ROLL NO : MS ☐☐☐☐☐ NAME :

PHY202 Jan–Aug 2019: Midsem 2 Dated March 15, 2019: Inst: Dipanjan Chakraborty

- **Time : 60 minutes**
- **Max Marks : 40**
- Attempt all questions.

Question	1	2	3	4	T
Marks					

1. Each of the following question has 1 marks for the correct tick mark and the rest for justification.

(a): The Legendre transformation of the function $y = ax^2$ is $c(m) = -m^2/4a$

[5]

✓ YES | NO ①

Justification

$m = \frac{dy}{dx} = 2ax \qquad x = \frac{m}{2a}$ — ①

① ← $\boxed{c(m) = y - mx} = a\left(\frac{m}{2a}\right)^2 - \frac{m^2}{2a}$

$\boxed{= \frac{m^2}{4a} - \frac{m^2}{2a} = -\frac{m^2}{4a}}$ → ②

(b): The specific heat at constant volume C_V is given by $C_V = T\left(\frac{\partial^2 F}{\partial T^2}\right)_V$, where F is the Helmholtz free energy.

[5]

YES | NO ①

Justification

① ← $C_V = T \left.\frac{\partial S}{\partial T}\right)_V \qquad S = -\left.\frac{\partial F}{\partial T}\right)_V$ → ①

$\Rightarrow C_V = -T \left.\frac{\partial^2 F}{\partial T^2}\right)_V$ → ②

(c): For a rubber band the force–extension curve is given by $F = AL/T$, where A is a constant, T is the temperature. The difference in the specific heats $C_F - C_L$ is AF^2. [5]

YES | ~~NO~~ → 1

Justification

$$\boxed{C_F - C_L = \frac{TL\left(\frac{1}{L}\frac{\partial L}{\partial T}\right)_F^2}{+\frac{1}{L}\frac{\partial L}{\partial F}\Big)_T}} \rightarrow 1$$

$$= \frac{+\cancel{TL}\left(\frac{FA}{L}\right)^2}{\cancel{\left(\frac{AT}{L}\right)}}$$

$L = \frac{FT}{A}$

$\frac{1}{L}\frac{\partial L}{\partial T}\Big)_F = \frac{F}{AL} \rightarrow 1$

$\frac{1}{L}\frac{\partial L}{\partial F}\Big)_T = \frac{T}{LA} \rightarrow 1$

$$= \cancel{\frac{TL^2F^2A^2}{ATL^2}}$$

$$= \cancel{AF^2} \rightarrow 1$$

$$C_F - C_L = TL\frac{(F/AL)^2}{T/LA} = \cancel{T}\cancel{L}\frac{F^2\cancel{LA}}{A^2\cancel{L^2}\cancel{T}} = F^2/A$$

(b): An ideal monoatomic gas undergoes a free expansion. Upon reaching final state, the gas cools down. [5]

YES | ~~NO~~ → 1

Justification

Internal energy ~~is~~ depends only on temperature. Hence, C_v is constant number.

The temperature remains the same. → 4

2. Consider a simple magnetic system. The generalized coordinate is the magnetization M and the generalized force is the external magnetic field B. Write down the thermodynamic potentials for such a system and indicate the possible experimental conditions. Determine the corresponding Maxwell's relations.

[Marks=10] $\frac{1}{2}$

Thermodynamic Potential	Differential	Maxwell Relation	Experimental Condition
$U = TS + BM$	$dU = T\,dS + B\,dM$	$\left.\frac{\partial T}{\partial M}\right)_S = \left.\frac{\partial B}{\partial S}\right)_M$ 1	Isolated System
$F = U - TS$ 1/2	$dF = -S\,dT + B\,dM$ 1/2	$\left.\frac{\partial T}{\partial B}\right)_S = -\left.\frac{\partial M}{\partial S}\right)_B$ 1	Magnetic system + thermostat 1/2
$G = U - TS - MB$ 1/2	$dG = -S\,dT - M\,dB$ 1/2	$\left.\frac{\partial S}{\partial B}\right)_T = \left.\frac{\partial M}{\partial T}\right)_B$ 1	Magnetic System in presence of thermostat and constant magnetic Field 1/2
$H = U - MB$ 1/2	$dH = T\,dS - M\,dB$ 1/2	$\left.\frac{\partial S}{\partial B}\right)_T = -\left.\frac{\partial B}{\partial T}\right)_M$ 1	Magnetic System in presence of only constant magnetic field. 1/2

