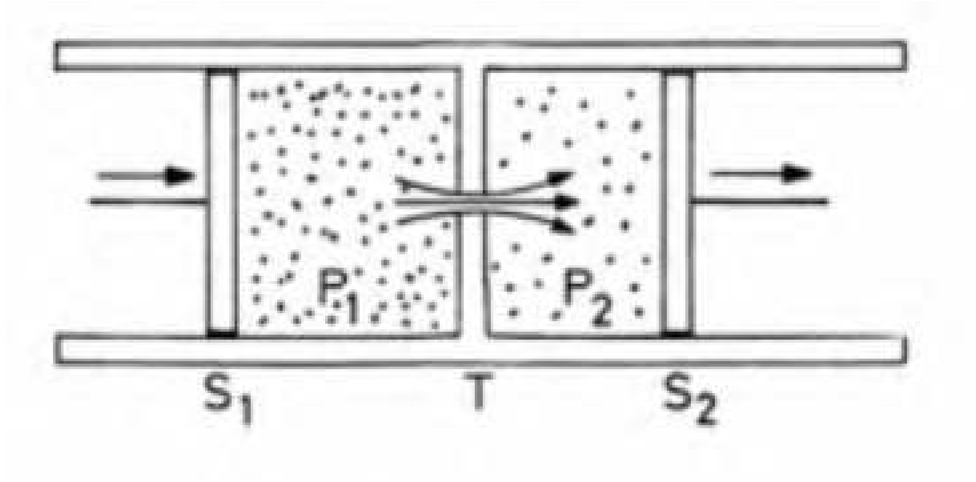
1. The Joule–Thomson process consists of the controlled expansion of a gas. Here, the stream of expanding gas is limited by a throttle valve. The gas volume is bounded to the left and the right of the throttle by the two sliding pistons S_1 and S_2, which produce the pressures P_1 and P_2 in the left and right chambers, with $P_1 > P_2$. The process is assumed to occur adiabatically, i.e. $dQ = 0$ during the entire process.

   In the initial state (1), the in the left-hand chamber has the volume V_1 and the energy E_1 . In the final state, the gas is entirely in the right-hand chamber and has a volume V_2 and energy E_2. The left piston performs work on the gas, while the gas performs work on the right piston and thus on the environment.



   a) Show that the enthalpy H remains constant during the process.

   b) We would like to know if the temperature of the gas increases/decreases during the process. The relevant quantity is called the Joule–Thomson coefficient $\left(\frac{\partial T}{\partial P}\right)_H$. Show that

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{T\left(\frac{\partial S}{\partial P}\right)_T + V}{T\left(\frac{\partial S}{\partial T}\right)_P}$$

   c) Show that

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial S}{\partial T}\right)_P / \left(\frac{\partial P}{\partial T}\right)_S$$

   d) Using all the above information, show that for an ideal gas the Joule–Thompson coefficient is zero.

2. We derived the following expression for the entropy of an ideal monoatomic gas

$$S = Nk_{\mathrm{B}}\left[\frac{5}{2} + \ln\left(\frac{U^{3/2}V}{\kappa N^{5/2}}\right)\right]$$

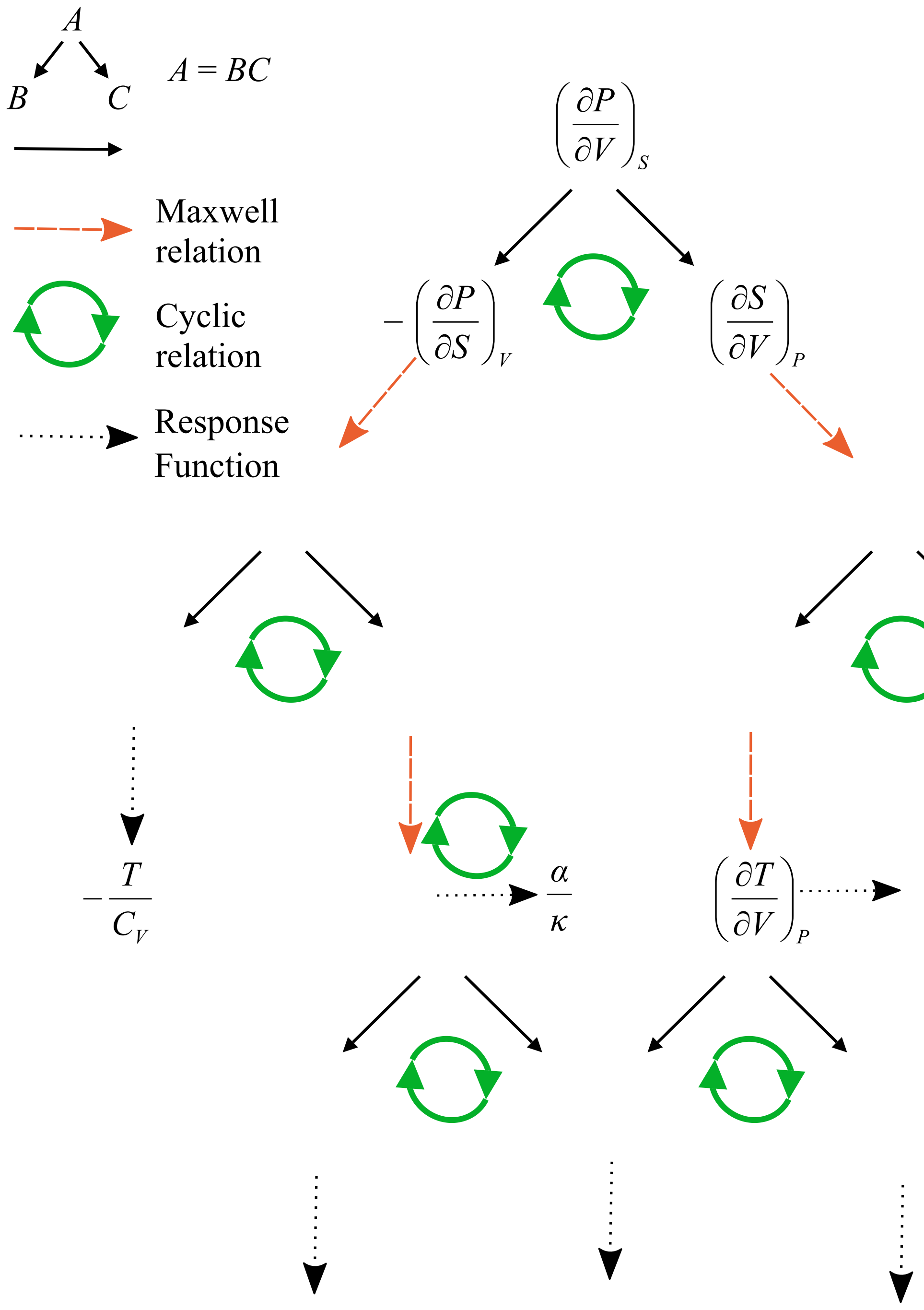
   Consider two ideal monoatomic gases at temperatures T_1 and T_2. The two gases are separated by a sliding piston and insulated from the environment. The pressure of the two gases are taken to be equal $P_1 = P_2 = P$.

   a) Determine the equilibrium temperature T after joint equilibration.

   b) Calculate the change in entropy for the process and show that it is positive.

c) Calculate the total change in enthaply.

3. Complete the diagram. The first one has been done for you as an example.



$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$