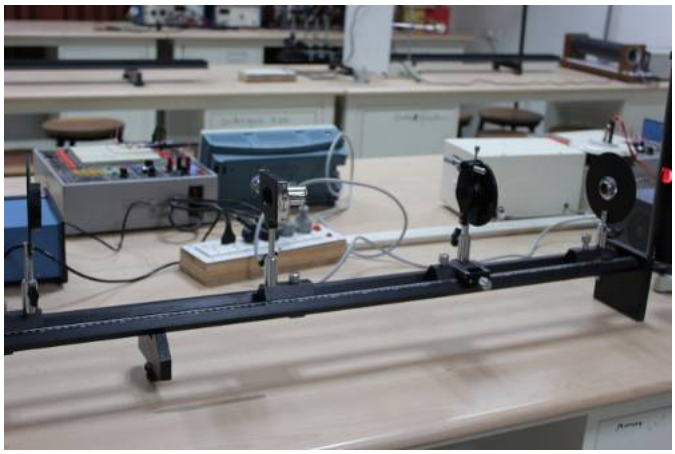

Figure 7.1: The Fresnel biprism experimental setup.

The distance d cannot be measured directly since the two slits are virtual. So it is determined by placing a converging lens between the biprism and the screen and forming real images of the virtual slits on the screen; d can then be found from the lens magnification formula. The experiment hence proceeds in two parts: first the fringes are setup and the fringe width is measured; then the distance between the virtual sources is measured. The optical bench has a heavy metal base with a metal scale along which four adjustable mounts can be moved, to carry a slit, a biprism, a lens and a micrometer eyepiece. A tangent screw can rotate the metal jaws clamping the slit and the biprism so that the slit and the edge of the biprism can be made exactly parallel to each other.

7.4 Experimental Procedure

- Adjust the mounts of the optical bench and check that the centre of the slit, the centre of the biprism and the centre of the eyepiece are all at the same height. Move the biprism close to the slit and first adjust the height of the prism and then the eyepiece till they are at the same height as the slit.
- Switch on the sodium lamp and wait a while for it to heat up. Adjust the slit width to be narrow.
- When you look at the biprism you should see two "virtual" light sources. Move your eye from one side of the biprism to the other, across the bench. One of the images will appear to cross the edge of the biprism from one side to the other. If the refracting edge of the biprism is exactly parallel to the slit, the image as a whole will appear to cross the edge. If you do not see this, the adjustment is not correct. Rotate the biprism in its own plane by the tangent screw till the full image crosses the edge. At this point the slit and the biprism edge are parallel.
- Focus the eyepiece on the crosswires (at right angles to each other). Turn the crosswires till one of them appears vertical to the eye.

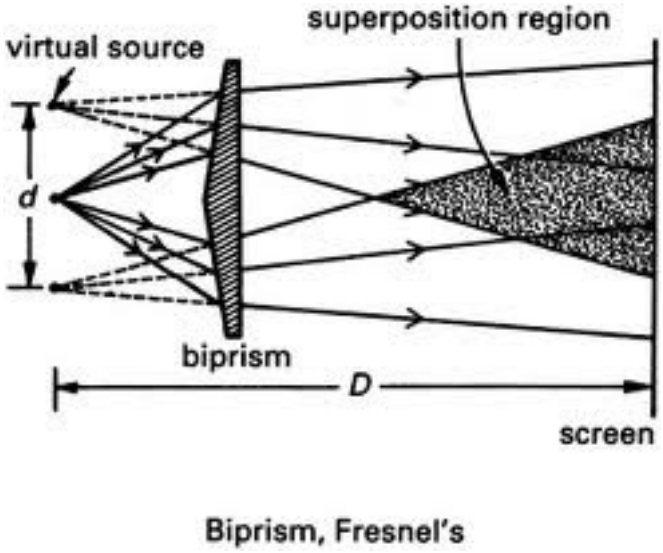

Figure 7.2: The source and division of wavefront leading to the interference pattern.

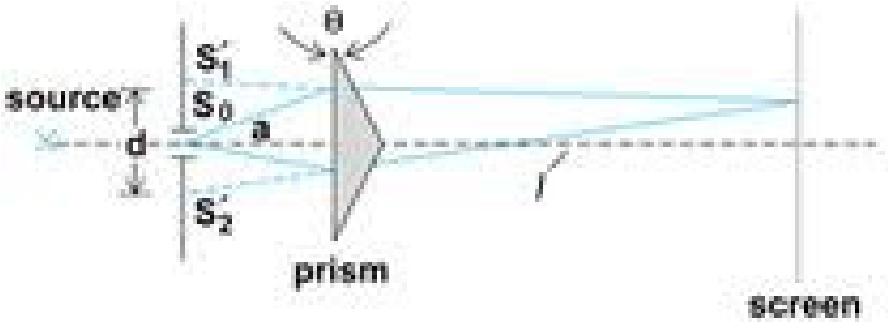

Figure 7.3: The prism arrangement.

- The central edge of the biprism must be parallel to the slit. On looking through the eyepiece a bright patch of light should appear in the field of view. If you cannot see this bright patch, try rotating the top of the upright mount with the biprism about its vertical axis and move the mount along the bench till you see the bright patch.
- Look through the eyepiece to see the fringes. If the fringes cannot be seen, move the biprism along the optical bench till the fringes appear in the field of view of the eyepiece. Rotate the prism in its own plane by the tangent screw till the fringes are seen clearly.
- Fix the vertical crosswire of the eyepiece on the central bright fringe and move the eyepiece away from the biprism and see if there is any lateral displacement of the fringes. If there is a lateral shift, move the biprism along the optical bench till the shift stops.
- **Measurement of fringe width** β: Find the least count of the eyepiece micrometer (as x/n where x is one main scale division and n the number of divisions on the circular scale). Adjust the distance of the eyepiece along the optical bench till the fringes are clear and distinct enough to be counted. Set the vertical crosswire on the centre of one of the bright fringes and note the main scale and micrometer readings. Set the crosswire on the next consecutive 20 fringes and note down the readings and determine the mean fringe width (if you cannot clearly distinguish 20 fringes, then try for the maximum number that you can clearly make out). Without disturbing the position of the slit and the biprism, change the position of the eyepiece and take two more sets of readings to measure the fringe width.
- Measure the distance D between the slit and the eyepiece from the attached scale on the optical bench.
- **Measurement of d:** Next, place the convex lens between the prism and eyepiece, close to the eyepiece. Move the lens towards the biprism till you see two sharp images of the slit. Measure the distance d_1 between the two images of the slit using the eyepiece micrometer. Fix the vertical crosswire on one image and note the micrometer reading and move the screw till the crosswire falls on the second image. The difference between the two readings gives the distance d_1 between the two virtual sources. Again move the lens toward the biprism till another set of two slit images are seen and measure the distance d_2 between the second set of images; $d = \sqrt{d_1 d_2}$. Take a set of readings (when lens is near the slit and when lens is near the eyepiece) and find the mean d.
- Find the wavelength of the sodium light from the formula and compare value you got with the actual value of 589.3 nm.