Gibbs–Duhem relation using $U = U(S, M)$	Gibbs–Duhem relation using $S = S(U, M)$
$S\,dT + M\,dB = 0$ 1	$U\,d(1/T) - M\,d(B/T) = 0.$ 1

Sol

$$S = U/T - \frac{B}{T} M$$

$$dS = \frac{1}{T} dU + U\,d(1/T) - \frac{B}{T} dM - M\,d(B/T)$$

or $U\,d(1/T) - M\,d(B/T) = 0$

3. For such a simple magnetic system, it is observed that if the magnetic field changes from B to $B + \Delta B$ **at fixed temperature**, the change in entropy is given by $\Delta S = -NAB\Delta B/T^2$, where A is a constant. What is $\left(\frac{\partial S}{\partial B}\right)_T$? From this information, show that the magnetization depends on temperature as $M = NAB/T$. Note that for zero magnetic field, the magnetization is also zero in a paramagnetic system. *Hint: you will need one of the Maxwell's relation that you have derived in the earlier question.*

[Marks=~~5~~] $4\frac{1}{2}$

$$\left.\frac{\partial S}{\partial B}\right)_T = \lim_{\Delta B \to 0} \frac{\Delta S}{\Delta B} = -\frac{NAB}{T^2} \rightarrow 1$$

$$\left.\frac{\partial S}{\partial B}\right)_T = \left.\frac{\partial M}{\partial T}\right)_B = \frac{-NAB}{T^2} \rightarrow 1$$

$$\left.\frac{\partial M}{\partial T}\right)_B = -\frac{NAB}{T^2} \rightarrow 1$$

$$M = \frac{NAB}{T} \rightarrow 1$$

The constant of integration is zero since at zero magnetic field Magnetization is zero.

$\rightarrow 1/2$

4. A paramagnetic needle is dipped in liquid helium at a temperature T_0. The whole system is placed in a weak magnetic field B_0 directed along the needle's long axis. Then the magnetic field is suddenly decreased to B_ℓ. Determine the temperature of the needle, given that the specific heat at constant magnetic field goes as $C_B = NAB^2/T^2$ (A is the same constant as mentioned earlier) and you can use the relation between M and T derived in the earlier question. *The sudden decrease means – too fast for heat exchange. Hence the process is adiabatic.*

[Marks=5]

Adiabatic Process

$S(T,B)$

$$dS = \left.\frac{\partial S}{\partial T}\right)_B dT + \left.\frac{\partial S}{\partial B}\right)_T dB \longrightarrow 1$$

$dS = 0$

$$\Rightarrow \quad \frac{C_B}{T}\, dT = -\left.\frac{\partial S}{\partial B}\right)_T dB$$

$$\frac{C_B}{T}\, dT = -\left.\frac{\partial M}{\partial T}\right)_B dB$$

$$\boxed{dT = \frac{T}{C_B}\left(-\left.\frac{\partial M}{\partial T}\right)_B\right) dB} \quad 2$$

Alternatively

$$dT = \left.\frac{\partial T}{\partial B}\right)_S dB \longrightarrow 1$$

$$\left.\frac{\partial T}{\partial B}\right)_S \left.\frac{\partial S}{\partial T}\right)_B \left.\frac{\partial B}{\partial S}\right)_T = -1$$

$$\left.\frac{\partial T}{\partial B}\right)_S = -\frac{\left.\frac{\partial S}{\partial B}\right)_T}{\left.\frac{\partial S}{\partial T}\right)_B}$$

$$\left.\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C_B}\cdot\left.\frac{\partial M}{\partial T}\right)_B$$

→ If this equation is derived in either way then you get 2 marks.

$$\left.\frac{\partial M}{\partial T}\right)_B = -\frac{NAB}{T^2}$$

$$\therefore \quad dT = \frac{T}{C_B}\,\frac{NAB}{T^2}\, dB$$

$$= \frac{T^3}{NAB^2}\,\frac{NAB}{T^2}\, dB = \frac{T}{B}\, dB$$

$$\frac{dT}{T} = \frac{dB}{B} \longrightarrow 1$$

$$\Rightarrow \ln T/B = \text{const} \quad \Rightarrow \quad T/B = \text{const.}$$

$$\Rightarrow \quad \frac{T_0}{B_0} = \frac{T_f}{B_\ell} \quad \Rightarrow \quad \boxed{T_f = \frac{B_\ell}{B_0}\, T_0} \longrightarrow 1$$

Rough Work