ROLL NO : MS | | | | | NAME :

PHY202 Jan–Aug 2019: Midsem 1 Dated February 4, 2019: Inst: Dipanjan Chakraborty

- **Time : 60 minutes**
- **Max Marks : 40**
- Attempt all questions. No aids (Books/Notes/Gadgets).

Question	1	2	3	4	T
Marks					

1. Each of the following question has 1 marks for the correct tick mark and the rest for justification.

(a): The differential $dz = 2xydx + (x^2 + 2y)dy$ is not an exact differential [4]

YES | NO ✓

Justification

$$dz = A(x,y)dx + B(x,y)\,dy \quad — 1$$

$$\frac{\partial A}{\partial y} = 2x \qquad \frac{\partial B}{\partial x} = 2x \quad — 1$$

$$dz = d\left(x^2 y + y^2\right) \quad — 1$$

(b): For an ideal gas, the internal energy per unit volume is $P/(\gamma - 1)$, where P is the pressure and γ is the ratio of specific heats. [4]

✓YES | NO

Justification

$$U = \frac{3}{2} nRT \qquad PV = nRT$$

$$C_V = \frac{3}{2} nR \qquad C_P = \frac{5}{2} nR$$

$$\frac{C_P}{C_V} = \gamma \qquad C_P - C_V = nR$$

$$C_V(\gamma - 1) = nR$$

1

$$U = C_V T = \frac{nRT}{(\gamma-1)} = \frac{PV}{(\gamma-1)} \quad — 1$$

$$U/V = \frac{P}{(\gamma - 1)} \quad — 1$$

(c): For n moles of an ideal gas, initially confined in volume V and at temperature T is expanded to a volume αV (i) by a reversible isothermal expansion (ii) removing a partition and allowing a free expansion. The change in entropy is the same in the two cases. [4]

YES	NO

Justification

Entropy is a state function. Change in entropy is $\Delta S = nR \ln \alpha$.

(b): An ideal monoatomic gas in three dimensions is initially at a temperature T. The temperature is changed to $4T$. The root-mean square velocity $v_{\mathrm{rms}} = \sqrt{\langle \vec{v}^2 \rangle}$ changes by a factor 3. [4]

YES	NO

Justification

$$V^1_{rms} = \sqrt{\langle \vec{v}^2 \rangle} = \sqrt{\frac{3k_BT}{m}} \quad 1 \qquad V^2_{rms} = \sqrt{\frac{12k_BT}{m}} = 2\sqrt{\frac{3k_BT}{m}} \quad 1$$

$$\therefore \frac{V^2_{rms}}{V^1_{rms}} = 2 \quad 1$$

Ideal monoatomic Gas. & specific heats constant.

2. Consider a hydrostatic system with fixed particle number. The system undergoes a change in state going from a state-1 (P_1, V_1) to a state-2 (P_2, V_2) by an isothermal expansion. Now consider another process in which we first go from (P_1, V_1) to (P_2, V_1) and then to (P_2, V_2). Estimate the entropy change in going from state 1 to state 2 along each of these paths. What do you conclude?

[Marks=12]

For isothermal expansion from (P_1,V_1) to (P_2,V_2)

$$\Delta S = nR \int_{V_1}^{V_2} \frac{dV}{V} = nR \ln V_2/V_1 \qquad \text{— (2)}$$

Change in entropy (P_1,V_1) to (P_2,V_1)

$$\Delta S = C_p \int_{T_\phi}^{T'} \frac{dT}{T} = C_v \ln \frac{T'}{T_\phi} \qquad \text{— (2)}$$

Change in entropy for (P_2,V_1) to (P_2,V_2)

$$\Delta S = C_p \int_{T'}^{T_2} \frac{dT}{T} = C_p \ln \frac{T_2}{T'} \qquad \text{— (2)}$$

$\therefore$ ~~ΔS~~ Total Change $\Delta S = C_v \ln \frac{T'}{T_1} + C_p \ln \frac{T_2}{T'}$

$C_p - C_v = nR$

and $T_1 = T_2 = T$ — (1)

