

Tutorial-6 (PHY201) Due on Wednesday

1. Explain key features of the phenomenon of Rayleigh scattering of a plane EM radiation. Discuss by making careful diagram, how and when the Rayleigh scattering converts unpolarized light into a perfectly linear polarized light.

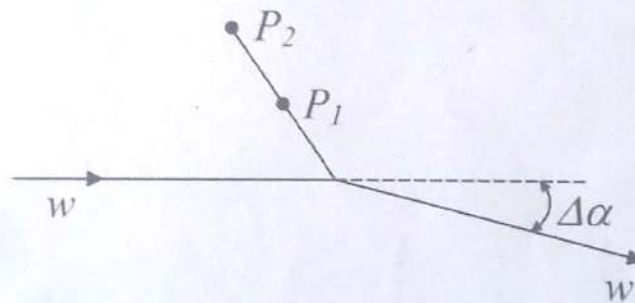
Discuss five daily life phenomena that exploit the Rayleigh scattering of EM radiation.

2. A copper box with dimensions as shown in the figure acts as a cavity resonator. The electric field

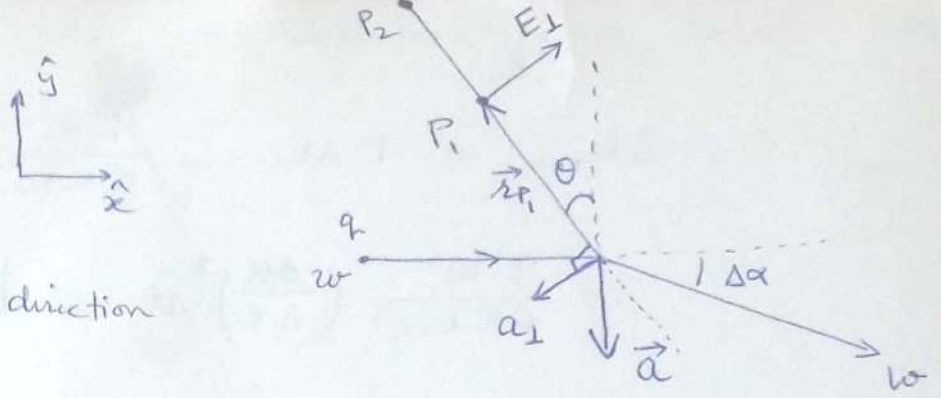
$$E_z = E_0 \sin(k_x x) \sin(k_y y) \sin(\omega t), \quad E_x = E_y = 0$$

is a possible solution of the wave equation for this case.

- (a) Find the lowest resonance frequency  $\omega_1$  and the corresponding free space wavelength  $\lambda_1$ .
  - (b) Find the next-to-lowest resonance frequency  $\omega_2$  and the corresponding free space wavelength  $\lambda_2$ .
3. A point charge  $q$  has been moving with constant velocity  $w$  along a straight line until the time  $t=t_0$ . In the short time interval from time  $t_0$  to  $t_0+\Delta t$ , a force perpendicular to the trajectory changes the direction without changing the magnitude of the velocity. After the time  $t=t_0+\Delta t$  the charge again moves with the velocity  $w$  along a straight line making a small angle  $\Delta\alpha$  with the initial trajectory.



- (a) What is the direction of E-field caused by the acceleration, at the distant point  $P_1$ .
- (b) In what direction is the radiation intensity of the accelerated charge the most intense?
- (c) Where is it least intense?
- (d) Point  $P_2$  is twice as far from the bend of trajectory as  $P_1$ . By what fraction does the amplitude of magnetic disturbance decrease as the radiation pulse move from  $P_1$  to  $P_2$ ?
- (e) What is the total energy radiated?



(a) Acceleration along  $-y$  direction

$$\vec{a} = \frac{\Delta v_y}{\Delta t} \hat{y}$$

for  $\Delta\alpha \ll 1 \Rightarrow \Delta v_y = w \sin \Delta\alpha \approx w \Delta\alpha$

