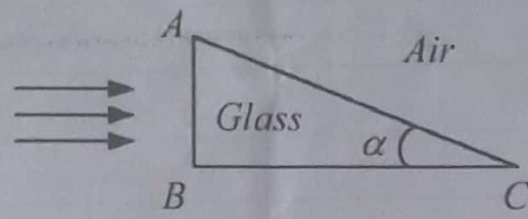
Tutorial-7 (PHY201) Due on Saturday

1. Derive the four boundary conditions for light at the surface of a dielectric medium by applying Maxwell's equations.

   Using the Fresnel's equations explain the Brewester's angle for air-glass interface.

2. A plane light wave is incident normally on the face AB of a glass prism as shown in the figure. The index of refraction of glass is 1.50.



   (a) Find the value of the angle α such that the wave will be totally reflected at the surface AC.

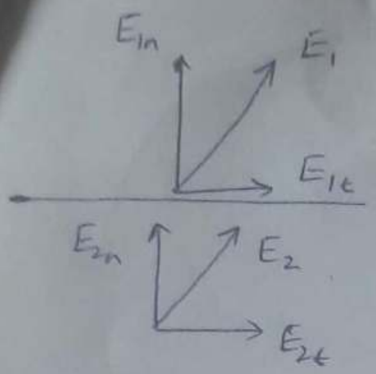
   (b) Is this the smallest or largest permissible angle for total reflection?

3. (a) Red light ($\lambda \approx 650$ nm) is incident on the surface of the water in a swimming pool. What is the wavelength λ' of the light inside the water? If you swim under water and look up at the refracted light coming from the surface, what color do you see? Justify your answer in a few words. Assume the refractive index of water is 1.33.

   (b) Using the Fresnel equations at normal incidence, show that the sum of the power in the reflected light and the power in the refracted light is the same as the power in the incident light. Thus, energy is conserved.

4. A right circularly polarized light is incident on a perfect conductor. Show that the reflected light is left circularly polarized.

BC at dielectric surface

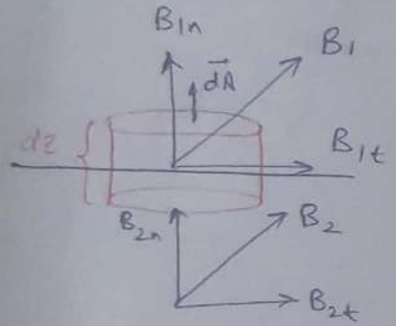


$$\begin{cases} \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{E} = \frac{\rho_s}{\epsilon} \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 J_s + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

① $\underset{\text{closed loop}}{\oint} \vec{E} \cdot \vec{dl} = -\frac{\partial \phi}{\partial t} \quad \Rightarrow \quad \boxed{E_{1tan} = E_{2tan}}$

② $\underset{\text{closed surface}}{\int} \vec{E} \cdot \vec{dA} = \frac{1}{\epsilon} \underbrace{\int \rho \, dV}_{\text{charge inside}} \quad \Rightarrow \quad \boxed{k_{e_1} E_{1n} - k_{e_2} E_{2n} = \frac{\rho_s}{\epsilon_0}}$

③ $\int \vec{B} \cdot \vec{dA} = 0$



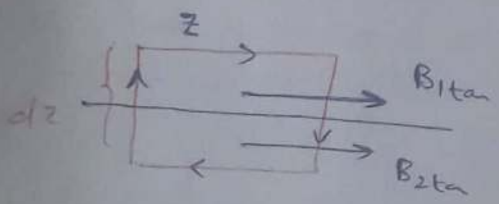
$B_{1n} \, dA - B_{2n} \, dA + \phi_{cyl} = 0$

Limits $dz \to 0$

$\boxed{B_{1n} = B_{2n}}$

④ $\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t} + \mu_0 \, i_{enclosed}$ ← current through surface Amp/meter

$\Downarrow$



$$\oint \frac{\vec{B}}{\mu_{m_1}} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{\partial \phi_n}{\partial t} + \mu_0 i_{an}$$

$$\frac{B_{1tan}}{K_{m_1}} z - \frac{B_{2tan}}{K_{m_2}} = \overset{0}{\cancel{\frac{\partial A}{\partial t}}} + \overset{0}{\cancel{\mu_0 i_{an}}}$$

