PHY112: ELECTRICITY & MAGNETISM LABORATORY MANUAL

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Chapter 0

Notes on Error Analysis

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0.1 Introduction

Error or uncertainty about a particular experimental measurement is the best estimate of the quantitative range within which you can trust your results. Any experimental measurement you make in the laboratory is meaningless unless quoted with an uncertainty/error. We are not talking about errors like misreading a scale or slipping a decimal point while taking a reading. Experimental uncertainties are a statement about the **resolution** of your measurement i.e. how far from the "true" value you are likely to be. There are two kinds of uncertainties associated with the measurement of an experimental quantity:

- **Random uncertainty**: associated with unpredictable variations in the experimental conditions. For example changes in room temperature, vibrations from nearby machinery, error in time period measurement when the experimenter does not start/stop the stopwatch at exactly the same point in the swing of the pendulum etc. So if a measurement is repeated a number of times with sufficient precision, a slightly different value of the measured quantity is obtained each time and if the experiment is free from bias these variations will be random and the measurements will group symmetrically about the "true" value.

- **Systematic uncertainty**: associated with inherent faults in measuring instrument or in measurement technique. This is an error that is consistent from measurement to measurement. For example, measuring length of a table with a tape that has a kink in it, a weak spring in a current meter, a calibration error in the measuring device, a clock that runs too fast etc. So if there is an experimental bias, the measurements will group around the wrong value and are said to contain a systematic error. If you always round down to the nearest tic mark on a meter stick while measuring length, you will make a systematic error of measuring a slightly shorter length.

Random uncertainties are easier to quantify and deal with. There is no general procedure for estimating the magnitude of systematic uncertainties.

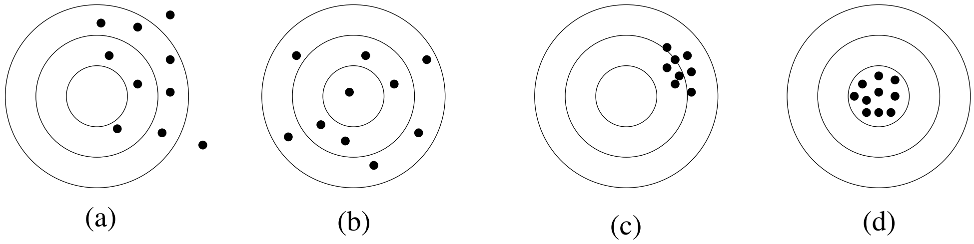


Figure 1: Random vs systematic errors.

0.2 Precision vs Accuracy

Random uncertainty decreases the **precision** of an experiment whereas **systematic** uncertainty decreases the **accuracy** of the experiment.

NOTE:- Systematic uncertainty does NOT mean that the uncertainty is repeatable. It means that the uncertainty has not been accounted for in the analysis.

Accuracy refers to the degree to which your value is correct within uncertainty. It is largely a matter of having the correct calibration of all reference measurements. If you used a uncalibrated meter stick that was shorter than the official length of a meter, you might measure the length of an object with great precision (lots of decimal places) but poor accuracy (what you think is a meter is not really a meter).

Precision can be thought of as the number of meaningful digits to a measurement. A measurement of a length as being 1.023405 meters is more precise than a measurement of 1.02 meters.

As an illustration of the concepts of precision and accuracy, consider the analogy shown in Fig. 1. The measured quantity's true value lies at the center of all circles and the various dots represent the data points measured by the same apparatus.

- In the first experiment [Fig.1(a)], the data points show very different values and are scattered over the circles. In this case, the random as well as systematic errors are large and so the *measurement is neither precise, nor accurate.*

- In the second experiment [Fig.1(b)], the random errors are large but the systematic errors are small. The uncertainty in each measurement is large, so the *measurements are accurate but not precise.*

- In the third experiment [Fig. 1(c)], the values lie within an experimental uncertainty, that is, the random errors are small but since all the measurements are away from the center, the systematic errors are large. Therefore, the *measurements are precise but not accurate.*

- In the final experiment [Fig. 1(d)], the values lie both within an experimental uncertainty and the actual value, that is, the measured value is *precise and accurate.*

If we remove the circles from Fig. 1, we do not know the true value of the quantity being measured. In this situation, we can still assess the random errors (i.e., the precision of the measured quantity) easily but it is impossible to estimate systematic errors, i.e., we do not know if our measured quantity is accurate!

0.3 Three major sources of errors

0.3.1 Reading Error

Almost all direct measurements involve reading a scale (ruler, caliper, stopwatch, analog voltmeter, etc.) or a digital display (e.g., digital multimeter or digital clock). Sources of uncertainty depend on the equipment we use. One of the unavoidable sources of errors is a reading error. Reading Error refers to the uncertainties caused by the limitations of our measuring equipment and/or our own limitations at the time of measurement (for example, our reaction time while starting or stopping a stopwatch). This does not refer to any mistakes you may make while taking the measurements. Rather it refers to the uncertainty inherent to the instrument and your own ability to minimize this uncertainty. A reading error affects the precision of the experiment. The uncertainty associated with the reading of the scale and the need to interpolate between scale markings is relatively easy to estimate. For example, consider the millimeter (mm) markings on a ruler scale. For a person with a normal vision it is reasonable to say that the length could be read to the nearest millimeter at best. Therefore, a reasonable estimate of the uncertainty in this case would be $\Delta l = \pm 0.5$ mm which is half of the smallest division. A rule of thumb for evaluating the reading error on analogue readout is to use half of the smallest division (in case of a meter stick with millimeter divisions it is 0.5 mm), but only the observer can ultimately decide what is his/her limitation in error evaluation. Note that it is wrong to assume that the uncertainty is always half of the smallest division of the scale. For example, for a person with a poor vision the uncertainty while using the same ruler might be greater than one millimeter. If the scale markings are further apart (for example, meter stick with markings 1 cm apart), one might reasonably decide that the length could be read to one-fifth or one-fourth of the smallest division. It is an estimate of systematic differences between different scales of the multimeter. However it is the random error that determines the precision, and gives you an idea of the scatter that you might expect in your readings. Thus, the "$\pm$ digit" quoted by the manufacturer might be a better estimate of the random error. Though you should quote the systematic error at the end of your experiment when you are comparing your result with some "standard", it is better to use 1 digit for the random error in each reading. For example, if your reading is 3.48 mA, you should quote (3.48 ± 0.01) mA. It is usually difficult or impossible to reduce the inherent reading error in an instrument. In some cases (usually those in which the reading error of the instrument approximates a "random error distribution") it is possible to reduce the reading error by repeating measurements of exactly the same quantity and averaging them.

0.3.2 Random Error

Random Error refers to the spread in the values of a physical quantity from one measurement of the quantity to the next, caused by random fluctuations in the measured value. For example, in repeating measurements of the time taken for a ball to fall through a given height, the varying initial conditions, random fluctuations in air motion, the variation of your reaction time in starting and stopping a watch, etc., will lead to a significant spread in the times obtained. This type of error also affects the precision of the experiment.