7.5 Safety & Precautions

- The refracting edge of the biprism should be parallel to the slit.
- Avoid backlash error and turn the micrometer screw in one direction only while taking readings.

- Place the vertical crosswire in the centre of a bright fringe to find the fringe width.
- Measure as many fringes as possible to find the mean fringe width.

Chapter 8

Newton's Rings

8.1 Aim

To study the fringes of equal thickness in the Newton's ring setup and hence determine the wavelength of sodium light.

8.2 Experimental Setup

Sodium vapor lamp, traveling microscope, lens assembly consisting of a plane glass plate and a planoconvex lens, spherometer, tiltable glass plate assembly.

8.3 Physics Concepts

A thin wedge shaped air film is created by placing a planoconvex lens on a flat glass plate. The thickness of the film is very small at the point of contact of the lens and plate and gradually increases from the center outwards. When a monochromatic beam of light (from say a sodium vapor lamp) is incident on the arrangement, interference fringes in the form of concentric bright and dark circles called "Newton's rings" are observed in the reflected light. The wavelength of the source can be calculated by measuring the diameter of the rings and the radius of curvature of the spherical lens surface.

The beam of light is incident on the glass plate at an angle of 45^0. The plate reflects a part of the incident light onto the air film formed between the planoconvex lens and small glass plate below it. Interference occurs between the light reflected at the lower surface of the lens and the upper surface of the glass plate below the lens. For normal incidence the optical path difference between the two waves is nearly $2\mu t$, where μ is the refractive index of the air film and t is its thickness. The ray that gets reflected from the upper surface of the glass plate picks up an extra phase difference of π as the incident beam goes from a rarer to a denser medium.

The conditions for constructive and destructive interference are ($\mu = 1$ for air):

$$2t = m\lambda \quad m = 0, 1, 2, 3....\text{minima} \tag{8.1}$$

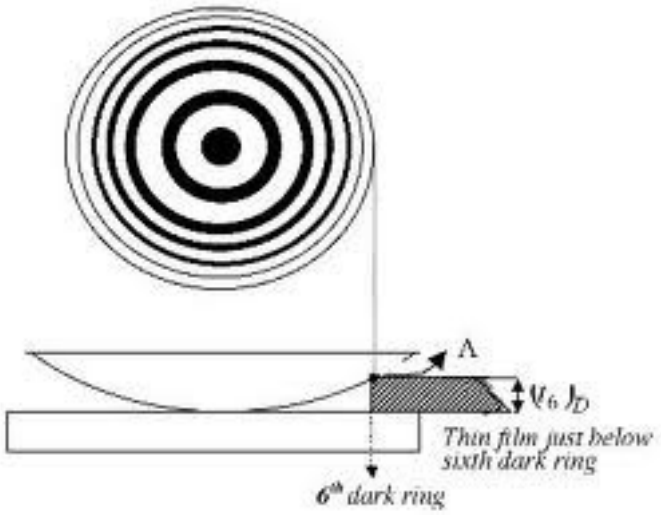

Figure 8.1: Interference paths and appearance of Newton's rings.

$$2t = (m + \frac{1}{2})\lambda \quad m = 0, 1, 2, 3....\text{maxima}$$

The air film enclosed between the spherical surface with radius of curvature R and the plane glass surface gives rise to circular rings of radius

$$r_m^2 = (2R - t)t \tag{8.2}$$

where r_m is the radius of the mth order dark ring (central dark spot is counted as the zeroth order ring).

Using the approximation $R >> t$,

$$(R - t)^2 + r_m^2 = R^2 \tag{8.3}$$

Neglecting the t^2 term leads to

$$2t \approx \frac{r_m^2}{R} \tag{8.4}$$

Substituting the value of $2t$ back in the expressions for minima and maxima gives the radius of the dark rings

$$r_m = \sqrt{m\lambda R} \tag{8.5}$$

and for the radius of the bright rings

$$r_m = \sqrt{(m + \frac{1}{2})\lambda R} \tag{8.6}$$

The diameter D_m of the mth order dark ring can be measured by a traveling microscope. The wavelength of light is given by

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR} \tag{8.7}$$

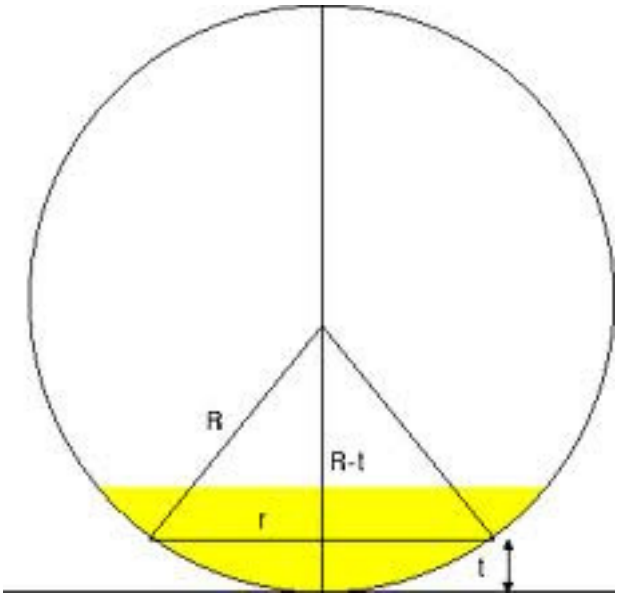

Figure 8.2: Geometry of the rings diameter and thickness of air wedge.

where D_{n+m} is the diameter of the $(n+m)$th ring, D_n is the diameter of the nth ring.

Measuring the radius of curvature of the plano-convex lens using a spherometer leads to the estimation of R:

$$R = \frac{l^2}{6h} + \frac{h}{2} \tag{8.8}$$

where l is the distance between two legs of the spherometer (i.e the mean length of one side of the equilateral triangle formed by joining the tips of the three outer spherometer legs) and h is the thickness of the lens at the center (the height of the central screw of the spherometer above the plane of the outer legs).

