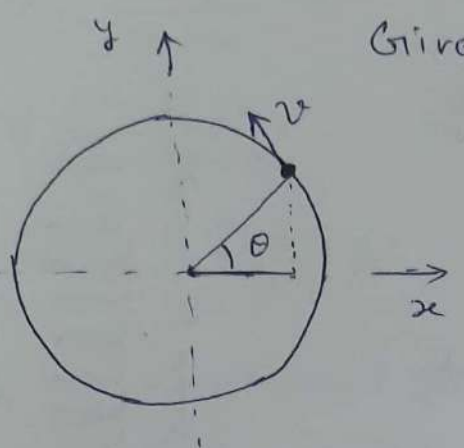
Solutions : Tut. # 1.

1



Given : $v = 50$ cm/s

$T = 6$ s

$\theta\,(t=0) = 33°$

$\omega = 60°/s$

(a) x- co-ordinate follows: $x = A\cos(\omega t + \alpha)$

Amp. ≡ Radius of circle

$$A = \frac{v}{\omega} = \frac{v}{2\pi/T} = \frac{50}{(6.28/6)} \simeq 50 \text{ cm}$$

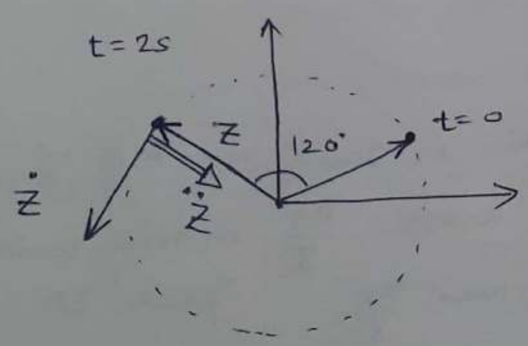
$$\omega = 2\pi/T \quad \& \quad \alpha = 30°$$

(b)
$$\left.\begin{aligned} x &= A\cos(\omega t+\alpha) \\ \dot{x} &= -A\omega\,[\sin(\omega t+\alpha)] \\ \ddot{x} &= -A\omega^2\cos(\omega t+\alpha) \end{aligned}\right\} \text{ Sose } \quad t = 2\text{ S}$$

(c) $\theta\,(t=0) = 30°$

$\theta\,(t=2s) = 30 + 120$

$= 150°$



Z - position

$\dot{Z}$ = velocity

$\ddot{Z}$ = Acceleration

Q-2 Eqⁿ. of motion

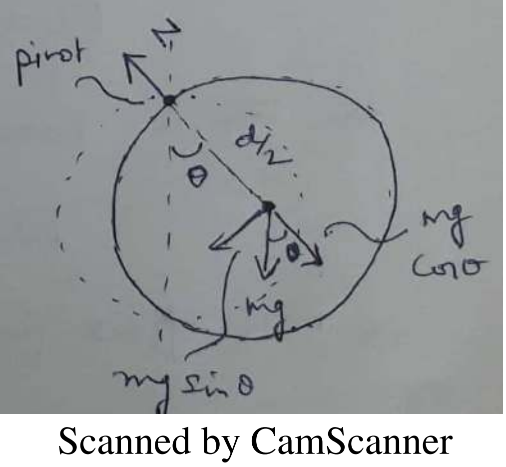
Torque $I\ddot{\theta} = -mg\sin\theta \cdot \frac{d}{2}$