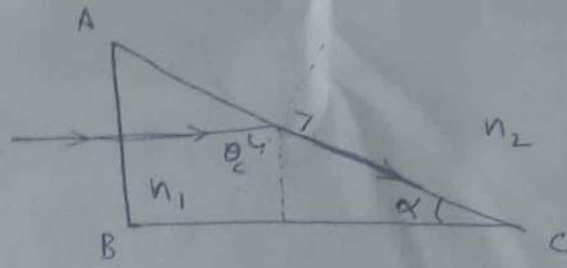
$$\boxed{\frac{B_{1tan}}{K_{m_1}} = \frac{B_{2tan}}{K_{m_2}}}$$

Tutorial-7

2 Snell's law

$n_1 \sin\theta_c = n_2 \sin\theta_2$ ($\theta_2 = 90°$)

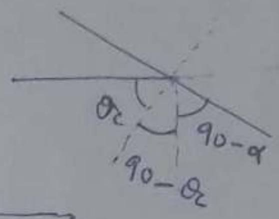
$n_1 = 1.5$, $n_2 = 1.0$



$1.5 \sin\theta_c = 1$

$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) =$

$\theta_c = \frac{\pi}{2} - \alpha_c$



$\sin\theta_c = \sin\left(\frac{\pi}{2} - \alpha_c\right) = \frac{1}{n_1}$

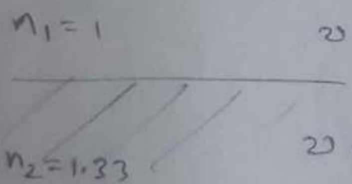
$\cos\alpha_c = \frac{1}{n_1} \Rightarrow \boxed{\alpha_c = 48.2°}$

(b) for ~~Grate~~ TIR $\theta > \theta_c$

& $\alpha = \frac{\pi}{2} - \theta \Rightarrow \alpha < \alpha_c$

Its a maximum angle & light can undergo TIR for $\alpha < \alpha_c$

Q-3 Red $\lambda = 650\,nm$



$\nu = \frac{c}{\lambda}$

$\nu = \frac{v}{\lambda'} = \frac{c}{n\lambda'}$

$\Rightarrow$ freq. remains same

$\frac{c}{\lambda} = \frac{c}{n\lambda'}$

$\Rightarrow \boxed{\lambda' = \frac{\lambda}{n}} = \frac{650}{1.33}\,nm \approx 488\,nm$

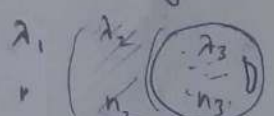
wavelength reduces

$\Rightarrow$ If we see under ~~bright~~ water, we see still red light.

(i) Freq. & therefore energy remains same.

(ii) λ when it reach retina remain sa

$\frac{\lambda_1}{n_1} = \frac{\lambda_2}{n_2} = \frac{\lambda_3}{n_3} \Rightarrow \boxed{\lambda_3 = \lambda_1\left(\frac{n_3}{n_1}\right)}$



$\lambda_2 = \frac{\lambda_1}{n}$; $\lambda_3 =$

(b) Fresnel's eq. for Normal incidence

$r_\perp = \frac{n_1 - n_2}{n_1 + n_2}$; $t_\perp = \frac{2n_1}{n_1 + n_2}$

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Power of EM wave in a medium of index n

$$P = \frac{n}{c} |E|^2$$

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}} \; W/m^2$$

Prove: $\boxed{P_i = P_r + P_t}$

$$P_r + P_t = \frac{n_1}{c} |E_r|^2 + \frac{n_2}{c} |E_t|^2$$
$$= \frac{n_1}{c} r_\perp^2 |E_i|^2 + \frac{n_2}{c} t_\perp^2 |E_i|^2$$

$r_\perp = \frac{E_r}{E_i}$

$t_\perp = \frac{E_t}{E_i}$

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$$= \frac{|E_i|^2}{c}\left[n_1 r_\perp^2 + n_2 t_\perp^2\right]$$

$$= \frac{|E_i|^2}{c}\left[n_1\left(\frac{n_1-n_2}{n_1+n_2}\right)^2 + n_2\left(\frac{2n_1}{n_1+n_2}\right)^2\right]$$

$$= \frac{|E_i|^2}{c}\left[\frac{n_1(n_1^2+n_2^2-2n_1n_2) + 4n_1^2n_2}{(n_1+n_2)^2}\right]$$

$$= \frac{|E_i|^2}{c}\left[\frac{n_1(n_1^2+n_2^2+2n_1n_2)}{(n_1+n_2)^2}\right]$$

$$= n_1\frac{|E_i|^2}{c}$$

$$\boxed{P_t + P_r = P_i}$$