8.4 Experimental Procedure

- Clean the glass plate and lens gently and thoroughly with a piece of tissue paper.
- Using the naked eye, find out which is the glass plate and which is the planoconvex lens.
- **Measure the radius of curvature:** Find the least count of the spherometer (using the pitch of the screw and the number of circular scale divisions). Place the lens with its curved surface upwards on the glass plate. Take the spherometer reading when it just touches the surface. Remove the lens. Take the reading on the plane surface. Place the spherometer on your lab notebook and press gently to obtain the impression of the three legs of the spherometer. Join the three points and determine the mean distance between the legs. This is however not an accurate estimate of the distance l between the legs; use a vernier calipers to find the accurate distance (note down the difference in the two readings). Take several readings at different points on the two glass surfaces and take a mean of your readings.

- Put the lens and plate assembly back in the holder and place on the Newton's ring setup.

- Switch on the sodium lamp and wait a while for it to heat up.

- Look down vertically from above the lens and see that the center is well illuminated; adjust the positioning of the setup if required, so that a dark spot with rings around it is seen on properly focusing the microscope.

- Adjust the tiltable magnifying lens so that it makes an angle of 45^0 with the direction of the incident light coming from the source.

- Determine the least count of the traveling microscope.

- Adjust the microscope (you can make it move vertically by moving the appropriate screws) till you clearly see the circular fringes.

- Once you are able to see nice sharp rings, rotate the eyepiece such that out of the two perpendicular crosswires, one has its length parallel to the direction of travel of the microscope. This crosswire should pass through the center of the ring system.

- Focus the crosswire tangentially on a ring. Do this by moving the microscope in the horizontal direction to one side of the fringes such that one of the crosswires becomes tangential to say the 20th ring. Note down the main and vernier scale readings on the microscope.

- Before measuring the diameters of the rings, properly adjust the range of the microscope.

- Move the microscope and make the cross wire tangential to the next ring nearer to the center and continue with this procedure till you pass through the center and move to the other side upto the 20th ring. Note down the microscope scale readings in each case.

- Plot a graph between D_n^2 and the number of the rings (n). Do a least squares fitting and draw a best-fit line through your data. Measure the slope of the best fit line. Find the wavelength λ from the slope and the radius of curvature.

- Given that the mean wavelength of sodium light is 5893×10^{-10}m, use the graph between D_n^2 and n to find the radius of curvature of the surface of the lens.

- To obtain the value of R very accurately, use a traveling microscope to measure l instead of a pair of dividers.

- Perform a rigorous error analysis of your data and write out your results in the Results and Discussion part of your lab notebook.

Figure 8.3: Newton's ring apparatus.

Figure 8.4: Glass plate and plano-convex lens arrangement.

8.5 Safety & Precautions

- The optics arrangement for the Newton's rings is very delicate. Be very careful with unscrewing and removing (and putting back) the lens and plane glass plate from the holder.
- Clean the glass surfaces gently yet firmly. Do not rub your hands/fingers on the glass surfaces and always hold the lens and plate from the edge.
- The sodium vapor lamp takes time to heat up; do not be impatient and switch on/off frequently.
- To avoid any error due to backlash of the screw in the travelling microscope, move the micrometer screw in only one direction while measuring the ring diameter.
- The amount of incident light should be adjusted or regulated since too much light decreases the contrast between bright and dark fringes.

Chapter 9

Diffraction at a single slit

9.1 Aim

To study the diffraction pattern of a slit using a laser source and a photodiode based detection system on an optical bench. To measure the intensity distribution due to the single slit and measure the slit width. To observe the diffraction pattern from an obstacle(thin wire).

9.2 Experimental Setup

Optical bench, red semiconductor laser, photo cell, multimeter, single slit with adjustable width, optical mounts for wire, screen to observe pattern.

9.3 Physics Concepts

Diffraction may be thought of as the deviation of a light wave caused by partial obstruction of a wavefront by a mask or a sharp edge. Sound waves are diffracted in the same manner as light waves. Diffraction patterns are characterized by a rapid decrease in intensity with increasing distance from the centre of the pattern. This experiment studies the Fraunhofer diffraction pattern produced by a single slit of width a. A diffraction pattern for which the phase of the light at the point of observation is a linear function of the position, for all points in the diffracting aperture is Fraunhofer diffraction. In effect, Fraunhofer diffraction occurs when the source and screen are effectively at infinity with respect to the slit/obstacle causing the light to diffract, and the diffraction pattern is formed in the image plane of the optical system. A plane wave is incident normally on the slit from a laser source and the intensity distribution produced on the screen is calculated. The diffraction patterns formed by opaque objects (slits, wires, disks etc) are the result of geometrical path differences. A single slit will produce a complicated diffraction pattern if the slit width is comparable to the wavelength.

Assume the slit can be decomposed into a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets that interfere with the wavelets emerging from the other points. If the distance between consecutive points is Δ

Figure 9.1: Experimental setup for single slit diffraction showing the optical bench and mountings.

and if n is the number of point sources

$$a = (n-1)\Delta \tag{9.1}$$

Each source sends light in all directions to a distant screen and we take light rays reaching any particular point on the screen to be parallel, with all rays making an angle θ with the horizontal. The approximation $\Delta << \lambda$ holds, so that all the light from a point source is in phase. The point sources are of equally size and equally spaced and if the whole slit is uniformly illuminated, we can take the electric field wave amplitudes from each point source to be equal. The resultant field produced by n sources at some point P on a screen can be computed; in the final expression n will tend to infinity and Δ to zero (to mathematically describe the slit as a continuous distribution of sources) such that $n\Delta$ tends to a. The object of the calculation is to compute the amplitude E_θ and the phase angle ϕ of the coherent sum of waves in the limit $n \longrightarrow \infty$ and $\Delta \longrightarrow 0$ such that the product $a = n\Delta$ is held fixed. The amplitude of the wave emerging from each point source is given by

$$\Delta E_0 = \frac{E_0}{n} \tag{9.2}$$

where E_0 is the amplitude of the wave incident on the single slit. At a point P on the screen, because of slightly different path lengths, the field produced by a point source at A_1 will differ in phase from the field produced by a point source at A_2 with the phase difference being

$$\Delta\beta = \frac{2\pi\Delta \sin\theta}{\lambda} \tag{9.3}$$

where the diffracted rays make an angle θ with the normal to the slit. The total amplitude on the screen for the angle θ will be the coherent sum of the n waves

$$E_\theta = E_{\theta 0} \sin(\omega t + \phi) \tag{9.4}$$

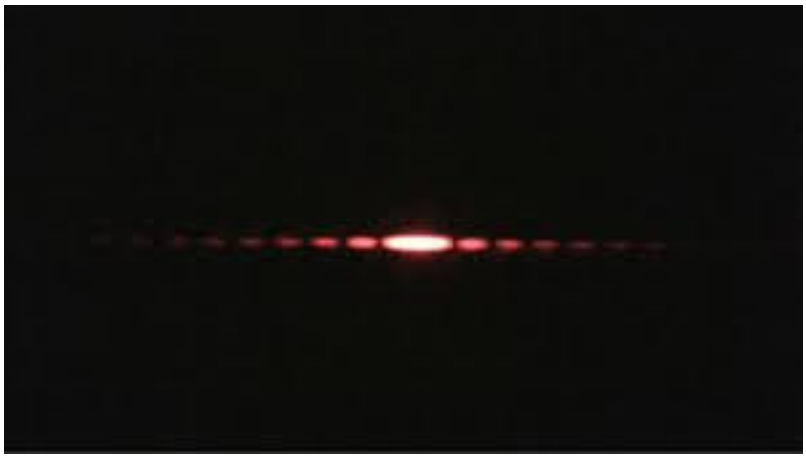

Figure 9.2: Single slit diffraction image from a laser source.