0.3.3 Systematic Error & Instrument Calibration

Systematic Error refers to an error which is present for every measurement of a given quantity; it may be caused by a bias on the part of the experimenter, a miscalibrated or even faulty measuring instrument, etc. Systematic errors affect the accuracy of the experiment. After evaluating the reading error or the standard error, or both if necessary, we have to make sure that the scale of our measuring instrument is checked against an internationally established measuring standard. Such comparison is called calibration. In the real world, we frequently find that our measuring scale is in slight disagreement with the standard. For example, if you inspect such simple tools as rulers, you will find out that no two rulers are exactly the same. It is not uncommon to find a discrepancy of 1 mm or even more among meter sticks. The correct calibration of measuring instruments is obviously of great importance. However, in the first year laboratory, the instruments you will use are usually calibrated by the laboratory staff and ready to use (unless explicit lab instructions tell you otherwise). In addition to all the errors discussed above, there can be other sources of error that may pass unnoticed: variations in temperature, humidity or air pressure, etc. Such disturbances are more or less constant during our measurements (otherwise they would appear as random error when the measurement is repeated) and are generally referred to as the systematic errors. Systematic errors are very difficult to trace since we do not know where to look for them. It is important to learn to notice all the irregularities that could become the sources of systematic errors during our experimental work. Moreover, it is particularly important in data-taking always to record some information about the surrounding physical conditions. Such information may help us later on if we discover a serious discrepancy in our experimental results. As a rule, the place, date and time of measurements, and the type and serial numbers and specifications of the instruments which were used must be recorded. Estimate all your reading errors while you take your data and write them down with your data. Do the same for all manufacturers' error specifications. These usually cannot be guessed later on.

0.4 Mean & Standard Deviation

Mean

If the sources of error in a measurement (say measuring the length of a table) are random, the values of the length will vary randomly above and below the "true" value of the table length, and will not be biased/skewed toward the lower/higher values. The procedure to get the most precise value for the length is to take the average or arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + ... + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1}$$

where N is the number of measurements and x_i is the value of one measurement. This definition of mean assumes that each measurement of x is independent and has the same experimental uncertainty.

Standard Deviation

Now that the mean ("best" value) is known, it is important to quantify how much the individual measurements are scattered about the mean or how "good" each individual measurement is. If the experiment is precise, all measurements will be very close to the mean value. So the extent of scatter about the mean is a measure of the precision and a way to quantify the **random uncertainty**.

For unbiased measurements (all data points have equal weights), the standard deviation σ is

$$\sigma = \sqrt{\frac{1}{(N-1)}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{2}$$

σ becomes larger if the data is more scattered about the mean.
NOTE:- Convince yourself at this stage that more scatter of data means a larger standard deviation and also that σ has the same units as x_i.

Most Probable Value:

For unbiased measurements, the standard deviation of the *mean of a set of measurements*, σ_m, is

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}} = \frac{\sigma}{\sqrt{N}}. \tag{3}$$

This is important since it states that the uncertainty in the **mean** of N measurements decreases as $1/\sqrt{N}$.
NOTE:- Convince yourself that σ_m is necessarily smaller than σ. Also think about the difference between σ and σ_m: σ is the standard deviation associated with individual data points whereas σ_m is the standard deviation of *the mean value of a set of data points*, that is, the uncertainty of a set of measurements made under identical conditions.
EXERCISE:- For a Gaussian distribution, convince yourself that the mean will be within the range $\bar{x_i} \pm \sigma_i$ 68% of the time, i.e., if another set of N measurements is made, the mean of this new set has a 68% likelihood of being within the range $\bar{x_i} \pm \sigma_i$.

Random errors and Gaussian distributions

In some measurements, there is a random element involved. Say that you measure the fraction of times that a coin lands face up. You might refuse to make the measurement, saying that you know the answer: its going to land face up exactly 50% of the time. What if you make two measurements? If you flip the coin twice, do you expect it to land face up once, and face down once, every time you flip it twice? Of course not! Since each flip of the coin is uncorrelated with the previous flip (the coin has no reason to remember how it landed last time), there is an intrinsic measurement error which we can approximate as being equal to the square root of the number of events $\sqrt{N}$. If we flip a coin $N = 100$ times, we would expect to have $\mu = 50$ heads. About 2/3 of the time we will find that the number of heads we get is within the range $50 - \sqrt{50} \approx 43$ and $50 + \sqrt{50} \approx 57$, and 1/3 outside this range. In the continuum limit, we expect to get something like a Gaussian (or the normal) distribution of obtaining heads x:

$$P(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \tag{4}$$

about a mean value μ, with standard deviation σ. These two quantities completely define the Gaussian (or the normal) distribution.

In Fig. 2(a) to 2(d), we have plotted the probability distribution of obtaining heads in a coin tossing experiment (consisting of $N = 100$ tosses per trial) when the experiment is repeated $M = 10^2, 10^3, 10^4$, and 10^6 times, respectively. The average values of heads, μ, and the standard deviation, σ, for each case is reported in the brackets. We can clearly see that μ approaches the value $N/2 = 50$ as the number of trials M increases.

0.5 Stating your results: Absolute & Relative Uncertainty

In general, the result of any measurement of physical quantity must include both the value itself (best value) and its error (uncertainty). The result is usually quoted in the form

$$x = x_{best} \pm \Delta x \tag{5}$$

where x_{best} is the best estimate of what we believe is a true value of the physical quantity and Δx is the estimate of absolute error (uncertainty). Note that depending on the type of the experiment the prevailing error could be random or reading error. In case the reading error and random error are comparable in value, both should be taken into account and treated as two independent errors. You will learn how to calculate Δx in this case in the "Propagation of Errors" section. The meaning of the uncertainty Δx is that the true value of x probably lies between $(x_{best}\Delta x)$ and $(x_{best} + \Delta x)$. It is certainly possible that the correct value lies slightly outside this range. Note that your measurement can be regarded as satisfactory even if the accepted value lies slightly outside the estimated range of the measured value.

