Tutorial-5 (PHY201) Due on Wednsday

1. When a plane wave traverses a medium the displacement of particles is given by $y(x,t) = 0.01\sin(4\pi t - 0.02\pi x)$ where y is in meters and t is in seconds. Calculate: (i) Amplitude, wavelength, velocity and frequency, (ii) the phase difference between two positions of the same particles at a time interval of 0.25s, (iii) the phase difference between two particles 50m apart at same instant.

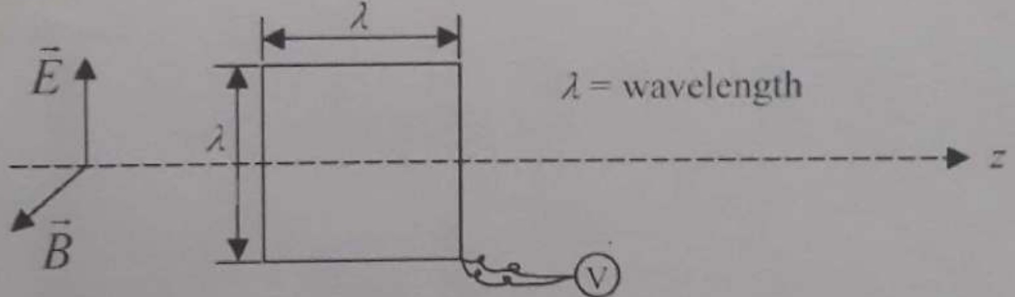
2. Assuming that all the energy from a 1000W street lamp is radiated uniformly, calculate the values of electric and magnetic fields of radiation at a distance 2m from the lamp. Explain if one can measure this Electric and Magnetic field in laboratory?

3. A pulse travelling along a stretched string is described by the following equation:

$$y(x,t) = \frac{b^3}{(2x - ut)^2 + b^2}$$

   (a) Sketch the graph of y against x at t=0
   (b) What are the speed of the pulse and its direction of travel?
   (c) The transverse velocity of a given point is of the string is defined by, $v_y = \partial y/\partial t$. Calculate it as a function of x at t=0, and show by means of a sketch what this tells us about the motion of pulse during a short time Δt.

4. The B field of a certain electromagnetic wave is given by, $\boldsymbol{B}(x,y,z,t) = B_0 \sin(\omega t - kz)\hat{x}$



   (a) Use Maxwell's equation to calculate the corresponding E field for this wave. A square sinle-turn loop of wire, with sides of length equal to λ is used to pick up signal from the wave by detecting the voltage V appearing between two ends. This will be of form $V = V_0 \sin(\omega t + \phi)$
   (b) The loop is placed as shown. With two sides parallel to **E** and the other two sides parallel to z. What is the value of V_0 in this situation?
   (c) What is the maximum possible value of V_0, and how should the loop be oriented to obtain it?

Tutorial - 5 (Solutions)

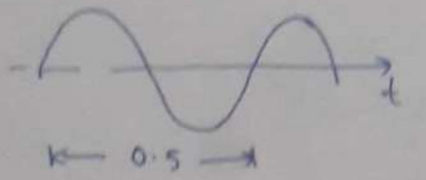
Q-1 Given $y(x,t) = 0.01 \sin(4\pi t - 0.02\pi x)$ m

$$\underset{A}{\uparrow} \qquad \underset{\omega}{\uparrow} \qquad \underset{k = \frac{2\pi}{\lambda}}{\uparrow}$$

Velocity $v = \frac{\omega}{k}$; $\frac{2\pi}{T} = 4\pi \Rightarrow T = 0.5$ sec

$\frac{2\pi}{\lambda} = 0.02\pi \Rightarrow \lambda = 100$ m

(ii)



phase diff = π

(iii) Since $\lambda = 100$ m $\Rightarrow$ phase diff = π

Q-2 Given Power = 10^3 W

Poynting vector at 2 m $S = \frac{10^3}{4\pi (2)^2}$ W/m^2

- Magnitude of E-field

$$S = \frac{1}{2} c \epsilon_0 E^2 \Rightarrow E = \sqrt{\frac{2S}{c\epsilon_0}}$$