The resultant field at the point P from all n sources will add upto

$$E_\theta = \lim_{n\longrightarrow\infty,\Delta\longrightarrow 0} \frac{E_0}{n}\left[\sin\omega t + \sin\left(\omega t + \Delta\beta\right) + \sin\left(\omega t + 2\Delta\beta\right) + ... + \sin\left(\omega t + (N-1)\Delta\beta\right)\right] \tag{9.5}$$

where $\Delta\beta = 2\pi\Delta\sin\theta/\lambda$.

In the limit $n \longrightarrow \infty$ and $\Delta \longrightarrow 0$ such that $n\Delta \longrightarrow a$ the resultant field becomes

$$E_\theta = E_0 \sin\left(\omega t + \beta/2\right)\frac{sin(\beta/2)}{\beta/2} \tag{9.6}$$

The time-averaged intensity of the resulting wave is proportional to E_θ^2 averaged over one full cycle of the wave and is given by

$$I(\theta) = I_0\left(\frac{\sin\left(\beta/2\right)}{\beta/2}\right)^2 \tag{9.7}$$

where $\beta = 2\pi a\sin\theta/\lambda$ and I_0 represents the intensity at $\theta = 0$.

The intensity is zero when

$$a\sin\theta = m\lambda \quad m = \pm 1, \pm 2.... \tag{9.8}$$

Differentiating the intensity expression wrt β and setting it equal to zero gives

$$\tan\beta = \beta \tag{9.9}$$

The root $\beta = 0$ corresponds to the central maximum. The other roots can be found from the points of intersections of the curves $y = \beta$ and $y = \tan\beta$. The intersections occur at $\beta = 1.43\pi, 2.46\pi$.. etc and are denoted the first maximum, second maximum etc. You can verify numerically that the intensities of the first and second maxima are around 4.96% and 1.88% of the central maximum respectively.

Mathematically the intensity of the single slit diffraction pattern is the Fourier transform of a square pulse. The diffraction pattern in the far field of the object ($D >> d^2/\lambda$) is actually the two-dimensional Fourier transform of the object profile.

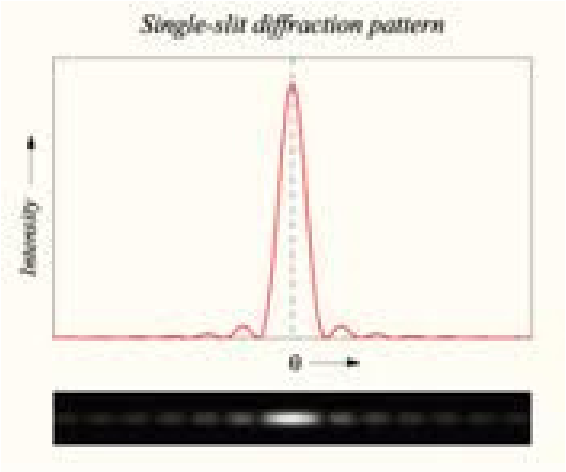

Figure 9.3: Expected single slit Fraunhofer diffraction pattern.

9.3.1 Coherence

Coherent light (laser beam) has constant phase while incoherent light (incandescent bulb) has randomly varying phase. The superposition at some fixed point in space of two harmonic waves u_1, u_2 with same angular frequency ω but time-varying phase $\phi(t)$ leads to

$$u = u_1(t) + u_2(t) = u_{01}\cos(\omega t - \phi_1(t)) + u_{02}\cos(\omega t - \phi_2(t)) \tag{9.10}$$

with a time varying phase difference $\delta(t) = \phi_2(t) - \phi_1(t)$ which tends to smear out the interference pattern. The degree of coherence $\gamma_{12}(\tau)$ is related to the phase fluctuations of a correlation function between the two waves combined at a relative time delay τ

$$\gamma_{12}(\tau) = \frac{\langle u_1(t+\tau)u_2(t)\rangle}{\sqrt{I_1 I_2}} \tag{9.11}$$

Even an incoherent source will show properties of **spatial coherence** if two different points 1 and 2 on the wavefront are compared. Even though the relative phases of the waves at points 1 and 2 are random, the relative phase differences depend only on geometry and become rather small far from the source. For **temporal coherence** the self-coherence of a wave can be represented by the coherence length Δr or coherence time Δt as

$$\Delta r = c\Delta t \tag{9.12}$$

where c is the speed of light.

9.3.2 How a laser works

Light (electromagnetic radiation) is produced when electrons in an atom jump from a higher to a lower energy level; whenever the electron makes such a transition the difference in energy is emitted (absorbed) as a discrete quantum of energy called a photon. If the transition happens randomly i.e. spontaneously, the emitted photons are incoherent or out of phase

with one another. The other way the transition can occur is via stimulated emission. If there are a number of photons in a field that are in phase and have the same frequency (and that frequency corresponds to the transition energy), the emission is no longer random. When an electron falls to the lower energy level, a photon is emitted which is in phase with the already existing photons. For stimulated emission to work there should be more electrons populating the higher state than the lower energy state (called a population inversion since at equilibrium atoms tend to be in their ground or lower energy states). Real lasers use at least three energy levels: the population inversion is achieved by pumping electrons from the lower energy state to a transient state (with an energy higher than the laser higher energy state) so that the electrons make transitions and keep replenishing the higher energy laser state. The laser setup is usually placed in a cavity with a mirror at one end and a semi-transparent mirror at the other end. This maintains a high density of light within the cavity and increases the efficiency of the laser.

