ROLL NO : MS NAME :

PHY202 Jan–Aug 2020: Midsem 1 Dated February 4, 2020: Inst: Dipanjan Chakraborty

- Time : 60 minutes
- Max Marks : 40
- Attempt all questions. No aids (Books/Notes/Gadgets).

Question	1	2	3	4	T
Marks					

1. Each of the following question has 1 marks for the correct tick mark and the rest for justification.

(a): The differential $dz = (2xy^3 + 2)dx + (3x^2y^2 + e^y)dy$, is not an exact differential. [4]

YES | NO ✓

Justification

$dz = A(x,y)\,dx + B(x,y)\,dy$

$\frac{\partial A}{\partial y} = 6xy^2$ — 1

$\frac{\partial B}{\partial x} = 6xy^2$ — 1

$\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$ — 1

Hence Exact differential.

$\frac{\partial z}{\partial x} = 2xy^3 + 2$

$z = 2 \cdot \frac{x^2}{2} y^3 + 2x + f(y)$ — 1

$\frac{\partial z}{\partial y} = 3x^2y^2 + e^y$

$z = x^2y^3 + e^y + g(x)$ — 1

By comparison

$z = x^2y^3 + 2x + e^y$ — 1

Hence exact.

Possiblity to show that $\oint_C dz = 0$. by choosing appropriate path.

(b): In an adiabatic process, the work done from taking a system from a volume V_1 to V_2 is more for n moles a diatomic gas than for n moles of monoatomic gas. [4]

1 ✓ YES | NO

Justification

$W = p\,dV = \alpha \int \frac{dV}{V^\gamma} = \alpha \left. \frac{V^{-\gamma+1}}{-\gamma+1} \right|_{V_1}^{V_2}$ — 1

$= \alpha \frac{V_2^{(1-\gamma)} - V_1^{(1-\gamma)}}{(1-\gamma)} = \frac{P_2V_2 - P_1V_1}{(1-\gamma)}$ — 1

$P_1 V_1^\gamma = \alpha$

$P_2 V_2^\gamma = \alpha$

γ for monoatomic is $5/3$

γ for diatomic is $7/5$

$\therefore W = \frac{3}{2}(P_1V_1 - P_2V_2)$ ~~mono~~ - monoatomic — 1

$W = \frac{5}{2}(P_1V_1 - P_2V_2)$ Diatomic.

(c): An ideal monoatomic gas, at a temperature T_0 is mixed with an equal volume of an ideal diatomic gas at a temperature $2T_0$. The final temperature of the mixture is $13T_0/8$. [4]

YES | NO

Justification

Both are possible answers.

If the work done is zero then heat given up is taken by other.
Assuming constant volume

$$C_1(T_f - T_1) = C_2(T_2 - T_f) \quad — (1)$$

$$T_f = \frac{C_1T_1 + C_2T_2}{C_1 + C_2} \quad — (1)$$

$C_1 = 3/2$; $T_1 = T_0$

$C_2 = 5/2$; $T_2 = 2T_0$ $\qquad T_f = \frac{13T_0}{8}$ —(1).

There are other possibilities.
For example $\Delta S = \Delta S_1 + \Delta S_2 = 0$.

(3)

(d): Given the fundamental equation for a thermodynamic system as $S = A(nVU)^{1/3}$, the equation of state is $P/T = (A/3)\sqrt{A/3}\sqrt{nT/V}$. [4]

YES | NO

Justification

$$S = A(nVU)^{1/3} \qquad \left.\frac{\partial S}{\partial V}\right)_{U,N} = \frac{P}{T} = \frac{1}{3} A \frac{n^{1/3} U^{1/3}}{V^{2/3}}$$

$$\left.\frac{\partial S}{\partial U}\right)_{N,V} = A V^{1/3} n^{1/3} \left(\frac{1}{3}\right) U^{-2/3} = \frac{1}{T} \quad (1).$$

$$\Rightarrow \frac{A}{3} \frac{V^{1/3} n^{1/3}}{U^{2/3}} = \frac{1}{T}$$

$$U^2 = \left(\frac{A}{3}\right)^3 V n T^3$$

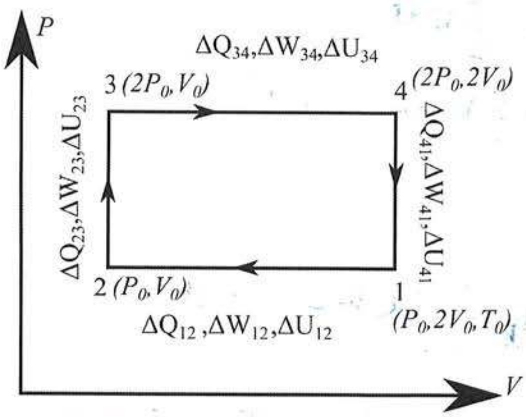
$$U = \left(\frac{A}{3}\right)^{3/2} V^{1/2} n^{1/2} T^{3/2}.$$