$$\vec{a} \approx w \frac{\Delta\alpha}{\Delta t} \hat{y}$$

- Component  $a_{\perp}$   $\perp$  to position vector to  $\vec{r}_{P_1}$

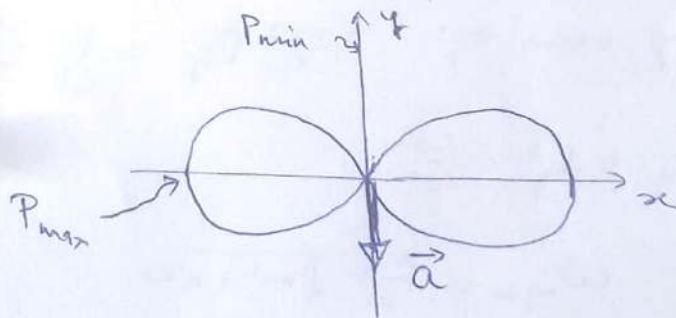
$$a_{\perp} = a_y \sin \theta = w \frac{\Delta\alpha}{\Delta t} \sin \theta$$

E-field at  $P_1$  is in the plane of  $a_{\perp}$  &  $\vec{r}_{P_1}$  & antiparallel to  $a_{\perp}$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \frac{a_{\perp}}{c^2} (\hat{r}_{P_1} \times \hat{z})$$

$$= \frac{q}{4\pi\epsilon_0 r} \frac{w \Delta\alpha \sin \theta}{c^2 \Delta t} (\cos \theta \hat{x} + \sin \theta \hat{y})$$

(b) Radiation intensity  $P \propto \sin^2 \theta$



Most intense  $xz$  plane

$P_{min} \equiv yz$ -plane

(d) Associated  $\vec{B}$ -field

$$\vec{B}(\vec{r}, t) = \hat{r} \times \frac{\vec{E}(\vec{r}, t)}{c}$$

$$\vec{B} \propto \frac{E_{\perp}}{c} \propto \frac{1}{r}$$

Amp. of B field reduces by a factor 2 at  $P_2$

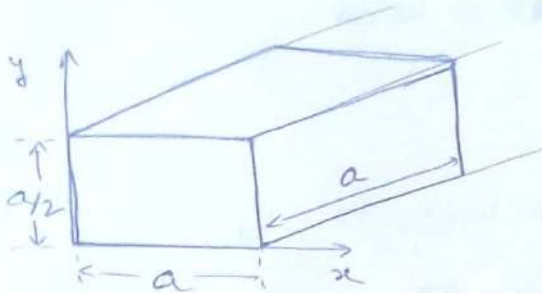
(e) Total energy radiated in  $\Delta t$  interval

$$\Delta E_{\text{rad.}} = P \Delta t = \frac{q^2 a^2 \Delta t}{6\pi \epsilon_0 c^3}$$

$$= \frac{q^2 \omega^2}{6\pi \epsilon_0 c^3} \left( \frac{\Delta x}{\Delta t} \right)^2 \Delta t$$

Radiated energy is integrated for entire sphere.

(2)



Box of dimensions

$$a \times a \times \frac{a}{2}$$

$$\vec{E} = E_0 \sin(k_x x) \sin(k_y y) \sin(k_z z)$$

Boundary condition

$$E_z(x=0) = E_z(x=a) = E_z(y=0) = E_z(y=a) = 0$$

$$k_x = n \frac{\pi}{a}, \quad k_y = m \frac{\pi}{a} \quad \& \quad k_z = 0$$

$m, n \geq 1$  & integers

Z - component of wave eq<sup>n</sup>:

$$\nabla^2 E_z = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

$$k_x^2 + k_y^2 = \frac{\omega^2}{c^2}$$

Resonance

$$\omega_{m,n} = \frac{\pi c}{a} \sqrt{m^2 + n^2}$$

(a) Lowest freq.

$$m = n = 1$$

$$\Rightarrow \omega_1 = \frac{\pi c \sqrt{2}}{a}; \quad \lambda_1 = \sqrt{2} a$$

(b) Next freq.

$$m = 1, n = 2$$

$$\text{OR } m = 2, n = 1$$

$$\omega_2 = \frac{\pi c \sqrt{5}}{a}; \quad \lambda_2 = \frac{2a}{\sqrt{5}}$$

Q-1

Tut. - 6: Solutions

Rayleigh scattering:

- (a) size of scatterer  $\ll \lambda$
- (b) Power  $\propto \omega^4 \propto \frac{1}{\lambda^4}$
- (c) for  $\vec{r} \perp$  to incoming beam light is perfectly linearly polarized for unpolarized radiation

Phenomena: Blue sky, Red sunset sunrise, blue smoke  
Scattering in water of laser, Red traffic sign etc.