9.4 Experimental Procedure

- Switch on the laser source and leave on for around 15 minutes so that the light intensity from the laser source is constant and does not flicker.
- Mount the slit and adjust its width so that you observe a diffraction pattern on the white screen.
- Play around with the slit adjustments and observe what happens to the diffraction pattern on the screen. Record your observations in your lab notebook. Sketch the diffraction patterns observed.
- Now quantify the intensity distribution with the help of the photocell.
- Mount the photocell on the optical bench, as far away from the slit as possible. Make suitable adjustments so that the light falls on the small aperture in the photocell. The photocurrent is proportional to the intensity of the incident light.
- Measure the photocurrent using the multimeter (in the μA range) and note down your readings. Move the photocell using the screws provided, till the entire diffraction pattern is covered from end to end, and note down the corresponding readings. Assume the wavelength of the red laser light = 623.8 nm. Measure the distance D between the slit and the photocell.
- Plot the intensity distribution and measure the slit width i.e. plot a graph of the current vs the position of the photocell. Measure the distance x from the centre of the diffraction pattern to the first minimum on the graph paper. Calculate the slit width using $m = 1$ as
$$a \sin\theta = a \frac{x}{\sqrt{x^2 + D^2}} = \lambda$$
- Tabulate the expected theoretical values of intensity versus position and compare them with your measured values.

- Adjust the distances between the laser, slit and screen to different lengths and again record the diffraction patterns in each case, and record the photocurrent intensities and make the plots. Also repeat for different slit widths.
- In the second part of the experiment, mount a thin wire onto the optical bench. Move the wire and laser source, till you observe the diffraction pattern from the wire. Record the pattern (and explain what you see) in your lab notebook. Measure the obstacle (wire) dimensions directly with a traveling microscope. Calculate the object dimensions from the information in the diffraction pattern. Compare your calculation with the actual dimensions measured directly.
- **Further experiments:** Experiment with different types of obstacles. You can even try measuring the thickness of one human hair!
- **Further reading:** Read up on how a photo cell works and briefly summarize in your lab notebook. Read up on how a laser works and briefly summarize in your lab notebook. Find out the differences between Fraunhofer and Fresnel diffraction and briefly summarize in your lab notebook.

9.5 Safety & Precautions

- NEVER look directly into the laser as this can damage your eyes permanently.
- Be very cautious with the laser especially while focusing it.

Appendix A

Notes on Error Analysis

A.1 Introduction

Error or uncertainty about a particular experimental measurement is the best estimate of the quantitative range within which you can trust your results. Any experimental measurement you make in the laboratory is meaningless unless quoted with an uncertainty/error. We are not talking about errors like misreading a scale or slipping a decimal point while taking a reading. Experimental uncertainties are a statement about the **resolution** of your measurement i.e. how far from the "true" value you are likely to be. There are two kinds of uncertainties associated with the measurement of an experimental quantity:

- **Random uncertainty**: associated with unpredictable variations in the experimental conditions. For example changes in room temperature, vibrations from nearby machinery, error in time period measurement when the experimenter does not start/stop the stopwatch at exactly the same point in the swing of the pendulum etc. So if a measurement is repeated a number of times with sufficient precision, a slightly different value of the measured quantity is obtained each time and if the experiment is free from bias these variations will be random and the measurements will group symmetrically about the "true" value.

- **Systematic uncertainty**: associated with inherent faults in measuring instrument or in measurement technique. This is an error that is consistent from measurement to measurement. For example, measuring length of a table with a tape that has a kink in it, a weak spring in a current meter, a calibration error in the measuring device, a clock that runs too fast etc. So if there is an experimental bias, the measurements will group around the wrong value and are said to contain a systematic error. If you always round down to the nearest tic mark on a meter stick while measuring length, you will make a systematic error of measuring a slightly shorter length.

Random uncertainties are easier to quantify and deal with. There is no general procedure for estimating the magnitude of systematic uncertainties.

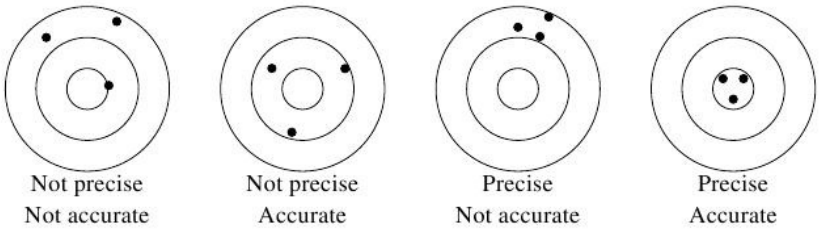

Figure A.1:

A.2 Precision vs Accuracy

Random uncertainty decreases the **precision** of an experiment whereas **systematic** uncertainty decreases the **accuracy** of the experiment.

NOTE:- Systematic uncertainty does NOT mean that the uncertainty is repeatable. It means that the uncertainty has not been accounted for in the analysis.

Accuracy refers to the degree to which your value is correct within uncertainty. It is largely a matter of having the correct calibration of all reference measurements. If you used a miscalibrated meter stick that was shorter than the official length of a meter, you might measure the length of an object with great precision (lots of decimal places) but poor accuracy (what you think is a meter is not really a meter).

Precision can be thought of as the number of meaningful digits to a measurement. A measurement of a length as being 1.023405 meters is more precise than a measurement of 1.02 meters.

As an illustration of the concepts of precision and accuracy, look at the figure of a Bulls-Eye chart. The measured quantity's (say the acceleration due to gravity $g = 9.8m/s^2$) true value lies at the center of all three circles and the three dots represent the data points (the results of the three times g was experimentally measured by the same apparatus). In the first experiment, the data points show very different values and are scattered over the circles - the measurement is neither precise, nor accurate. In the second measurement, the values all lie close to the actual value (ranging between $9.9 - 10.5m/s^2$) but the uncertainty in each measurement is large, so the experiment is accurate but not precise. The converse happens in the next experiment, where all the values lie within an experimental uncertainty of $\pm 0.5m/s^2$, but the value of g measured is $12m/s^2$ which is not accurate. The final experiment of course gives the best results - i.e both precise and accurate.

