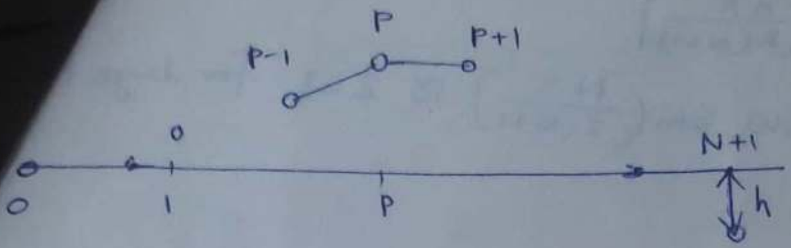
Tutorial - 3



$y_0 = 0$

$y_{N+1} = h \cos \omega t$

Eq: for p^{th} particle

$$m \ddot{y}_p = T \sin \alpha_p - T \sin \alpha_{p-1}$$

$$\boxed{\ddot{y}_p = \omega_0^2 (y_{p+1} - 2y_p + y_{p-1})} \quad \text{— (1)} \quad ; \omega_0^2 = \frac{T}{ml}$$

BC $y_0 = 0$; $y_{N+1} = h \cos \omega t$

Normal mode solution:

one-freq.

$$\begin{cases} y_p = A_p \cos(\omega t) & \text{— (2)} \\ A_p = C \sin(\alpha p) & \text{— (3)} \end{cases}$$

&

unknown

for $p = N+1 \Rightarrow A_{N+1} = h = C \sin(\alpha(N+1))$

$$\Rightarrow \boxed{C = h / \sin[\alpha(N+1)]} \quad \text{Amplitude}$$

Substituting (2) into (1)

$$\Rightarrow \frac{A_{p+1} + A_{p-1}}{A_p} = \frac{-\omega^2 + 2\omega_0^2}{\omega_0^2} \quad \text{— (4)}$$

& using eq. (3)

$$\frac{A_{p+1} + A_{p-1}}{A_p} = \frac{C\{\sin[\alpha(p-1)] + \sin[\alpha(p+1)]\}}{C \sin(\alpha p)}$$

$$= \frac{2C \sin \alpha p \cos \alpha}{C \sin \alpha p} = 2 \cos \alpha \quad \text{— (5)}$$

Equating (4) & (5) $\quad 2 \cos \alpha = -\frac{\omega^2}{\omega_0^2} + 2$

$$\Rightarrow \boxed{\cos \alpha = 1 - \frac{\omega^2}{2\omega_0^2}}$$

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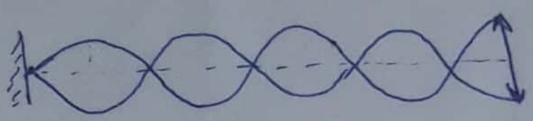
$\Rightarrow$ What happens if driving $\omega >$ highest normal mode freq ?

$$\omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right]$$

$$(\omega_n)_{max} = 2\omega_0 \sin\left(\frac{N\pi}{2(N+1)}\right) \approx 2\omega_0 \text{ for large } N$$



if $\omega > 2\omega_0$

$\Rightarrow$ No standing waves

wave damp out.

$\Rightarrow$ Normal mode amp. ~ Complex (Re & Im. part)

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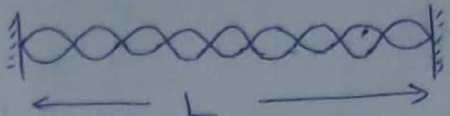
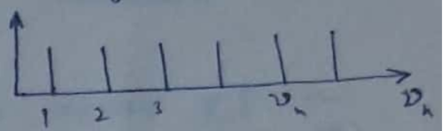
Normal modes of a cavity ⟹ Standing light waves



$n \frac{\lambda}{2} = L \; ; \; n = 1, 2, \ldots \infty$

"freq. comb"

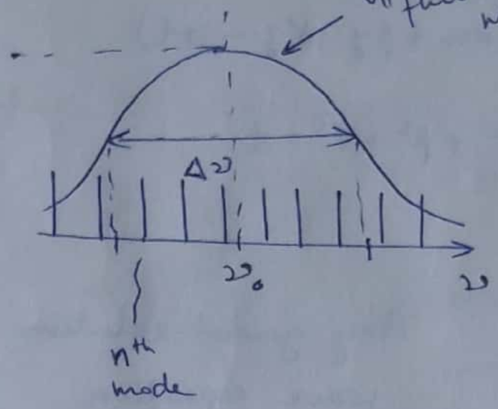
$\Rightarrow \quad \frac{1}{\lambda} = \frac{n}{2L} \quad \& \quad \boxed{\nu = \frac{c}{\lambda} = n\left(\frac{c}{2L}\right)}$