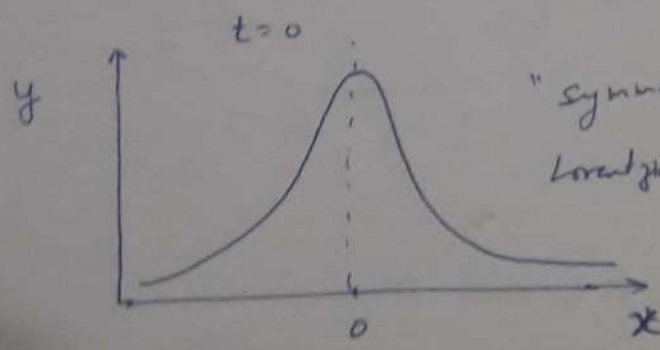
$$E = \left(\frac{2 \times 10^3 / 16\pi}{3 \times 10^8 \times 4\pi \, 10^{-7}} \right)^{1/2} \text{ V/m}$$

- IF measured in lab, the value of E will be much less because of broad spectrum of light

Q-3 Pulse shape

(a)

$$y(x,0) = \frac{b^3}{4x^2 + b^2}$$



"Symmetric" Lorentzian

(b) Compare $y(x,t) = \dfrac{b^3}{(2x-ut)^2+b^2} = f(x-vt)$ ← speed

$$= \frac{b^3/4}{(x-\frac{u}{2}t)^2+b^2/4}$$

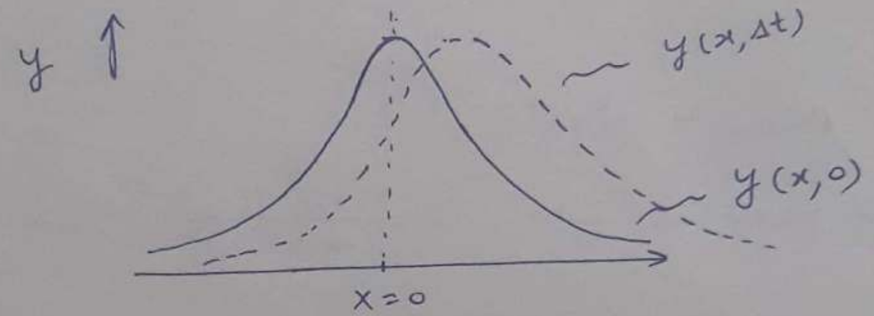
$\Rightarrow$ pulse speed $\frac{u}{2}$; travelling in $\oplus x$ direction

(c)

$$v_y(t=0) = \left.\frac{\partial y}{\partial t}\right|_{t=0}$$

$$= \left.\frac{2(2x-ut)u\,b^3}{[(2x-ut)^2+b^2]^2}\right|_{t=0}$$

$$v_y(t=0) = \frac{4b^3xu}{(b^2+4x^2)^2}$$



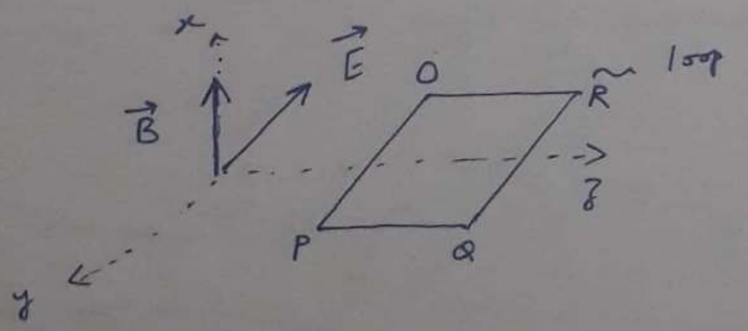
$$\frac{\partial y}{\partial t} = \frac{y(t+\Delta t)-y(t)}{\Delta t} = v_y(t)$$