$$U^{1/3} = \left(\frac{A}{3}\right)^{1/2} V^{1/6} n^{1/6} T^{1/2} \quad — (1).$$

$$\frac{P}{T} = \left(\frac{A}{3}\right) \frac{n^{1/3}}{V^{2/3}} V^{1/6} n^{1/6} T^{1/2} \left(\frac{A}{3}\right)^{1/2}$$

$$= \left(\frac{A}{3}\right)^{3/2} \frac{n^{1/2} T^{1/2}}{V^{1/2}} = \frac{A}{3}\sqrt{\frac{A}{3}}\sqrt{\frac{nT}{V}} \quad — (1)$$

2. Consider a system of ideal monoatomic gas that is taken through the cycle shown in the figure. The numerals and the corresponding arrows represents the direction of the thermodynamic processes. The corresponding values of pressure and volume are also indicated in the diagram. All processes are considered to be reversible. Let $\Delta Q_{ij}, \Delta U_{ij}, \Delta W_{ij}$ represent the change in heat (taken/or given up), the change in internal energy and the work done between the consecutive points i and j in the diagram. [Marks=6+6]



(a): Calculate the quantities $\Delta Q_{ij}, \Delta U_{ij}, \Delta W_{ij}$ between the consecutive points i and j as shown in the diagram. From this calculate the total heat change in the cyclic process.

$$\Delta W_{12} = -P_0 \int_{V_1}^{V_2} dV = -P_0(V_0 - 2V_0) = P_0 V_0 = RT_0/2.$$

$2P_0V_0 = RT_0$

$$\Delta U_{12} = \frac{3}{2}R(T_2 - T_1)$$

$P_0V_0 = RT_2 \Rightarrow \boxed{T_2 = T_0/2}$ *1

$$= -\frac{3}{4}RT_0$$

1

$$\Delta Q_{12} = \Delta U_{12} - \Delta W_{12} = -\frac{3}{4}RT_0 - \frac{RT_0}{2} = -\frac{5RT_0}{4}.$$

$\Delta W_{23} = 0.$ $\Delta U_{23} = \frac{3}{2}R(T_3 - T_2)$

$2P_0V_0 = RT_3$

$\boxed{T_3 = T_0}$ *2

$$\Delta Q_{23} = \frac{3}{2}R(T_0 - T_0/2) = \frac{3}{4}RT_0.$$

1

$$\Delta W_{34} = -2P_0(V_4 - V_3) = -2P_0V_0 = -RT_0$$

$$\Delta U_{34} = \frac{3}{2}R(T_4 - T_3)$$

$4P_0V_0 = RT_4.$

$\boxed{T_4 = 2T_0}$ *3

1

$$= \frac{3}{2}RT_0$$

$\boxed{*1 + *2 + *3}$ $= 1$.

$$\Delta Q_{34} = \Delta U_{34} - \Delta W_{34} = \frac{3}{2}RT_0 + RT_0 = \frac{5}{2}RT_0.$$

$\Delta W_{41} = 0$ $\Delta U_{41} = \frac{3}{2}R(T_1 - T_4) = \frac{3}{2}R(T_0 - 2T_0) = -\frac{3}{2}RT_0.$ 1

$\Delta Q_{41} = -\frac{3}{2}RT_0.$

$$\Delta Q = \Delta Q_{12} + \Delta Q_{23} + \Delta Q_{34} + \Delta Q_{41} = \frac{RT_0}{2}$$

1

(b): Now calculate the change in the entropy explicitly from the definition $dS = đQ/T$. Is there a contradiction with the result for the heat change for the whole cycle that you obtained earlier?