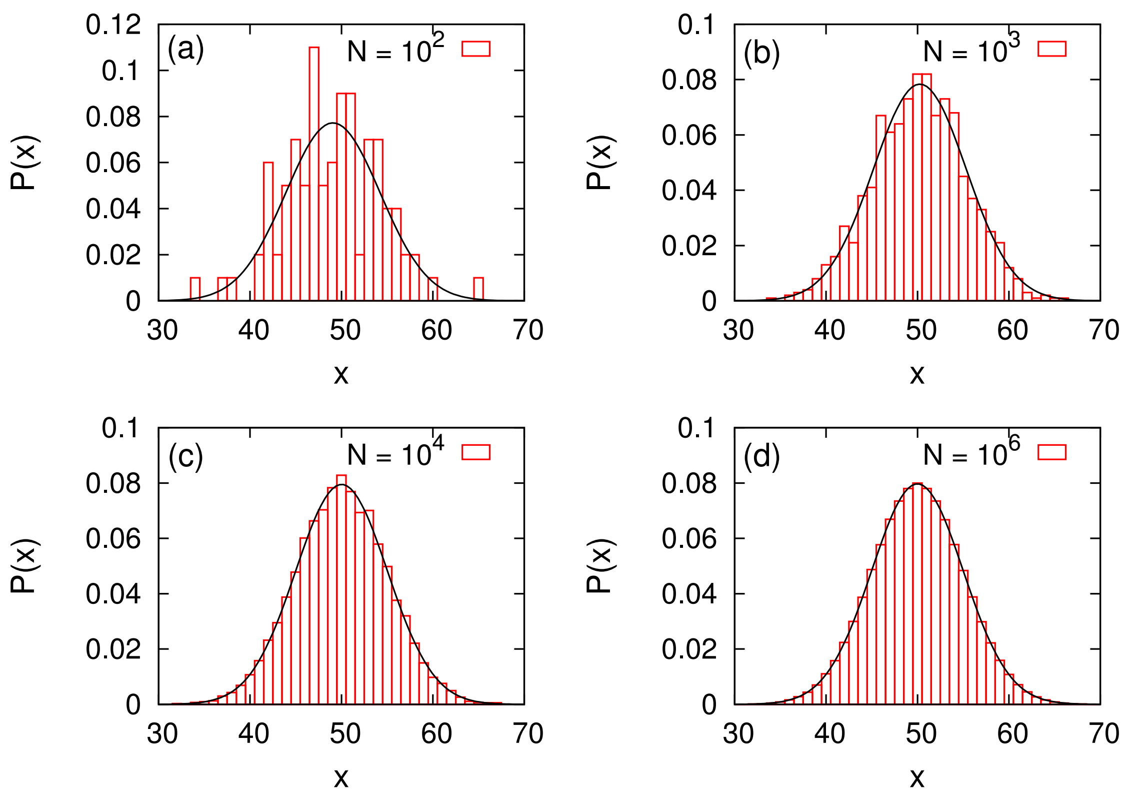


Figure 2: Probability distribution $P(x)$ of obtaining heads in a coin tossing experiment consisting of $N = 100$ tosses per trial. The histograms are the experimental data and the solid curve is the Gaussian fit, written in the brackets, to the data for (a) $M = 100$ trials ($\mu = 49.1 \pm 0.4$, $\sigma = 5.2 \pm 0.3$), (b) For $M = 10^3$ trials ($\mu = 50.3 \pm 0.1$, $\sigma = 5.09 \pm 0.08$) (c) For $M = 10^4$ trials ($\mu = 50.01 \pm 0.03$, $\sigma = 5.02 \pm 0.03$), (d) For $N = 10^6$ trials ($\mu = 49.996 \pm 0.003$, $\sigma = 5.005 \pm 0.002$).

Δx indicates the reliability of the measurement, but the quality of the measurement also depends on the value of x_{best}. For example, an uncertainty of 1 cm in a distance of 1 km would indicate an unusually precise measurement, whereas the same uncertainty of 1 cm in a distance of 10 cm would result in a crude estimate. Fractional uncertainty gives us an indication how reliable our experiment is. Fractional uncertainty is defined as $\Delta x / x_{best}$ where Δx is the absolute uncertainty. Fractional uncertainty can be also represented in percentile form $(\Delta x/x)100\%$. For example, the length $l = (0.50 \pm 0.01)$m has a best fractional uncertainty of $0.01/0.5 = 0.02$ and a percentage uncertainty of $0.02100 = 2\%$. Note that the fractional uncertainty is a dimensionless quantity. Fractional uncertainties of about 10% or so are usually characteristic of rather rough measurements. Fractional uncertainties of 1 or 2% indicate fairly accurate measurements. Fractional uncertainties much less than 1% are not easy to achieve, and are rare in an introductory physics laboratory.

Percentage disagreement: In some cases, you can compare the value of your experimental measurement with the standard value as

$$|\frac{x_{std} - x_{exp}}{x_{std}}| \times 100\% \tag{6}$$

If your percentage disagreement is more than ten percent, identify the reasons and explain why this is so in your report.

NOTE:- This percentage disagreement is to give you an idea of the accuracy of your experiment and in no case is to be used as a substitute for the detailed error analysis of your experiment.

0.6 Significant Figures

An uncertainty should not be stated with too much precision. The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty. For example, the answer 92.81 s with an uncertainty of 0.3 s should be rounded as (92.8 ± 0.3) s. If the uncertainty is 3 s, then the result is reported as (93 ± 3) s. However, the number of significant figures used in the calculation of the uncertainty should generally be kept with one more significant figure than the appropriate number of significant figures in order to reduce the inaccuracies introduced by rounding off numbers. After the calculations, the final answer should be rounded off to remove this extra figure.

- The uncertainty σ should have 1 digit or at most 2 digits (all uncertainty calculations are estimates; there is no such thing as exact uncertainty!). The result itself should be stated to the same precision as σ, for example, $10.25 \pm 0.15 sec$ or $10.3 \pm 0.2 sec$ but NOT $10.25 \pm 0.2 sec$.
- If σ is very large, you will lose significant digits. If the measurement is so bad that σ is larger than the value itself, you will have no significant digits but only know the order of magnitude!

0.6.1 Practical Hints

So far, we have found two different errors that affect the precision of a directly measured quantity: the reading error and the standard error. Which one is the actual error of precision in the quantity? For practical purposes you can use the following criterion. Take one reading of the quantity to be measured, and make your best estimate of the reading error. Then repeat the measurement a few times. If the spread in the values you obtain is about the same size as the reading error or less, use the reading error. If the spread in values is greater than the reading error, take three or four more, and calculate a standard error and use it as the error. In cases where you have both a reading error and a standard error, choose the larger of the two as "the" error. Be aware that if the dominant source of error is the reading error, taking multiple measurements will not improve the precision.

0.6.2 Mistakes and Misconceptions

In the introductory physics laboratory, it is almost always meaningless to specify the error to more than two significant digits; often one is enough. It is a mistake to write: $x = (56.7 \pm 0.914606)$ cm, or $x = (56.74057 \pm 0.9)$ cm. Instead, write: $x = (56.7 \pm 0.9)$ cm. You cannot increase either the accuracy or precision by extending the number of digits in your mean value beyond the decimal place occupied by the error. Keep in mind that the error, by its nature, denotes the uncertainty in the last one or two significant digits of the main number and therefore any additional digits obtained from multiplication or division should be rounded off at the meaningful position. So, first calculate your error; round it off to one significant figure; then quote the value of your measurement to the appropriate number of significant figures.

When quoting errors in a result do not use the flawed logic that "my result is x, the handbook gives a value for this quantity as y, thus the error in my result is $\pm(x-y)$". Your quoted error should be the result of your own analysis of your own experiment whereas $(x - y)$ relates to a comparison of your work to other people's work. (xy) represents the difference between your result and the accepted value. The discrepancy can be used to characterize the consistency between different sets of measurements, but has nothing to do with the estimate of error in your own experiment. If a result we produce differs significantly from the accepted value, we then are obligated to explain what has produced the difference. But in quoting our own result, we must provide the error of our own experiment.

0.7 Propagation of Errors

In the majority of experiments the quantity of interest is not measured directly, but must be calculated from other quantities. Such measurements are called indirect. The quantities measured directly are not exact and have errors associated with them. While we calculate the parameter of interest from the directly measured values, it is said that the errors of the direct measurements propagate. Errors can propagate in measurements. What happens to the final uncertainty in a measurement which depends on several variables, each with its own uncertainty? The answer is not obvious and two cases are possible: when the uncertainties in the individual variables are **independent** and when the individual uncertainties are **dependent**. In this lab, you will work with the assumption that the individual uncertainties are completely independent.