$$y(\Delta t) = y(0) + v_y(t=0)\,\Delta t$$

$\boxed{Q-4}$ (a) $\vec{B} = B_0 \sin(\omega t - kz)\,\hat{x}$; $\hat{k} = z$

using $\vec{E} = -c\,\hat{k}\times\vec{B}$

$$= -cB_0\sin(\omega t-kz)\,\hat{y}$$

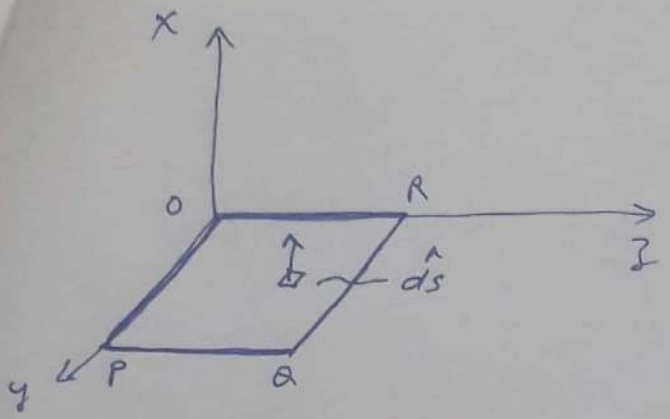


{ B - oscillates up-down
E - oscill sideways

EMF in the loop

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \oint \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

EMF in the loop :   Let origin at O



$$k = \frac{2\pi}{\lambda}$$

$$\vec{B} = B_0 \sin(\omega t - kz)\,\hat{x}$$

$$\vec{ds} = dy\,dz\,\hat{x}$$

"Choose surface enclosed by the square loop OPQRO

$$\mathcal{E} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

$$= -\frac{d}{dt}\int_S B_0 \sin(\omega t - kz)\,\hat{x}\cdot(dy\,dz\,\hat{x})$$

$$= \cancel{-B_0\,\omega} - \frac{d}{dt}\int_0^\lambda\int_0^\lambda B_0 \sin(\omega t - kz)\,dy\,dz$$

$$= -B_0\,\omega \int_0^\lambda\int_0^\lambda \cos(\omega t - kz)\,dy\,dz$$

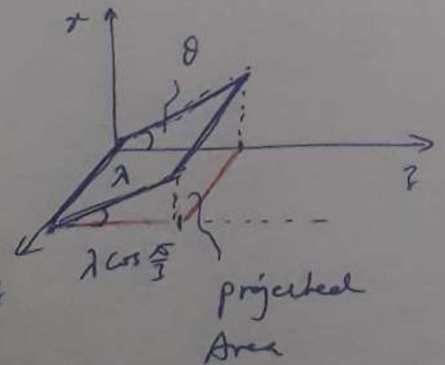
$$\mathcal{E} = -\lambda B_0 \omega \frac{\lambda}{2\pi} \sin\left(\omega t - \frac{2\pi}{\lambda} z\right)\Big|_{z=0}^{z=\lambda}$$

$$= 0$$

Alternatively, one can also compute it using E field of wave

(c) Rotating loop about z-axis ~~or about~~ leaves $\mathcal{E} = 0$

Similarly, rotating about x axis leaves $\mathcal{E} = 0$



loop should be oriented such that its projected Area on yz plane spans only half wavelength, to capture only ⊕ $\frac{\partial \vec{B}}{\partial t}$.

$\Rightarrow \theta = \frac{\pi}{3}$ as in Figure

$$\Rightarrow \mathcal{E} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{ds}$$

$$= -\frac{d}{dt}\int_0^{\lambda\cos\pi/3}\int_0^{\lambda} B_0 \cos(\omega t - kz)\,dy\,dz$$

$$= -B_0\,\omega\lambda\frac{\lambda}{\pi}\sin\left(\omega t - \frac{2\pi}{\lambda}z\right)\Big|_{z=0}^{z=\lambda\cos\pi/3} = \underbrace{2\lambda B_0 c}_{\text{Max. emf (amplitude)}}\cos(\omega t)$$

So maximum Emf

$$\mathcal{E} = \underbrace{2\lambda B_0 c}_{\mathcal{E}_0} \cos(\omega t)$$

$$\mathcal{E}_0 = 2\lambda B_0 c$$

Note: One can arrive at the same result using only $\vec{E}$ field of the wave