$\Rightarrow$ Normal mode spacing $\nu_{n+1} - \nu_n = \frac{c}{2L}$

(b)

gain curve "fluorescence measurement"



$\nu_0 = 5 \times 10^{14}$ Hz

$\frac{\Delta\nu}{2} = \pm 1 \times 10^{9}$

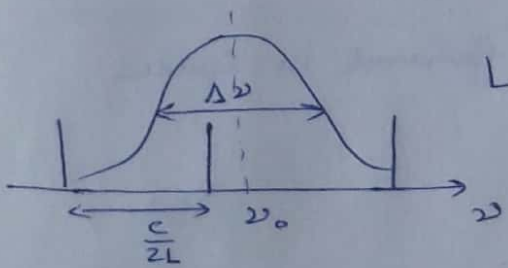
$\Delta\nu = 2 \times 10^{9}$ Hz

# of modes for $L = 1.5$ m within $\Delta\nu$

$$= \frac{\Delta\nu}{(c/2L)} = \frac{2 \times 10^9}{\frac{3 \times 10^8}{3}} \simeq 20 \text{ modes}$$

(c) For only 1-normal mode

$$\frac{c}{2L} = \Delta\nu \quad \Rightarrow \quad L = \frac{c}{2\Delta\nu} = \frac{3 \times 10^8}{2 \times 2 \times 10^9}$$



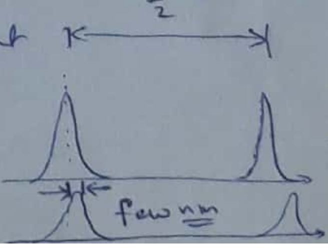
$L = \frac{3}{40}$ m. $= 7.5$ cm

Note: Principle of monochromatic laser

Application: Idea to measure < nm displacement

$\frac{\lambda}{2} \simeq 150$ nm

high Q-cavity ⟹ sharper reson



no fundamental limit.

$\boxed{10^{-15} \text{ m}}$

– doublet (yellow light)

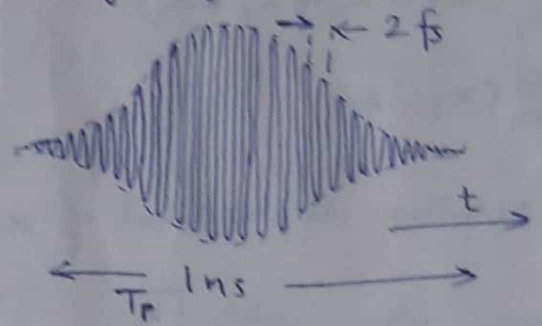
$\lambda_1 = 589\ nm$
$\lambda_2 = 589.6\ nm$ $\Rightarrow \Delta\lambda \simeq 0.6\ nm$

→ Period of one optical cycle in yellow

$$\frac{1}{T} = \nu = \frac{c}{\lambda} \Rightarrow T = \frac{\lambda}{c} = \frac{589 \times 10^{-9}}{3 \times 10^{8}}\ sec$$

$$T = \frac{589}{3} \times 10^{-17}$$
$$\simeq 1.96 \times 10^{-15}\ s \simeq 2\ fs$$

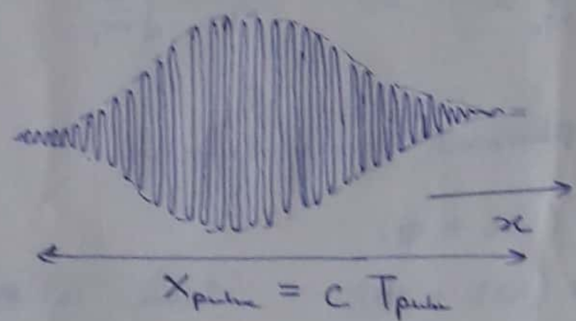
→ Shape of light pulse:



$T = 2\ fs$

Pulse width $= 1\ ns$

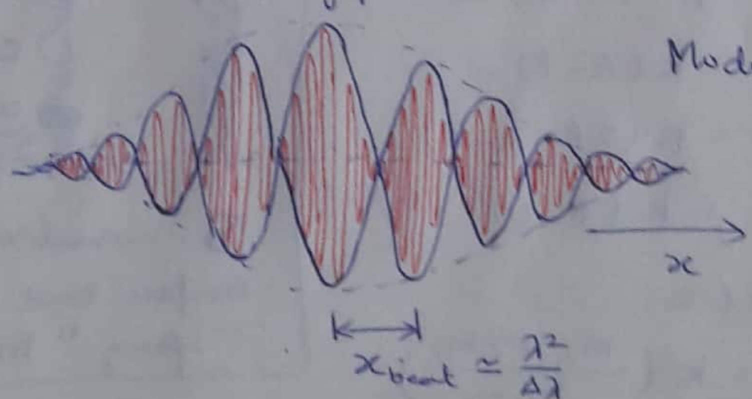
$$\#\text{ of oscill} = \frac{10^{-9}}{2 \times 10^{-15}} \sim 0.5 \times 10^{6}$$



Pulse width in space

$$X_{pulse} = 3 \times 10^{8} \times 10^{-9}\ m$$
$$= 0.3\ meter$$
$$= 30\ cm$$

(b) Composite structure of pulse



Modulated beats betn λ_1, λ_2

$$\text{Beat period in space} \simeq \frac{589 \times 589}{0.6}\ nm \simeq 0.6\ mm$$

$$\#\text{ of nodes} = \frac{X_{pulse}}{x_{beat}} = \frac{30}{0.06} \sim 500$$

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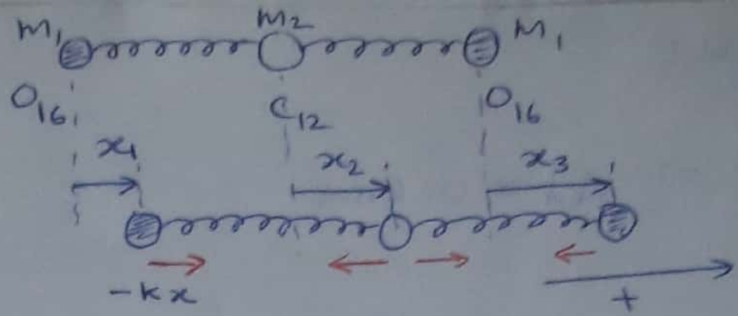
Q-2



let $x_3 > x_2 > x_1$

$$m_1 \ddot{x}_1 = k(x_2 - x_1) \quad — ①$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1) + k(x_3 - x_2) \quad — ②$$

$$m_3 \ddot{x}_3 = -k(x_3 - x_2) \quad — ③$$

Setting $m_1 = m_3$

$$(\ddot{x}_3 - \ddot{x}_1) = -\frac{k}{m_1}(x_3 - x_2 + x_2 - x_1)$$

$$= -\frac{k}{m_1}(x_3 - x_1)$$

Let $x = x_3 - x_1$ (relative displacement of x_3 & x_1

$$\Rightarrow \quad \ddot{x} = -\frac{k}{m_1}x \quad \Rightarrow \quad \omega_1^2 = \sqrt{\frac{k}{m_1}}$$

For other modes: Normal mode solution

$$\begin{cases} x_1 = A\cos(\omega t + \phi) \\ x_2 = B\cos(\omega t + \phi) \\ x_3 = C\cos(\omega t + \phi) \end{cases}$$

Same ω for all modes

- Substituting in eqⁿ ①, ②, ③

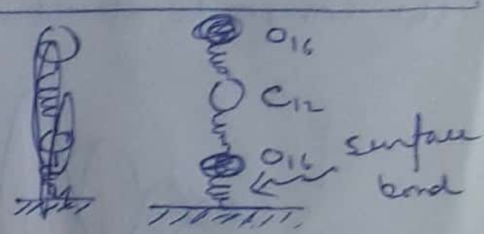
$$m_1 \omega^2 A = k(A - B)$$

$$m_2 \omega^2 B = k(2B - A - C)$$

$$m_3 \omega^2 C = k(B - C)$$

eliminating A, B & C

$$\omega_2^2 = k\left(\frac{m_2 + 2m_1}{m_1 m_2}\right)$$



3 - normal modes if surface bond is another spring " tripple pendulum

Ratio of two Normal modes $\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_2 + 2m_1}{m_2}} = \sqrt{\frac{12+32}{12}} \simeq 1.91$

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