As an example, consider the following problem. Suppose we have measured the value of a quantity x with an uncertainty, which we denote Δx. In order to test a theoretical formula, suppose that we need to calculate y as function of x i.e., $y = f(x)$. We want to know the uncertainty in y due to the uncertainty in the value of x. This is equivalent to asking what will be the variation in y (call it Δy) as x varies from x to $(x + \Delta x)$? Mathematically, this variation is given by $\Delta y = f(x + \Delta x) - f(x)$. The answer comes from the differential calculus: if $y = f(x)$ and Δx is small, then

$$\Delta y \approx \frac{dy}{dx}\Delta x = \frac{df}{dx}\Delta x \tag{7}$$

This argument can be extended for the calculation of quantities that are functions of several different measured quantities. All you will need at this point are the results that you can find below for different types of functions. Note that we neglect the sign in the differential, since the sign of all errors may take on numerical values which are either positive or negative.

0.7.1 Propagation of Independent Errors

Suppose various quantities $x_1, \cdots, x_n, w_1, \cdots, w_n$ with uncertainties $\Delta x_1, \cdots, \Delta x_n, \Delta w_1, \cdots, \Delta u$ are used to calculate a quantity y. The uncertainties in $x_1, \cdots, x_n, w_1, \cdots, w_n$ *propagate* through the calculation to cause an uncertainty in y, provided all errors are independent and random, as follows:

Sums and Differences: If

$$y = x_1 + \cdots + x_n - (w_1 + \cdots + w_n),$$

then

$$\Delta y = \sqrt{(\Delta x_1)^2 + \cdots + (\Delta x_n)^2 + (\Delta w_1)^2 \cdots + (\Delta w_n)^2}. \tag{8}$$

Product and Quotients: If

$$y = \frac{x_1 \times \cdots \times x_n}{w_1 \times \cdots \times w_n},$$

then

$$\frac{\Delta y}{|y|} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \cdots + \left(\frac{\Delta x_n}{x_n}\right)^2 + \left(\frac{\Delta w_1}{w_1}\right)^2 + \cdots + \left(\frac{\Delta w_n}{w_n}\right)^2}. \tag{9}$$

Measured Quantity Times Exact Number: If A is known exactly and

$$y = Ax,$$

then

$$\Delta y = |A|\Delta x \qquad \text{or, equivalently,} \qquad \frac{\Delta y}{|y|} = \frac{\Delta x}{|x|}. \tag{10}$$

Uncertainty in a Power: If n is an exact number and

$$y = x^n,$$

then

$$\frac{\Delta y}{|y|} = |n|\frac{\Delta x}{|x|}. \tag{11}$$

Uncertainty in a Function of One Variable: If $y = f(x)$ is any function of x, then

$$\Delta y = \left|\frac{df}{dx}\right| \Delta x.$$

If $f(x)$ is a complicated function, then instead of differentiating $f(x)$, one can use an equivalent formula

$$\Delta y = |f(x_{best} + \Delta x) - f(x_{best})|. \tag{12}$$

General Formula for Error Propagation: If $u = f(x, y, z, \ldots)$ is a function of several variables with the independent variables $x, y, z, \ldots$ having independent and random uncertainties $\Delta x, \Delta y, \Delta z$.... The uncertainty in u is then given by the formula

$$\Delta u = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2 + \cdots}, \tag{13}$$

where the partial derivatives are all evaluated at the best known values of $x, y, z, \ldots$.
NOTE:- This formula is based on a first-order Taylor series expansion of a function of many variables and is valid when the individual uncertainties Δx_i's are uncorrelated with each other and are small compared to the values of the quantities. The first-order Taylor series expansion of any function f at x_0 is given by:

$$f(x - x_0) \approx f(x_0) + (x - x_0)\frac{d}{dx}f(x)|_{x=x_0}. \tag{14}$$

0.7.2 Exercises

Write out the error propagation formula (in terms of $\Delta f/f$) when the function $f(x, y)$ is of the form:

1. $f = x * y$
2. $f = x/y$
3. $f = x + y$
4. $f = x - y$
5. $f = x^m y^n$
6. $f = kx$ (k is constant)
7. $f = \ln_e x$
8. $f = \log_{10} x$
9. $f = e^x$

0.8 Fitting Data: Least Squares Regression

Frequently in the lab you will perform a series of measurements of a quantity y at different values of x. This gives a more accurate determination of a physical parameter rather than a single measurement. If you have a linear relationship $y = mx + b$, you can determine the uncertainty in the measured slope m and the intercept b.

A common method to find the best curve to fit a set of data points is the "method of least squares". If all the data points have nearly the same weight/error, one can try to arrange the curve so that as many points lie below the line as above. However, such a visual method is not quantitative.

The least-squares method of curve fitting can be described qualitatively as follows: Let the data set be represented by the functional form $f(x; a, b, \ldots)$ where $a, b, ..$ are adjustable parameters that can be varied to get the best fit curve. The function, f, can be a straight line ($f(x) = mx + b$ where the adjustable parameters are m and b) or a higher order polynomial or any other complicated function. For each data point (x_i, y_i), the value $y_i - f(x_i; a, b...)$ is computed and then the "chi-square" value χ^2 is calculated from the expression

$$\chi^2(a,b,\ldots) = \sum_i \frac{[y_i - f(x_i;a,b,\ldots)]^2}{\sigma_i^2}, \tag{15}$$

where σ_i is the uncertainty of each data point. The best fit is found by adjusting the parameters $a, b, \ldots$ until the minimum value of χ^2 is achieved. For N data points and n adjustable parameters, the "reduced chi-square" can be calculated from

$$\chi_\nu^2 = \frac{\chi^2}{\nu} = \frac{\chi^2}{N-n}, \tag{16}$$

where ν is the "degrees of freedom" in the problem. If the parameters are adjusted so that $\chi_\nu^2 \approx 1$, a "good fit" is achieved i.e. the difference between the fitted curve and the data is on an average, as big as the uncertainty in the data itself.

0.8.1 Fitting to a straight line

As an example of the least squares method, consider the problem of fitting of a set of N data points (x_i, y_i) to a straight line $f(x) \equiv y = mx + c$. It is assumed that the uncertainty σ_i associated with each measurement y_i is known, and the values of the dependent variable x_i's are exactly known. The chi-square merit function given by Eq.(15) for this case is

$$\chi^2(m,c) = \sum_{i=1}^{N} \frac{(y_i - mx_i - c)^2}{\sigma_i^2}. \tag{17}$$

To determine the parameters m and c, we need to minimize $\chi^2(m,c)$. At its minimum, the derivatives of $\chi^2(m,c)$ with respect to m and c vanishes:

$$\frac{\partial \chi^2}{\partial m} = -2\sum_{i=1}^{N} \frac{(y_i - mx_i - c)\, x_i}{\sigma_i^2} = 0, \tag{18a}$$

and

$$\frac{\partial \chi^2}{\partial c} = -2\sum_{i=1}^{N} \frac{y_i - mx_i - c}{\sigma_i^2} = 0. \tag{18b}$$

Define,

$$w_i \equiv \frac{1}{\sigma_i^2}; \qquad S \equiv \sum_{i=1}^{N} w_i; \qquad S_x \equiv \sum_{i=1}^{N} w_i x_i; \qquad S_y \equiv \sum_{i=1}^{N} w_i y_i; \tag{19}$$

$$S_{xx} \equiv \sum_{i=1}^{N} w_i x_i^2; \qquad S_{xy} \equiv \sum_{i=1}^{N} w_i x_i y_i.$$