A.3 Three major sources of errors

A.3.1 Reading Error

Almost all direct measurements involve reading a scale (ruler, caliper, stopwatch, analog voltmeter, etc.) or a digital display (e.g., digital multimeter or digital clock). Sources of uncertainty depend on the equipment we use. One of the unavoidable sources of errors is

a reading error. Reading Error refers to the uncertainties caused by the limitations of our measuring equipment and/or our own limitations at the time of measurement (for example, our reaction time while starting or stopping a stopwatch). This does not refer to any mistakes you may make while taking the measurements. Rather it refers to the uncertainty inherent to the instrument and your own ability to minimize this uncertainty. A reading error affects the precision of the experiment. The uncertainty associated with the reading of the scale and the need to interpolate between scale markings is relatively easy to estimate. For example, consider the millimeter (mm) markings on a ruler scale. For a person with a normal vision it is reasonable to say that the length could be read to the nearest millimeter at best. Therefore, a reasonable estimate of the uncertainty in this case would be $\Delta l = \pm 0.5$ mm which is half of the smallest division. A rule of thumb for evaluating the reading error on analogue readout is to use half of the smallest division (in case of a meter stick with millimeter divisions it is 0.5 mm), but only the observer can ultimately decide what is his/her limitation in error evaluation. Note that it is wrong to assume that the uncertainty is always half of the smallest division of the scale. For example, for a person with a poor vision the uncertainty while using the same ruler might be greater than one millimeter. If the scale markings are further apart (for example, meter stick with markings 1 cm apart), one might reasonably decide that the length could be read to one-fifth or one-fourth of the smallest division. dings. It is an estimate of systematic differences between different scales of the multimeter. However it is the random error that determines the precision, and gives you an idea of the scatter that you might expect in your readings. Thus, the "± digit" quoted by the manufacturer might be a better estimate of the random error. Though you should quote the systematic error at the end of your experiment when you are comparing your result with some "standard", it is better to use 1 digit for the random error in each reading. For example, if your reading is 3.48 mA, you should quote (3.48 ± 0.01) mA. It is usually difficult or impossible to reduce the inherent reading error in an instrument. In some cases (usually those in which the reading error of the instrument approximates a "random error distribution") it is possible to reduce the reading error by repeating measurements of exactly the same quantity and averaging them.

A.3.2 Random Error

Random Error refers to the spread in the values of a physical quantity from one measurement of the quantity to the next, caused by random fluctuations in the measured value. For example, in repeating measurements of the time taken for a ball to fall through a given height, the varying initial conditions, random fluctuations in air motion, the variation of your reaction time in starting and stopping a watch, etc., will lead to a significant spread in the times obtained. This type of error also affects the precision of the experiment.

A.3.3 Systematic Error & Instrument Calibration

Systematic Error refers to an error which is present for every measurement of a given quantity; it may be caused by a bias on the part of the experimenter, a miscalibrated or even faulty measuring instrument, etc. Systematic errors affect the accuracy of the experiment. After evaluating the reading error or the standard error, or both if necessary, we have to make sure that the scale of our measuring instrument is checked against an internationally established

measuring standard. Such comparison is called calibration. In the real world, we frequently find that our measuring scale is in slight disagreement with the standard. For example, if you inspect such simple tools as rulers, you will find out that no two rulers are exactly the same. It is not uncommon to find a discrepancy of 1 mm or even more among meter sticks. The correct calibration of measuring instruments is obviously of great importance. In addition to all the errors discussed above, there can be other sources of error that may pass unnoticed: variations in temperature, humidity or air pressure, etc. Such disturbances are more or less constant during our measurements (otherwise they would appear as random error when the measurement is repeated) and are generally referred to as the systematic errors. Systematic errors are very difficult to trace since we do not know where to look for them. It is important to learn to notice all the irregularities that could become the sources of systematic errors during our experimental work. Moreover, it is particularly important in data-taking always to record some information about the surrounding physical conditions. Such information may help us later on if we discover a serious discrepancy in our experimental results. As a rule, the place, date and time of measurements, and the type and serial numbers and specifications of the instruments which were used must be recorded. Estimate all your reading errors while you take your data and write them down with your data. Do the same for all manufacturers' error specifications. These usually cannot be guessed later on.

A.4 Mean & Standard Deviation

Mean

If the sources of error in a measurement (say measuring the length of a table) are random, the values of the length will vary randomly above and below the "true" value of the table length, and will not be biased/skewed toward the lower/higher values. The procedure to get the most precise value for the length is to take the average or arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + ... + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{A.1}$$

where N is the number of measurements and x_i is the value of one measurement. This definition of mean assumes that each measurement of x is independent and has the same experimental uncertainty.

Standard Deviation

Now that the mean ("best" value) is known, it is important to quantify how much the individual measurements are scattered about the mean or how "good" each individual measurement is. If the experiment is precise, all measurements will be very close to the mean value. So the extent of scatter about the mean is a measure of the precision and a way to quantify the **random uncertainty**.

For unbiased measurements (all data points have equal weights), the standard deviation σ is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}} \tag{A.2}$$

σ becomes larger if there is more scatter of the data about the mean.

NOTE:- Convince yourself at this stage that more scatter of data means a larger standard deviation and also that σ has the same units as x_i.

Most Probable Value:

For unbiased measurements, the standard deviation of the mean value of a set of measurements σ_m is

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}} = \frac{\sigma}{\sqrt{N}} \tag{A.3}$$

This is important since it states that the uncertainty in the **mean** of N measurements decreases as $1/\sqrt{N}$.

NOTE:- Convince yourself that σ_m is necessarily smaller than σ. Also think about the difference between σ and σ_m: σ is the standard deviation associated with individual data points whereas σ_m is the standard deviation of the mean value of a set of data points i.e. the uncertainty of a set of measurements made under identical conditions.

EXERCISE:- For a Gaussian distribution, convince yourself that the mean will be within the range $\bar{x_i} \pm \sigma_i$ 68% of the time i.e if another set of N measurements is made, the mean of this new set has a 68% likelihood of being within the range $\bar{x_i} \pm \sigma_i$.

Random errors and Gaussian distributions

In some measurements, there is a random element involved. Say that you measure the fraction of times that a coin lands face up. You might refuse to make the measurement, saying that you know the answer: its going to land face up exactly 50% of the time. What if you make two measurements? If you flip the coin twice, do you expect it to land face up once, and face down once, every time you flip it twice? Of course not! Since each flip of the coin is uncorrelated with the previous flip (the coin has no reason to remember how it landed last time), there is an intrinsic measurement error which we can approximate as being equal to the square root of the number of events $\sqrt{N}$. If you flip a coin 10 times so that you would expect to have 5 heads, about two-thirds of the time you will find that the number of heads of you get is within the range $5\sqrt{5} \approx 3$ and $5 + \sqrt{5} \approx 7$, and that one third of the time you will get fewer than 3 or more than 7 heads! In other words, you expect to get something like a Gaussian distribution of events about a mean value of N of

$$P(N, N) = \frac{1}{\sqrt{2\pi N}} exp\left[-\frac{(N - \bar{N})^2}{2N}\right] \tag{A.4}$$

where $P(N, N)$ is the probability of measuring N this particular time and $\bar{N}$ is the average of many measurements. This has the same form as $exp[-(x - \bar{x})^2/2\sigma^2]$ where $\sigma = \sqrt{N}$ characterizes the width of the Gaussian distribution.