$$T\,dS = dU + P\,dV = \frac{3}{2}R\,dT + P\,dV. \qquad PV = RT \qquad P\,dV = R\,dT \quad — 1$$

$$T\,dS_{12} = \frac{3}{2}R\,dT + R\,dT = \frac{5}{2}R\,dT$$

$$\Delta S_{12} = \frac{5}{2}R \ln T_2/T_1 = -\frac{5}{2}R\ln 2 \quad — 1$$

$$T\,dS_{23} = dU_{23} = \frac{3}{2}R\,dT$$

$$\Delta S_{23} = \frac{3}{2}R \ln T_3/T_2 = \frac{3}{2}R\ln 2 \quad — 1$$

$$T\,dS_{34} = \frac{5}{2}R\,dT \Rightarrow \Delta S_{34} = \frac{5}{2}R\ln\frac{T_4}{T_3} = \frac{5}{2}R\ln 2. \quad — 1$$

$$T\,dS_{41} = \frac{3}{2}R\,dT \qquad \Delta S_{41} = \frac{3}{2}R\ln\frac{T_1}{T_4} = -\frac{3}{2}R\ln 2 \quad — 1$$

Total change in entropy $\Delta S = 0$. — 1

3. Consider a thermodynamic system described by the macroscopic variable S, U, V, N, where the symbols have their usual meaning. The equation of states are $U = 3Nk_BT$ and $PV = Nk_BT$. [Marks=4+4+4]

(a): Determine the dependence of the chemical potential μ as a function U, V, N.

Gibbs Duhem relation

$$U\,d(1/T) + V\,d(P/T) = N\,d(\mu/T) \quad — 1$$

$$U\,d\left(\frac{3Nk_B}{U}\right) + V\,d\left(\frac{Nk_B}{V}\right) = N\,d(\mu/T).$$

$$U\left[3Nk_B\left(-\frac{dU}{U^2}\right) + \frac{3k_B\,dN}{U}\right] + V\left[Nk_B\left(-\frac{dV}{V^2}\right) + \frac{k_B}{V}dN\right] = N\,d(\mu/T)$$

$$-3Nk_B\frac{dU}{U} + 4k_B\,dN - Nk_B\frac{dV}{V} = N\,d(\mu/T) \quad — 1$$

$$-3\frac{dU}{U} + 4\frac{dN}{N} - \frac{dV}{V} = d(\mu/k_BT) \quad — 1$$

$$\frac{\mu}{k_BT} = \ln \kappa \frac{N^4}{VU^3}$$ Where κ is a constant of integration.

$\hookrightarrow$ 1

(b): Now suppose I want the chemical potential μ to be zero, for all values of U, V. Determine N as function of U and V so that $\mu = 0$. Hence determine U and S as a function of T and V.

If $\mu = 0$, then $\frac{K N^4}{V U^3} = 1 \quad N^4 = K V U^3 \Rightarrow N = K V^{1/4} U^{3/4}$ — 1 .

Since $U = 3 N k_B T \Rightarrow U = 3 K V^{1/4} U^{3/4} k_B T$

$\Rightarrow U^{1/4} = 3 K k_B V^{1/4} T$

$\Rightarrow \boxed{U = A V T^4}$ — 1 $\quad \frac{PV}{T} = \cancel{3} N k_B = K V^{1/4} U^{3/4}$.

$S = \frac{U}{T} + \frac{PV}{T} - \frac{\mu N}{T} = \frac{U}{T} + \frac{PV}{T} = 3 N k_B + N k_B = 4 N k_B$ — 1

$N k_B = K k_B V^{1/4} (A V T^4)^{3/4} = K k_B V^{1/4} A^{3/4} V^{3/4} T^3$

$= \frac{1}{3} A^{1/4} V^{1/4} A^{3/4} V^{3/4} T^3$

$= \frac{A}{3} V T^3$

$\therefore \boxed{S = \frac{4}{3} A V T^3 = \frac{4}{3} \frac{U}{T}}$ — 1 .

(c): What would be the change in entropy of such a system (that is with $\mu = 0$) if the system is isothermally expanded. Verify that your answer is correct from the expression of S you obtained before.

Isothermal expansion $\rightarrow$ T is constant.

$\therefore \quad \Delta S = \frac{4}{3} A \Delta V T^3$ — 1

$\Delta U = A \Delta V T^4$.

$\frac{PV}{T} = N k_B = \frac{1}{3} A V T^3 \Rightarrow P = \frac{1}{3} A T^4$. — 1

$\Delta W = -P \int dV = -\frac{1}{3} A T^4 \Delta V. \Rightarrow dW = -\frac{1}{3} A T^4 dV$

$dU = A T^4 dV.$ — 1

$\Delta U = A \Delta V T^4$

$\therefore \quad dQ = dU + dW = \frac{4}{3} A T^4 dV.$

$dS = \frac{dQ}{T} = \frac{4}{3} A T^3 dV \Rightarrow \boxed{\Delta S = \frac{4}{3} A T^3 \Delta V}$ — 1

Rough Work