With the above definition, the above equations can be rewritten as simultaneous equations for m and c:

$$cS + mS_x = S_y, \tag{20a}$$

and

$$cS_x + mS_{xx} = S_{xy}. \tag{20b}$$

The solution of these two equations in two unknowns is calculated as

$$m = \frac{SS_{xy} - S_x S_y}{\Delta}, \tag{21a}$$

and

$$c = \frac{S_{xx}S_y - S_x S_{xy}}{\Delta}, \tag{21b}$$

with

$$\Delta \equiv SS_{xx} - (S_x)^2. \tag{21c}$$

This gives the best fit values of the parameters m and c. The next task is the estimation of the probable uncertainties in the estimates of m and c, which is introduced by the measurement errors in the data. If the data are independent, then each contributes its own bit of uncertainty to the parameters. Recall from the propagation of error section [Equation (13)] that the standard deviation σ_f in the value of any function f will be

$$\sigma_f = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \left(\frac{\partial f}{\partial y_i}\right)^2}. \tag{22}$$

For the straight line, the derivatives of m and c with respect to y_i can be directly evaluated from the solution:

$$\begin{aligned} \frac{\partial m}{\partial y_i} &= \frac{S_{xx} - S_x x_i}{\sigma_i^2 \Delta} \\ \frac{\partial c}{\partial y_i} &= \frac{Sx_i - S_x}{\sigma_i^2 \Delta}. \end{aligned} \tag{23}$$

Substituting these in Eq. (22) and summing over the points we get the standard deviations

$$\sigma_m = \sqrt{\frac{S_{xx}}{\Delta}} \qquad \text{and} \qquad \sigma_c = \sqrt{\frac{S}{\Delta}}, \tag{24}$$

in the estimates of m and c respectively.

If we assume that the uncertainties in y have the same magnitude σ_y for all the data points, then the above equations remain valid with $w_i = 1/\sigma_y^2$. For this case, the above equations take the form

$$
\begin{aligned}
m &= \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\Delta} \\
c &= \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta} \\
\Delta &= N\sum x_i^2 - \left(\sum x_i\right)^2.
\end{aligned}
\tag{25}
$$

The standard deviation in m and c is given by

$$
\sigma_c = \sigma_y\sqrt{\frac{\sum x_i^2}{\Delta}} \qquad \sigma_m = \sigma_y\sqrt{\frac{N}{\Delta}}, \tag{26}
$$

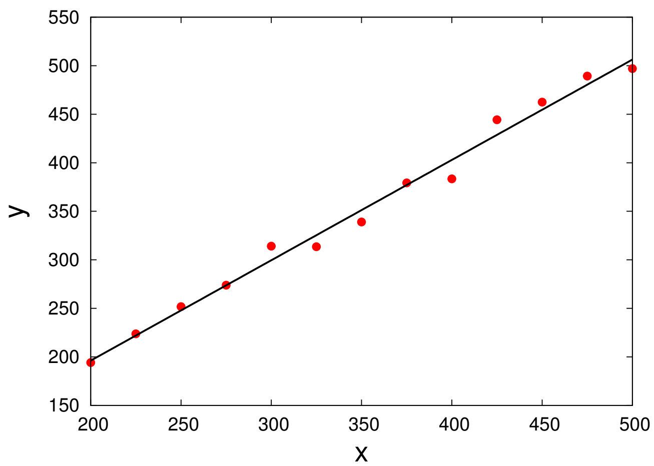
where, the uncertainty σ_y in the numbers $y_1, \ldots, y_N$ can be estimated by

$$
\sigma_y = \sqrt{\frac{1}{(N-2)}\sum_{i=1}^{N}(y_i - mx_i - c)^2}, \tag{27}
$$

assuming that the deviations $(y_i - mx_i - c)$ are normally distributed.

Example: Let us fit a straight line to a set of data (shown below) that is obtained by an arbitrary experiment. On the right hand side the data and the best straight line fit is plotted.

x	**y**
200.0	194.18
225.0	223.81
250.0	251.71
275.0	273.84
300.0	314.08
325.0	313.51
350.0	339.00
375.0	379.26
400.0	383.45
425.0	444.31
450.0	462.51
475.0	489.33
500.0	497.00



Step by step procedure to fit a straight line to a data set:

1. The data looks linear so we can try fitting a straight line $y = mx + c$ to it.
2. The above table does not mention the uncertainties of individual data points so we can assume that the uncertainties in y's have the same magnitude σ_y, which needs to be calculated.

3. To calculate m and c, we need to calculate the following sums

$$\sum x_i = 4550.0 \qquad \sum x_i^2 = 1706250.0 \qquad \sum y_i = 4565.99 \qquad \sum x_i y_i = 1715687.0$$

4. Using these values in Eq. (25), we get

$$\Delta = 1478750.0 \qquad m = 1.03 \qquad c = -10.59$$

5. Next we need to calculate the uncertainties in the constants m and c. We first calculate the uncertainty σ_y in y's by using Eq. (27). For the above set of data we get $\sigma_y = 11.19$. The uncertainties $\sigma_m = 0.03$ and $\sigma_c = 12.02$ in constant m and c respectively can then be calculated from Eq. (26).

Result: The slope m and the intercept c of the best fitted straight line to the above data is

$$m = 1.03 \pm 0.03 \qquad c = -10.59 \pm 12.02.$$

IMPORTANT NOTE:- You are expected to plot your data and do least squares analysis to find the best fit to your data and also estimate the goodness of fit. You may use **gnuplot** or other standard computer programs to find the best fit parameters and also the uncertainties in the parameters. Use the values of σ generated by the computer program in your analysis of error propagation in your experiment. LAB REPORTS WHICH DO NOT INCLUDE AN ANALYSIS OF ERRORS WILL NOT BE GIVEN A FULL GRADE.

References

1. Practical Physics, Third edition, by G. L. Squires, Cambridge University Press (1999).

2. An Introduction to Error Analysis, Second edition, by J. R. Taylor, University Science Books (1997).

Experiment 1

Terminal velocity of a magnet through a metal pipe

Manish Pareek, Prof. Arvind and Dr.Paramdeep Singh

1.1 Introduction

In this experiment, we study the motion of a magnet falling down a metal pipe, versus a plastic pipe. When a magnet falls through a plastic pipe, it falls under the action of gravity. However, when it falls through a metal pipe, in addition to gravity, it experiences an upward force due to electromagnetic induction. The force is velocity dependent and increases as the magnet speeds up, leading to a final terminal velocity of the magnet. We use small coils wound outside the pipe as sensors to locate the position of the magnet. The experiment can be used to measure acceleration due to gravity g when a plastic pipe is used and for the metal pipe, it can be used to study the Lenz's law. The experiment can also be extended to measure the conductivity of the material of the metallic pipe.

1.2 The Experiment

1.2.1 Aim of Experiment

1. To measure the velocity of a freely falling magnet through a tube fitted with sensors (sensor tube).
2. To find out the acceleration due to gravity g from data obtained in step (1).
3. To measure the terminal velocity of a falling magnet thorough a conducting tube (copper or aluminium) which is kept inside the sensor tube.
4. Calculate conductivity of material of pipe.
5. Calculate current flowing through the pipe due to motion of magnet.

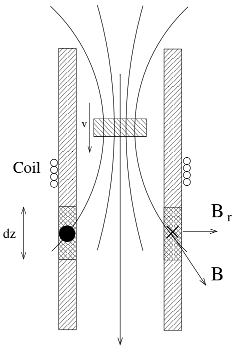


Figure 1.1: Schematic setup for the experiment

Apparatus used

A copper pipe, sensing unit, neodymium magnet, germanium diodes, wires, oscilloscope, and a gauss meter.