A.5 Stating your results: Absolute & Relative Uncertainty

In general, the result of any measurement of physical quantity must include both the value itself (best value) and its error (uncertainty). The result is usually quoted in the form

$$x = x_{best} \pm \Delta x \tag{A.5}$$

where x_{best} is the best estimate of what we believe is a true value of the physical quantity and Δx is the estimate of absolute error (uncertainty). Note that depending on the type of the experiment the prevailing error could be random or reading error. In case the reading error and random error are comparable in value, both should be taken into account and treated as two independent errors. You will learn how to calculate Δx in this case in the "Propagation of Errors" section. The meaning of the uncertainty Δx is that the true value of x probably lies between $(x_{best}\Delta x)$ and $(x_{best} + \Delta x)$. It is certainly possible that the correct value lies slightly outside this range. Note that your measurement can be regarded as satisfactory even if the accepted value lies slightly outside the estimated range of the measured value.

Δx indicates the reliability of the measurement, but the quality of the measurement also depends on the value of x_{best}. For example, an uncertainty of 1 cm in a distance of 1 km would indicate an unusually precise measurement, whereas the same uncertainty of 1 cm in a distance of 10 cm would result in a crude estimate. Fractional uncertainty gives us an indication how reliable our experiment is. Fractional uncertainty is defined as $\Delta x/x_{best}$ where Δx is the absolute uncertainty. Fractional uncertainty can be also represented in percentile form $(\Delta x/x)100\%$. For example, the length $l = (0.50 \pm 0.01)$m has a best fractional uncertainty of $0.01/0.5 = 0.02$ and a percentage uncertainty of $0.02100 = 2\%$. Note that the fractional uncertainty is a dimensionless quantity. Fractional uncertainties of about 10% or so are usually characteristic of rather rough measurements. Fractional uncertainties of 1 or 2% indicate fairly accurate measurements.

Percentage disagreement: In some cases, you can compare the value of your experimental measurement with the standard value as

$$|\frac{x_{std} - x_{exp}}{x_{std}}| \times 100\% \tag{A.6}$$

A.6 Significant Figures

An uncertainty should not be stated with too much precision. The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty. For example, the answer 92.81 s with an uncertainty of 0.3 s should be rounded as (92.8 ± 0.3) s. If the uncertainty is 3 s, then the result is reported as (93 ± 3) s. However, the number of significant figures used in the calculation of the uncertainty should generally be kept with one more significant figure than the appropriate number of significant figures in order to reduce the inaccuracies introduced by rounding off numbers. After the calculations, the final answer should be rounded off to remove this extra figure.

- The uncertainty σ should have 1 digit or at most 2 digits (all uncertainty calculations are estimates; there is no such thing as exact uncertainty!). The result itself should be stated to the same precision as σ, for example, $10.25 \pm 0.15 sec$ or $10.3 \pm 0.2 sec$ but NOT $10.25 \pm 0.2 sec$.

- If σ is very large, you will lose significant digits. If the measurement is so bad that σ is larger than the value itself, you will have no significant digits but only know the order of magnitude!

A.6.1 Mistakes and Misconceptions

In the introductory physics laboratory, it is almost always meaningless to specify the error to more than two significant digits; often one is enough. It is a mistake to write: $x = (56.7 \pm 0.914606)$ cm, or x = (56.74057 ± 0.9) cm. Instead, write: $x = (56.7 \pm 0.9)$ cm. You cannot increase either the accuracy or precision by extending the number of digits in your mean value beyond the decimal place occupied by the error. Keep in mind that the error, by its nature, denotes the uncertainty in the last one or two significant digits of the main number and therefore any additional digits obtained from multiplication or division should be rounded off at the meaningful position. So, first calculate your error; round it off to one significant figure; then quote the value of your measurement to the appropriate number of significant figures.

A.7 Propagation of Errors

In the majority of experiments the quantity of interest is not measured directly, but must be calculated from other quantities. Such measurements are called indirect. The quantities measured directly are not exact and have errors associated with them. While we calculate the parameter of interest from the directly measured values, it is said that the errors of the direct measurements propagate. Errors can propagate in measurements. What happens to the final uncertainty in a measurement which depends on several variables, each with its own uncertainty? The answer is not obvious and two cases are possible: when the uncertainties in the individual variables are **independent** and when the individual uncertainties are **dependent**. In this lab, you will work with the assumption that the individual uncertainties are completely independent.

As an example, consider the following problem. Suppose we have measured the value of a quantity x with an uncertainty, which we denote Δx. In order to test a theoretical formula, suppose that we need to calculate y as function of x i. e., $y = f(x)$. We want to know the uncertainty in y due to the uncertainty in the value of x. This is equivalent to asking what will be the variation in y (call it Δy) as x varies from x to $(x + \Delta x)$? Mathematically, this variation is given by $\Delta y = f(x + \Delta x) - f(x)$. The answer comes from differential calculus: if $y = f(x)$ and Δx is small, then

$$\Delta y \approx \frac{dy}{dx}\Delta x = \frac{df}{dx}\Delta x \tag{A.7}$$

This argument can be extended for the calculation of quantities that are functions of several different measured quantities. All you will need at this point are the results that you can find below for different types of functions. Note that we neglect the sign in the differential, since the sign of all errors may take on numerical values which are either + or -.

A.7.1 Propagation of Independent Errors

Here Δy and the various Δxs are either standard deviations, standard errors or reading errors, depending on the circumstances.

Rule 1: If two mutually independent quantities are being added or subtracted: $y = x1+x2$ or $y = x1x2$,

$$\Delta y = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2} \tag{A.8}$$

Rule 2: If two mutually independent quantities are being multiplied or divided i.e. $y = x_1x_2$ or $y = x_1/x_2$,

$$\frac{\Delta y}{y} \approx \sqrt{(\frac{\Delta x_1}{x_1})^2 + (\frac{\Delta x_2}{x_2})^2} \tag{A.9}$$

Rule 3: If a quantity is raised to a power i.e. $y = x^n$ and if Δx is small then $\Delta y \approx nx^{n-1}\Delta x$ and

$$\frac{\Delta y}{y} \approx n\frac{\Delta x}{x} \tag{A.10}$$

A general formula for the uncertainty in a measurement of a function $f(x, y, z...)$ where the independent variables $x, y, z...$ have individual (uncorrelated) uncertainties $\Delta_x, \Delta_y, \Delta_z...$ is given by the formula

$$\Delta_f = \sqrt{\Delta_x^2 \left(\frac{\partial f}{\partial x}\right)^2 + \Delta_y^2 \left(\frac{\partial f}{\partial y}\right)^2 + \Delta_z^2 \left(\frac{\partial f}{\partial z}\right)^2 + ...} \tag{A.11}$$

where the partial derivatives are all evaluated at the best known values of $x, y, z....$

NOTE:- This formula is based on a first-order Taylor series expansion of a function of many variables and is valid when the individual uncertainties Δ_i are uncorrelated with each other and are small compared to the values of the quantities. The first-order Taylor series expansion for any function f:

$$f(x - x_0) \approx f(x_0) + (x - x_0)\frac{d}{dx}f(x)|_{x=x_0} \tag{A.12}$$

A.7.2 Exercises

Write out the error propagation formula (in terms of $\Delta f/f$) when the function $f(x, y)$ is of the form:

1. $f = x * y$ and $f = x/y$
2. $f = x \pm y$
3. $f = x^m y^n$

4. $f = kx$ (k is a constant)
5. $f = \ln_e x$
6. $f = \log_{10} x$
7. $f = e^x$

A.8 Fitting Data: Least Squares Regression

Frequently in the lab you will perform a series of measurements of a quantity y at different values of x. This gives a more accurate determination of a physical parameter rather than a single measurement. If you have a linear relationship $y = mx + b$, you can determine the uncertainty in the measured slope m and the intercept b.

