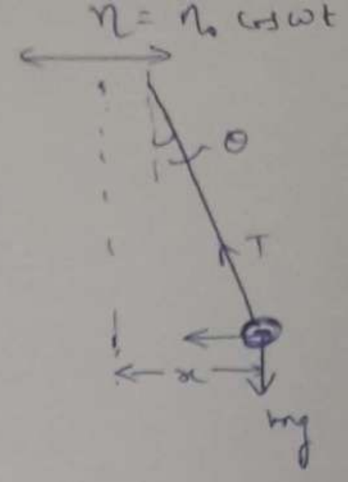
Q-1



(a) Pls follow derivation in class to arrive at

$$\ddot{x} + \omega_0^2 x + \gamma \dot{x} = \omega_0^2 \eta_0 \cos\omega t$$

(b) Steady state:

$$x = A\cos(\omega t - \delta)$$

$$A = \frac{\omega_0^2 \eta}{\sqrt{(\omega_0^2-\omega^2)^2 + (\omega\gamma)^2}}$$

$$\tan\delta = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$$

(c) At exact resonance ($\omega \simeq \omega_0$) for $Q >> 1$

$$A_{Max} = Q\eta_0$$

$$Q = N.\pi = 50 \times 3.14 = 157.0$$

$$\Rightarrow A_{max} = 157\,mm.$$

Q-2 Power dissipated against frictional force

(a) $$P = \frac{dW}{dt} = -F.v \quad \text{(work done against } bv\text{)}$$
$$= +bv^2$$

Instantaneous power $\quad P(t) = +bv^2$

(b) $$x = A\cos(\omega t - \delta)$$
$$\Rightarrow \dot{x} = -A\omega \sin(\omega t - \delta)$$

Mean power $\langle P \rangle = b\langle v^2 \rangle = b(A\omega)^2 \langle \sin^2(\,)\rangle$
$$= \frac{1}{2} b (A\omega)^2$$

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Q.3: Given $\frac{dE(t)}{dt} = P(t) = \underbrace{\frac{ke^2}{c^3}}_{C \leftarrow \text{ constant}} a^2$

(a)

$$P(t) = C\, a^2(t)$$

for damped H.O.: $E(t) = E_0 e^{-\gamma t}$

$$\Rightarrow \quad \frac{dE}{dt} = -\gamma E$$

Cycle avg. loss $\frac{d}{dt}\langle E(t)\rangle = \gamma \langle E(t)\rangle = C\langle a^2(t)\rangle$

→ for 1d SHM:

$$x(t) = x_0 \cos\omega t\,; \qquad \omega = 2\pi\nu$$

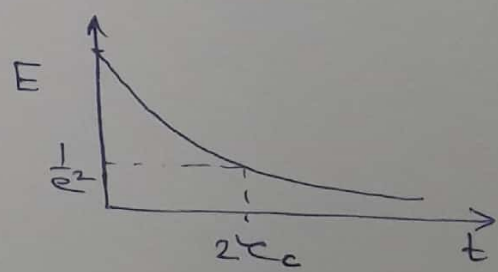
$$\ddot{x} = a(t) = -\omega_0^2 x_0 \cos\omega t$$

$$\langle a^2(t)\rangle = \omega_0^4 x_0^2 \langle \cos^2\omega t\rangle = \frac{1}{2}\omega_0^4 x_0^2$$

→ Coeff. of damping

$$\gamma = \frac{C\langle a^2(t)\rangle}{\langle E\rangle} = \frac{C\left(\frac{1}{2}\omega_0^4 x_0^2\right)}{\frac{1}{2}m\omega_0^2 x_0^2} = \frac{C\omega_0^4}{m}$$

$$\boxed{\gamma = \frac{C\omega_0^2}{m}}$$



⇒ Energy decays to $\frac{1}{e^2}E_0$ in $\tau_c = \frac{1}{\gamma}$ sec

(b) Q-factor : $Q = \frac{\omega_0}{\gamma} = \frac{m}{C\omega_0}$

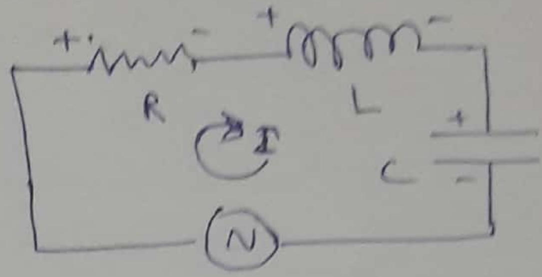
Plugging in value $Q \sim 10^5$

⇒ It is very good oscillator.

Note: Recall, this radiative loss of ~~charge~~ energy was an argument going against stability of H-atom.

Q-4

Using voltage drop across R, L, C



$V(t) = V_0 \cos \omega t$

$$IR + L\frac{dI}{dt} + \frac{Q}{C} = V_0 \cos \omega t$$

$\Rightarrow$ Use $I = \frac{dQ}{dt}$ & $\omega_0^2 = \frac{1}{LC}$

$\gamma = \frac{R}{L}$

$$\boxed{\ddot{Q} + \omega_0^2 Q + \gamma \dot{Q} = \frac{V_0}{L} \cos \omega t}$$

(b) In steady state

$$Q = Q_0 \cos(\omega t - \delta)$$

$$Q_0 = \frac{\omega_0^2 (V_0/L)}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} = \frac{V_0}{\omega\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\tan \delta = \frac{\omega\gamma}{\omega_0^2 - \omega^2} = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)$$

(c) Power delivered to the ckt by the voltage source

$$P(t) = V(t) \cdot I(t)$$

$$V(t) = V_0 \cos \omega t$$

$$I(t) = I_0 \sin(\omega t - \delta)$$

Max. power is consumed near resonance $\omega \approx \omega_0$

$$P(t) = V_0 I_0 \cos \omega t \sin(\omega t - \delta)$$

power factor

$$\langle P(t) \rangle = \frac{1}{T}\int_0^T P(t)\, dt = \frac{V_0^2}{2Z} \cos \phi \;; \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

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$$\langle P \rangle = I_{rms}^2 R = \frac{1}{2}\frac{V_0^2}{R}$$

$\Rightarrow$ All the power will be dissipated in the resistor R