1.2.2 Theory

The strong magnet falling through a conducting pipe experiences an opposing magnetic damping force which gradually increases. If the pipe is long enough, the magnet eventually reaches a constant terminal speed. The damping force on the magnet arises from circular currents flowing inside the tube wall known as eddy or Foucault currents. These eddy currents are generated due to the electromotive force (e.m.f.), ε, induced in the pipe due to the time variation of the magnetic flux caused by the motion of magnet inside the tube.

Let us start with a short conductive ring of radius a is moving with velocity $\boldsymbol{v}$ in a region where non-uniform magnetic field $\boldsymbol{B}$ exists we observe

1. A transient e.m.f. ε is induced in the ring which is given by

$$\varepsilon = \int \boldsymbol{v} \times \boldsymbol{B} \cdot d\boldsymbol{\ell} = vB_r(2\pi a), \tag{1.1}$$

   where B_r is the radial component of the magnetic field, and the integral was evaluated along the ring.

2. A variable retarding magnetic force $\boldsymbol{F}$ appears on the short ring which opposes its motion. The axial component F_z opposing the motion of the ring along the z-axis is given by

$$F_z = i(\boldsymbol{\ell} \times \boldsymbol{B})_z = 2\pi i a B_r. \tag{1.2}$$

Now consider the motion of a short and strong cylindrical magnet through a vertical conducting pipe, whose inner and outer radii are a, and b, respectively, under gravity. The velocity of magnet is $\boldsymbol{v} = v\hat{z}$. It is assumed that the magnet axis of symmetry is always coincident with the vertical pipe symmetry as shown in Fig. 1.1.

The field $\boldsymbol{B}$ due to the magnet can be approximated as being produced by a simple magnetic dipole. The axial component B_z and the radial component B_r of the field are respectively given by

$$B_z = \frac{\mu_0 \mu}{4\pi} \left(\frac{2z^2 - r^2}{(r^2 + z^2)^{5/2}} \right), \tag{1.3}$$

$$B_r = \frac{3\mu_0 \mu z r}{4\pi (r^2 + z^2)^{5/2}}. \tag{1.4}$$

Inserting B_r in Eq. 1.1, we get the induced e.m.f.

$$\varepsilon = v(2\pi a) \frac{3\mu_0 \mu z r}{4\pi (r^2 + z^2)^{5/2}}. \tag{1.5}$$

If σ is the conductivity of the material of the pipe wall and dA denotes the cross-sectional are of a small ring element of length $\ell = 2\pi a$, then the conductance of this ring is given by $dC = \sigma dA/\ell$, and the induced current di along such a ring is

$$di = \varepsilon dC = \varepsilon \frac{\sigma dA}{\ell} = B_r v \sigma dA. \tag{1.6}$$

If we denote τ as the thickness of the pipe, the magnetic force dF on the small ring of height dz is given by

$$dF = \ell B_r di = (2\pi a) B_r^2 \sigma v \tau dz \tag{1.7}$$

or

$$dF = (2\pi a) \sigma v \tau \left(\frac{3\mu_0 \mu}{4\pi a^3} \right)^2 \frac{u^2 du}{(1 + u^2)^5}, \tag{1.8}$$

where we have inserted value of B_r from Eq. 1.4 and introduced the new variable u with $z = au$. Integrating Eq. 1.8 along the pipe gives the effective retarding force on the magnet

$$F = \int_{-\infty}^{\infty} (2\pi a) \sigma v \tau \left(\frac{3\mu_0 \mu}{4\pi a^3} \right)^2 \frac{u^2 du}{(1 + u^2)^5} = (2\pi a^2) \sigma v \tau \left(\frac{3\mu_0 \mu}{4\pi a^3} \right)^2 \frac{f}{\pi}, \tag{1.9}$$

where f is a constant whose value is given by

$$f = \int_{-\infty}^{\infty} \frac{u^2 du}{(1 + u^2)^5} = \frac{5\pi}{256}. \tag{1.10}$$

We can obtain the force experienced by the falling magnet using Eq. 1.9 if we know the magnetic moment $\tilde{\mu} = \mu_0 \mu / 4\pi$ (in SI units) of the magnet. The magnetic moment can be obtained by measuring the radial field B_r of the magnet (using a Gaussmeter) as a function of distance r from its axis. [1] If we plot B_r as a function of $1/r^3$, we get a straight line with slope $\tilde{\mu}/2$. The magnetic drag force on the magnet is given by

$$F = \left(\frac{45\pi^2 \sigma \tau \tilde{\mu}^2}{64 a^4} \right) v = kv, \tag{1.11}$$

[1] The magnetic magnetic field B_r due to a magnetic dipole varies as $B_r = \tilde{mu}/2r^3$

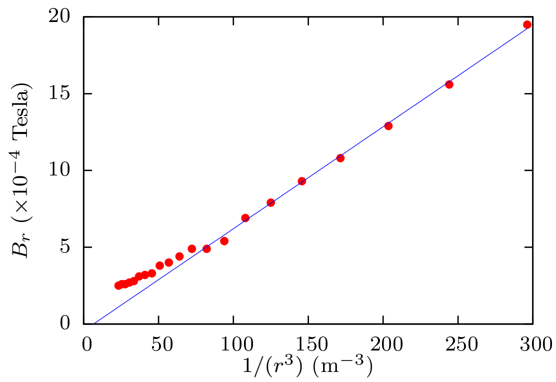


Figure 1.2: The radial magnetic field B_r (in Tesla) vs $1/r^3$ (in m^{-3}) for the magnet. The slope of the line gives the magnetic moment $\tilde{\mu}/2$.

where

$$k = \left(\frac{45\pi^2\sigma\tau\tilde{\mu}^2}{64a^4}\right), \tag{1.12}$$

is the magnetic drag constant. When the magnetic drag force becomes equal to the weight mg of the magnet, the magnet attains a terminal velocity v_T given by

$$v_T = \frac{mg}{k}. \tag{1.13}$$

In deriving Eq. (1.11) it was assumed that the thickness of the pipe is much smaller than the pipe's inner radius a, i.e., $\tau \ll a$. If we consider a pipe of finite thickness $\tau = b - a$, where b is the external radii of the pipe. A tube of infinitesimal thickness da exerts on the falling magnet an infinitesimal force dF given by Eq. (1.11),

$$dF = \left(\frac{45\pi^2\sigma v\tilde{\mu}^2}{64a^4}\right) da. \tag{1.14}$$

Integrating across the wall, i.e., from a to b, we get

$$F = \int_a^b \frac{45\pi^2\sigma v\tilde{\mu}^2}{64a'^4} da' = \frac{45\pi^2\sigma v\tilde{\mu}^2}{64\cdot 3}\left(\frac{1}{a^3} - \frac{1}{b^3}\right). \tag{1.15}$$

Introducing the *thickness parameter* λ defined so that $b = \lambda a$, the above equation becomes

$$F = \frac{45\pi^2\sigma v\tilde{\mu}^2}{64a^4}\frac{a}{3}\left(1 - \frac{1}{\lambda^3}\right). \tag{1.16}$$

It is therefore useful to define the *effective thickness* τ' of the cylindrical wall as

$$\tau' = \frac{a}{3}\left(1 - \frac{1}{\lambda^3}\right) \tag{1.17}$$

Now replacing the thickness τ by τ' in Eq. (1.12) we get the magnetic drag constant as

$$k = \frac{45\pi^2\sigma\tau\tilde{\mu}^2}{64a^4}\frac{a}{3}\left(1 - \frac{1}{\lambda^3}\right). \tag{1.18}$$

This value of k should be used in Eq. (1.13) to obtain the terminal velocity.