A common method to find the best curve to fit a set of data points is the "method of least squares". If all the data points have nearly the same weight/error, one can try to arrange the curve so that as many points lie below the line as above. However, such a visual method is not quantitative.

The least-squares method of curve fitting can be described qualitatively as follows: Let the data set be represented by the functional form $f(x; a, b...)$ where $a, b, ..$ are adjustable parameters that can be varied to get the best fit curve. The function f may be for example a line ($f(x) = mx+b$ where the adjustable parameters are m, b) or a higher order polynomial or any complicated function. For each data point (x_i, y_i), the value $y_i - f(x_i; a, b...)$ is computed and then the "chi-square" value χ^2 is calculated from the expression

$$\chi^2 = \sum_i \frac{[y_i - f(x_i; a, b, ...)]^2}{\sigma_i^2} \tag{A.13}$$

where σ_i is the uncertainty of each data point. The best fit is found by adjusting the parameters $a, b...$ until the minimum value of χ^2 is achieved. For N data points and n adjustable parameters, the reduced chi-square can be calculated from

$$\chi^2_\nu = \frac{\chi^2}{\nu} = \frac{\chi^2}{N - n} \tag{A.14}$$

where ν is the degree of freedom in the problem. If the parameters are adjusted so that $\chi^2_\nu \approx 1$, a "good fit" is achieved i.e the difference between the fit curve and the data is on average, as big as the uncertainty in the data itself.

Fitting to a straight line As an example of the least squares method, consider fitting a set of data points to a straight line $f(x) \equiv y = mx + b$. The best fit to the curve is when the sum of the squares of the deviations of the fitted curve from the data points at each x_i is a minimum i.e the parameters m and b are adjusted to get a minimum of the χ^2 term which in this case is

$$\chi^2(m, b) = \sum_i \frac{[y_i - (mx_i + b)]^2}{\sigma_i^2} \tag{A.15}$$

A minimization of this function yields

$$\begin{aligned} \frac{\partial \chi^2}{\partial m} &= 2\sum_i \left(\frac{y_i + mx_i + b}{\sigma_i} \right) \frac{x_i}{\sigma_i} = 0 \\ \frac{\partial \chi^2}{\partial b} &= 2\sum_i \left(\frac{y_i + mx_i + b}{\sigma_i} \right) \frac{x_i}{\sigma_i} = 0 \end{aligned} \tag{A.16}$$

These are a pair of linear simultaneous equations in m and b that can be solved to find the best fit values of the parameters.

You can use a simple graphical method to find the least squares fit to your data if you do not wish to use standard computer programs. Plot the measured points (x, y) and mark for each point (or at least for some representative points) the errors Δx and Δy as bars that extend from the plotted point in the x and y directions. Draw the line that best describes the measured points (i.e., the line that minimizes the sum of the squared distances from the line to the points to be fitted; the least-squares line). This line will give you the best value for slope m and intercept b. Next, draw the steepest and flattest straight lines that are consistent with the error bars of at least two-thirds of the data points. The difference in slope m between the flattest (m_{min}) and steepest (m_{max}) lines should indicate the 2σ range of the slope m, so that the 1σ error Δm in the slope will be given by $\Delta m = (m_{max} m_{min})/2$.

Appendix B

Notes on Fourier Analysis

B.1 Fourier analysis in waves and optics

Fourier series and transforms are ubiquitous in the study of waves and optics. All wave propagation (and light is an electromagnetic wave) can be described using Fourier analysis. Basically, any periodic function can be expressed as a sum of sines and cosines (the sum may be finite or infinite, depending on the periodic function). The result is that the periodic function in the time domain can be completely characterized by the information in the frequency domain (the Fourier domain) i.e. by frequencies and amplitudes of the sine and cosine terms. This means that you can build up (or decompose) any complicated waveform from just a linear superposition of sine and cosine waves of appropriate amplitudes and frequencies. As an example, you would have worked out in the single slit diffraction experiment and concluded for yourself that the Fraunhofer diffraction pattern is nothing but the Fourier transform of the amplitude function in the diffracting aperture.

B.2 Fourier Series

Fourier series are used to represent a given periodic function $f(x)$ in terms of cosine and sine functions. If $f(x)$ is a periodic function of period 2π and integrable over a period, it can be represented by a trigonometric series called the Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \tag{B.1}$$

so the series converges and has $f(x)$ as its sum. The Fourier coefficients a_0, a_n, b_n are given by

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \end{aligned} \tag{B.2}$$

where $n = 1, 2....$

The Fourier series of a function $f(x)$ of period $p = 2L$ has a Fourier series given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \tag{B.3}$$

The Fourier coefficients a_0, a_n, b_n are given by

$$\begin{aligned} a_0 &= \frac{1}{2L} \int_{-L}^{L} f(x) dx \\ a_n &= \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} dx \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} dx \end{aligned} \tag{B.4}$$

where $n = 1, 2....$

The calculation of the Fourier coefficients hinges on the orthogonality of sine and cosine functions i.e.

$$\begin{aligned} \int_0^T \sin m\omega t \cos n\omega t dt &= 0 \text{ for all } m, n \\ \int_0^T \sin m\omega t \sin n\omega t dt &= 0 \ m \neq n \\ \int_0^T \cos m\omega t \cos n\omega t dt &= 0 \ m \neq n \\ \int_0^T \sin^2 m\omega t dt &= \frac{T}{2} \text{ for all } m \\ \int_0^T \cos^2 m\omega t dt &= \frac{T}{2} \text{ for all } m \end{aligned} \tag{B.5}$$

B.3 Fourier Transform

The Fourier transform $F(\omega)$ of a function $f(t)$ is given by

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{B.6}$$

The inverse Fourier transform is given by

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \tag{B.7}$$