1. Suppose you are given the following relation among the entropy S, volume V , internal energy U , and number of particles N of a thermodynamic system (hydrostatic): $S = A[NVU]^{1/3}$ where A is a constant. Derive a relation among:

   a) U,V,N and T.

   b) the pressure p,N,V, and T.

   c) calculate the specific heat at constant volume.

   d) Now imagine that two bodies made up of this material are initially at temperatures T_1 and T_2. They are brought in contact to each other. Calculate the final temperature T_f. Assume, that N and V for both the bodies are same.

2. A system, maintained at constant volume, is brought in contact with a thermal reservoir at temperature T_f. The initial temperature of the system is T_i.

   a) Calculate ΔS , change in the total entropy of the system +reservoir. You may assume that c_v, the specific heat of the system, is independent of temperature.

   b) Assume now that the change in system temperature is brought about through successive contacts with N reservoirs at temperature $T_i+\Delta T$,$T_i+2\Delta T$, $\cdots$,$T_f-\Delta T$,T_f,where $N\Delta T = T_f - T_i$. Show that in the limit $N \to \infty$, $\Delta T \to 0$ with $N\Delta T = T_f - T_i$ fixed, the change in entropy of the system +reservoir is zero.

   c) Comment on the difference between (a) and (b) in the light of the second law of thermodynamics.

3. Consider an engine working on a reversible cycle and using an ideal gas with constant heat capacity (c_P) as the working substance. The cycle consists of two processes at constant pressure joined by two adiabats.

   a) Sketch the process in the $p-V$ plane.

   b) Find the efficiency of the engine as a function of the pressures.

   c) Denote the initial state of the cycle as A and go clockwise renaming the states as B,C,D. Let T_A, T_B, T_C, T_D be the corresponding temperatures. Which of these is the highest and which one the lowest?

4. A cylinder contains a perfect gas in thermodynamic equilibrium at p, V, T, U and S. The cylinder is surrounded by a large heat reservoir at temperature T. The cylinder walls and piston can be either perfect thermal conductors or perfect thermal insulators. The piston is moved to produce a small volume change $\pm\Delta V$. "Slow" or "fast" means that during the volume change the speed of the piston is very much less than, or very much greater than, molecular speeds at temperature T. For each of the five processes below make a table showing whether the changes (after the reestablishment of equilibrium) in T, p, U and S have been positive, negative, or zero.

   a) $+\Delta V$ done slowly with conducting walls.

   b) $+\Delta V$ done slowly with insulating walls.

   c) $+\Delta V$ done fast with conducting walls.

   d) $+\Delta V$ done fast with insulating walls.

5. In the big-bang theory of the universe, the radiation energy initially confined in a small region adiabatically expands in a spherically symmetric manner. The radiation cools down as it expands.
   a) Derive a relation between the temperature T and the radius R of the spherical volume of radiation, based purely on thermodynamic considerations.
   b) Find the total entropy of a photon gas as a function of its temperature T , volume V , and the constants k , $\hbar$, c .
6. Two finite, identical, solid bodies of constant total heat capacity per body, C, are used as heat sources to drive heat engine. Their initial temperatures are T_1 and T_2 respectively. Find the maximum work obtainable from the system.