Experiment 2

Black body radiation and Stefan's law using an incandescent tungsten lamp

Prof. Arvind and Dr. Paramdeep Singh

2.1 Introduction

Black body radiation experiments are slightly tricky to set-up in an undergraduate Physics laboratory owing to the requirement that the object under study needs to be at very high temperatures ($T > 10^3 K$). In addition, such an object needs to be isolated from the environment to prevent oxidation. The incandescent lamps, either gas filled or having vacuum inside, provide a very convenient source of black body radiation. In this experiment we will use a lamp with vacuum inside it.

2.2 The Experiment

2.2.1 Aim

The aim of the experiment is:

1. To study the variation of total power radiated by the lamp with temperature.
2. To investigate if this source follows the Stefan's law.
3. To what extent this source emulate black body.

2.2.2 Circuit Diagram

The setup designed to provide all the requirements is shown in Fig. 2.1 (Left) while the connection diagram for the experiment is shown in Fig. 2.1(Right). The lamp is connected to a variable power supply through an ammeter. The voltmeter is connected parallel to the lamp.

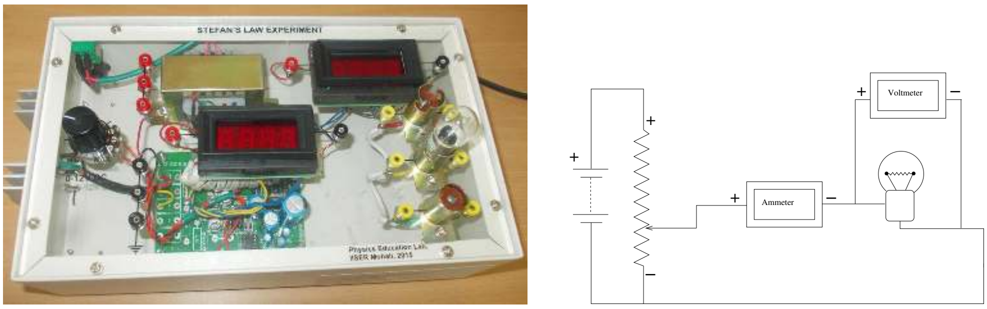


Figure 2.1: Left: The experimental setup. Right: The connections required for the experiment.

2.2.3 Theory

We know that an object at an elevated temperature T radiates energy which we can feel also if the temperature is significantly higher than the environment. The Power P radiated by such an object is give by the following relation called Stefan-Boltzmann Equation:

$$P = \sigma A e T^4, \tag{2.1}$$

where, P is the power or the rate at which energy is radiated by the source, $\sigma = 5.6696 \times 10^{-8} W m^{-2} K^{-4}$ is a constant called Stefan's constant, A is the surface of radiating object, T is the temperature in Kelvin and e is the emissivity and is considered constant with temperature. For a perfect black body $e = 1$.

The following assumptions are being made for this experiment:

1. The loss of heat due to conduction is negligible. This is because the lamp used in this experiment contains a filament in an evacuated chamber.
2. The loss of heat through the wires connecting the filament is assumed to be negligible. This means that all of the energy supplied by the battery is radiated as heat.
3. The filament is considered perfect black body with emissivity $e = 1$.

Taking logarithm of Eq. 2.1 we get

$$\log(P) = \log(\sigma A) + \log(T^4). \tag{2.2}$$

Since we have considered that all the electrical energy supplied to the lamp is radiated, the power radiated can be obtained from the $V - I$ data of the lamp as $P = VI$. Where I is the current through the lamp and V is the potential difference across it. If we are able to obtain T, then we can plot $\log(P)$ vs $\log(T)$ with the following observations

1. The slope of graph will allow us to validate the Stefan-Boltzmann Equation. The expected value of slope is 4.
2. The graph allows the measurement of $\log(\sigma A)$.

3. If A can be estimated the above measurement will allow to us estimate σ.

Except power, P, the other variables will be obtained indirectly using $V - I$ data.

2.2.4 Procedure

The experimental measurements are divided into two parts:

1. Measurement of $V - I$ data at high temperatures and calculation of required quantities.
2. Measurement of lamp filament resistance at room temperature.

Measurement of Resistance and calculation of Resistivity at different Temperatures

We know that the resistance of a metal varies with temperature as

$$R_t = R_0(1 + \alpha t) \tag{2.3}$$

Where R_0 is the resistance at 0 °C and R_t resistance at t °C. For temperatures t_1 and t_2, the Eq. 2.3 can be written as

$$R_{t_1} = R_0(1 + \alpha t_1), \tag{2.4a}$$

and

$$R_{t_2} = R_0(1 + \alpha t_2). \tag{2.4b}$$

Dividing Eq. 2.4b by Eq. 2.4a we get

$$\frac{R_{t_2}}{R_{t_1}} = \frac{(1 + \alpha t_2)}{(1 + \alpha t_1)}. \tag{2.5}$$

The R_{t_1} is the value of lamp resistance at room temperature t_1. The temperature coefficient of resistance α for tungsten is 0.0045/°C. The temperature t_1 is known (through measurement). R_{t_2} is the value of resistance obtained from the $V - I$ data. Remember that you are not to find resistance from $V - I$ graph, but calculate individual values using $R = V/I$ (Why?). Once the values of R_{t_2} representing the resistance measured at different temperatures are known, the resistivity corresponding to those resistance values can be calculated, which in turn allow us to obtain temperature values from the standard data.

Calculation of Resistivity

We know that the resistance R of a conductor is given by

$$R = \rho \frac{\ell}{a}, \tag{2.6}$$

where ℓ is the length of the conductor and a is its are of the cross section. In our case, the conductor is a filament (which is usually a cylindrical coil for large and a wire for

S. No.	Voltage V	Current I	Resistance R_t	Resistivity ρ_t	Temperature T

Table 2.1: Sample table for the calculation of temperature.

small filaments). The factor ℓ/a is usually constant over a temperature range. Let us take $\ell/a = k$, then from Eq. 2.6, we get

$$k = \frac{R}{\rho}. \tag{2.7}$$

Remember that R_{t_1} is the resistance at room temperature t_1. We will set $R = R_{t_1}$ and $R_t = R_{t_2}$. Similarly $\rho = \rho_{t_1}$ and $\rho_t = \rho_{t_2}$. For tungsten $\rho = 5.6 \times 10^{-8}$ohm-m at 20°C. This value of k allows us to calculate ρ_t at different temperatures

$$\rho_t = \frac{R_t}{k}. \tag{2.8}$$

Calculation of Temperature

Equation 2.5 can be written as

$$\frac{\rho_t}{\rho} = \frac{(1 + \alpha t)}{(1 + \alpha\ 20)}. \tag{2.9}$$

In Eq. 2.9, ρ_t, ρ and α are known, and for the room temperature the value of ρ at 20°C can be used. From Eq. 2.9 we get

$$t = \frac{1}{\alpha}\left[\frac{\rho_t}{\rho}(1 + 20\alpha) - 1\right]. \tag{2.10}$$

Equation 2.10 yields temperature values in °C which needs to be converted to Kelvin and tabulated as shown in Table. 2.1.