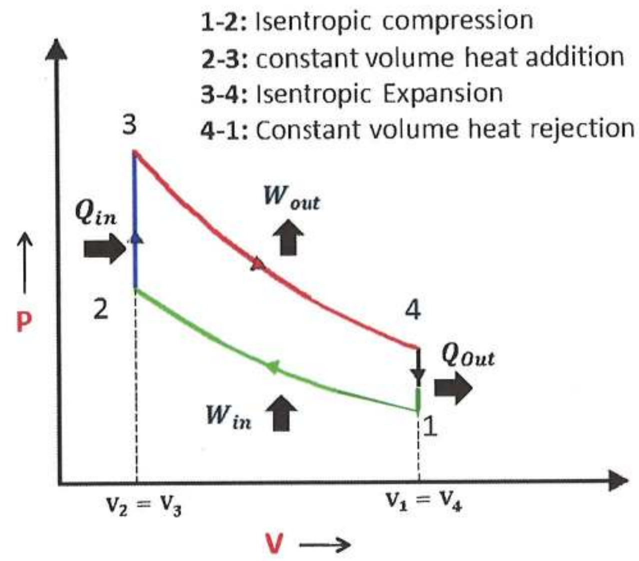
$$\Delta S = (C_v \cancel{+nR}) \ln \frac{T'}{T} + C_v \ln \frac{T}{T'} + nR \ln \frac{T}{T'} \qquad \text{— (2)}$$

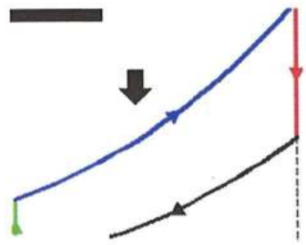
$$= nR \ln \frac{T}{T'} = nR \ln \frac{P_1}{P_2}$$

$$\Delta S = nR \ln \frac{V_2}{V_1} \qquad \text{— (2)}$$

Entropy ~~change~~ is a state function and it's change depends only on the initial and final points. (1)

3. Consider an ideal heat engine operated in a cycle using two adiabatic processes and two isochoric processes. The cycle in the $p-V$ diagram is shown below: The heat engine is operated with an ideal gas as a working substance. Calculate





the efficiency of the heat engine and show that this efficiency is less than a Carnot engine working between the same hot and cold reservoirs.

[Marks=8+4]

$$Q_{in} = C_V (T_3 - T_2) \quad \text{and} \quad Q_{out} = C_V (T_4 - T_1) \quad - \ 2$$

$$\therefore \ \eta = \frac{W}{Q_{in}}, \quad \text{where} \quad W = W_{in} - W_{out}$$

$$= 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad - \ 4$$

$$\left.\begin{aligned} T_3 V_2^{\gamma-1} &= T_4 V_1^{\gamma-1} \\ T_2 V_2^{\gamma-1} &= T_1 V_1^{\gamma-1} \end{aligned}\right\} \Rightarrow \quad (T_3 - T_2) V_2^{\gamma-1} = (T_4 - T_1) V_1^{\gamma-1}$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{1}{\alpha^{\gamma-1}}$$

$$\therefore \ \eta = 1 - \frac{1}{\alpha^{\gamma-1}} = 1 - \alpha^{1-\gamma} \qquad \text{where } \alpha = \frac{V_1}{V_2}$$

2

Let us compare the temperatures.

$$\left.\begin{aligned} \frac{P_2}{T_2} &= \frac{P_3}{T_3} \qquad T_3 = T_2 \frac{P_3}{P_2} \qquad P_3 > P_2 \Rightarrow T_3 > T_2 \\ \frac{P_4}{T_1} &= \frac{P_1}{T_1} \qquad T_4 = T_1 \frac{P_4}{P_1} \qquad P_4 > P_1 \Rightarrow T_4 > T_1 \end{aligned}\right] \ 1$$

$$T_3 V_2^{\gamma-1} = T_4 V_1^{\gamma-1}$$

$$T_3 = T_4\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Even for a monoatomic gas $\gamma = 5/3$

$\gamma - 1 = 2/3$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} > 1$$

$T_3 > T_4$

Similarly $T_2 > T_1$. 1 Hence we have

$$T_3 \quad T_2 \quad T_4 \quad T_1$$

This means $T_3 - T_1 > T_2 - T_1$

Now, the maximum Carnot efficiency is obtained when we operate a heat engine between T_1 & T_3. The efficiency is

$$\eta_c = 1 - \frac{T_1}{T_3}$$

For the given heat-engine the efficiency is

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad 1 \quad = 1 - \frac{T_1}{T_2}\frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)} = 1 - \frac{T_1}{T_2}$$

Since $T_3 V_2^{\gamma-1} = T_4 V_1^{\gamma-1}$

$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

Hence since $T_3 - T_1 > T_2 - T_1$

$$\frac{T_3}{T_1} > \frac{T_2}{T_1} \Rightarrow \frac{T_1}{T_3} < \frac{T_1}{T_2}$$

or $$1 - \frac{T_1}{T_3} > 1 - \frac{T_1}{T_2}$$

$$\boxed{\therefore \quad \eta_c > \eta}$$ — 1