$$\Rightarrow \quad \omega = \sqrt{\frac{mgd}{2I}}$$

Moment of inertia $I = I_{cm} + m\left(\frac{d}{2}\right)^2$

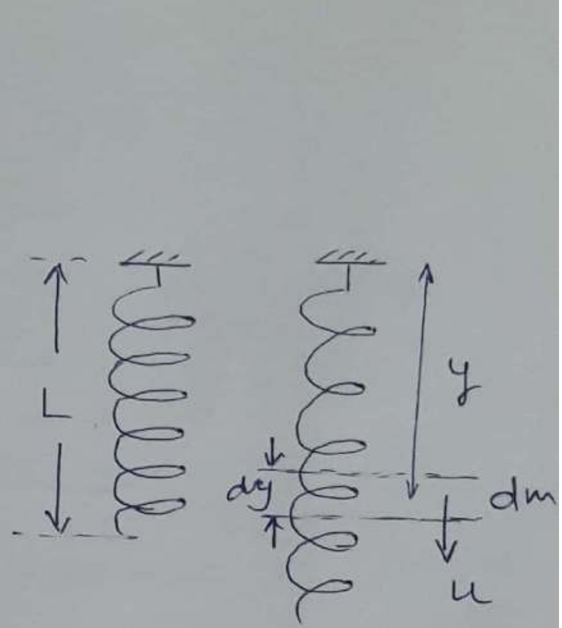
$$= m\left(\tfrac{d}{2}\right)^2 + m\left(\tfrac{d}{2}\right)^2$$

Force Balance



Using above: $\omega = \sqrt{\frac{g}{d}}$

Q-3 Consider a real spring of uniform mass densiy



- Let a mass element $\underline{dm}$ at distance y from pivot moves with u

Total KE:

$$T = \int_0^L \frac{1}{2} u^2 \, dm$$

Uniform spring $dm = \frac{dy}{L} m$

&

$u = \frac{v}{L} y$ (lower point moves faster)

$$\Rightarrow \quad T = \int_0^L \frac{1}{2} u^2 \frac{m}{L} dy$$

$$= \frac{m}{2L} \int_0^L u^2 dy$$

$$= \frac{1}{2} \frac{m}{L} \int_0^L \frac{1}{L^2} y^2 v^2 dy$$

$$\boxed{T = \frac{1}{2} \frac{m}{3} v^2}$$

This is same as KE of a mass $\frac{m}{3}$ moving with speed v

$\Rightarrow$ For real spring effective mass $\frac{m}{3}$

$$\omega_0 = \sqrt{\frac{k}{m/3}}$$

$\Rightarrow$ For a loaded real spring

$$\omega_0 = \sqrt{\frac{k}{M + m/3}}$$

$\Rightarrow$ Freq offset for $m = M/10$

$$\omega_0^2 = \frac{k}{M'}$$

$$2 \frac{\Delta\omega_0}{\omega_0} = \frac{\Delta M'}{M'}$$

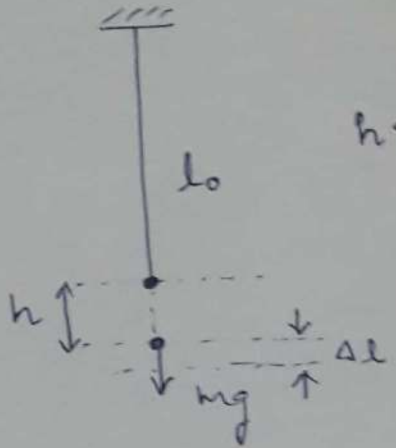
$$\Rightarrow \frac{\Delta\omega_0}{\omega_0} = \frac{1}{2}\left(\frac{0.1}{3}\right) \approx 2\%$$

Q-5

Young's mod.

$$Y = \frac{\text{stress}}{\text{strain}} \quad N m^{-2}$$

$$Y = \frac{-\Delta F / A}{\Delta l / l}$$



$h << l_0$

$$\Rightarrow \quad dF = -\left(\frac{YA}{l}\right) \Delta l$$

if $\Delta l \rightarrow$ displacement from equill.

$$m \ddot{x} = -\left(\frac{YA}{l_0}\right) x$$

$$\Rightarrow \quad \omega_0^2 = \frac{YA}{m l_0} \quad \& \quad \boxed{T = 2\pi \sqrt{\frac{l_0 m}{AY}}}$$

<u>At equillibrium:</u> $mg = \frac{YA}{l_0} h$ (permanant elongation)

$$\Rightarrow \quad \frac{mg}{AY} = \frac{h}{g}$$

$$\boxed{T = 2\pi \sqrt{\frac{h}{g}}}$$

Period same as simple pendulum of length h!

(some kind of micro-pendulum)

<u>Example:</u> IF $h = 10^{-3}$ m & $l_0 = 1$ m

$$T = 2\pi \sqrt{\frac{10^{-3}}{10}}$$

$$= 20\pi \text{ ms}$$