Measurement of lamp filament resistance at room temperature

This part needs more attention. The most convenient way is using $V-I$ data, but even very small values of current through filament can raise its temperature significantly so as to change its resistance by a considerable amount. Using lower potential ensures very small amount of power dissipation in the filament, so that filament temperature does not increase and its resistance remains constant over the range of measurement. The preferable current range is μA, therefore, voltage range should be kept very small (mV). It is recommended that the measured resistance be marked on the lamps and if

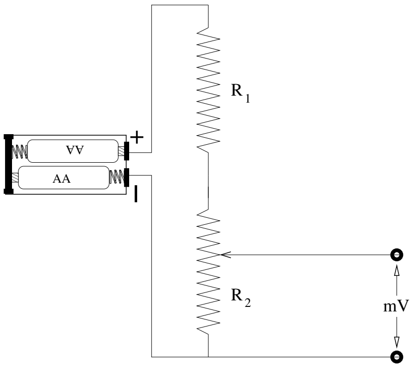


Figure 2.2: Connections for millivolt power source to measure the resistance of the bulb at room temperature.

available previously marked lamps can be used. Such low voltage supplies are usually not available in the lab. To work at such low voltages your supply should be very clean with no ripples. Best option is to use two AA size batteries with a holder and potential divider arrangement as shown in Fig. 2.2. Alternatively a good quality multimeter can be used to measure the filament resistance using the Ohm range (the requirement here is the current during this measurement should not be more than couple of hundred μA).

2.2.5 Representation of Data

The following graphs needs to be plotted

1. V vs I for high temperatures.
2. V vs I for millivolt data (if acquired).
3. $\log(P)$ vs $\log(T)$ for high temperatures.

Calculate the slope of $\log(P)$ vs $\log(T)$. Ideally the slope should be 4. Measure intercept from the same graph to find $\log(\sigma A)$. Look at the lamp and see if you can optically estimate the length ℓ of the filament. If ℓ can be measured, it will allow us to calculate cross-sectional area of the coil using Eq. 2.6. Using this data the surface area of the filament can be estimated, which in turn can be used to estimate σ using $\log(\sigma A)$ calculated from the intercept or alternatively effective area of the coil can be estimated using value of σ.

The plot of $V - I$ characteristics at high temperatures and the plot of $\log(P)$ vs $\log(T)$ obtained for the experiment are shown in the left and right panels of Fig. 2.3, respectively.

References

1. B. S. N. Prasad and Rita Mascarenhas, Am.J.Phys. 46, 420 (1978).
2. I. R. Edmonds, Am.J.Phys. 36, 845 (1968).

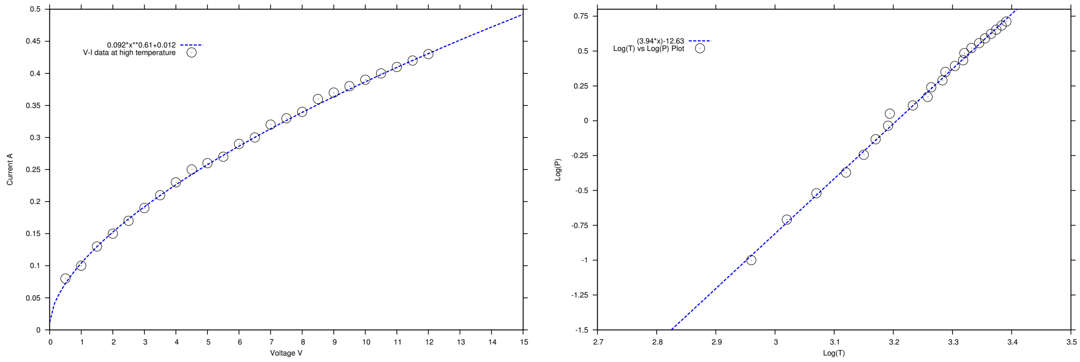


Figure 2.3: Left panel: V-I characteristics at high temperatures. Right panel: The plot of $\log(P)$ vs $\log(T)$.

3. D. H. Jaecks and R. Du Bois, Am.J.Phys. 40, 1179 (1972).

Experiment 3

Random Sampling of an AC source

Prof. Arvind and Dr. Paramdeep Singh

3.1 The Experiment

3.1.1 Aim

The aim of the experiment is to perform random sampling of an AC source using a capacitor and use the obtained data to study the following

1. Plotting the probability distribution of mathematical function $V = V_0 \sin(\omega t)$ and comparing it with the observed distribution.
2. Study the effect of sampled data size on the probability distribution by taking
   (a) 100 observations,
   (b) 500 observations,

   and comparing them.
3. Study the effect of bin size on the probability distribution by plotting the same set of data with different bin sizes and comparing them.
4. Reconstructing the sinusoidal waveform using the information recovered from the random data in this experiment.
5. Plotting the probability distribution of the square wave and compare the result with the sine wave.

Apparatus

A capacitor (nonpolar) 50-100 $\mu F/25$V, a DC voltmeter (preferably peak reading), a low frequency AC source (a step down transformer 0-6V), a DPDT switch.

3.1.2 Circuit diagram

The circuit diagram is shown in Fig. 3.1.

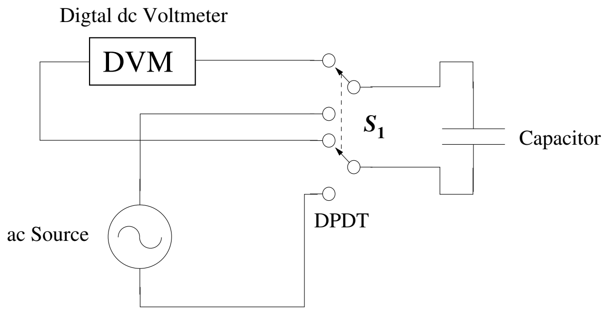


Figure 3.1: Circuit Schematic

3.2 Procedure

1. Connect the circuit diagram as shown in Fig. 3.1.
2. Switch on the power.
3. Using the toggle switch S_1 connect the capacitor to the AC source.
4. Toggle the switch S_1 to disconnect the capacitor from ac source and connect it to the digital voltmeter. The following precautions should be taken while taking voltage readings
   - If a peak reading voltmeter is connected, note down the final reading of the display.
   - If any simple digital multimeter is connected then note down the maximum which appears on the display.
   - The input impedance of the meter should be $\geq 1\text{M}\Omega$.
5. Repeat the procedure in step 3 and 4 to get a data set preferably 500 observations.

3.2.1 Data Analysis

Let us analyse the data obtained in the previous step. The first step is to obtain the distribution function $n(V)$, which is defined as the number of times the measurement of V results in a value between V and $V + \Delta V$.

Distribution Function

To obtain $n(V)$, we need to bin the data. Let us assume that in a particular set the maximum and minimum voltage measured are 8.0 V and -8.0 V. Take V along the x-axis. We will divide the whole range in a number of bins as follows:

- Let us take 1 V = 1 cm on the graph paper. Using this scale create bins of width 1 V from $-8.0V$ to $+8.0V$.

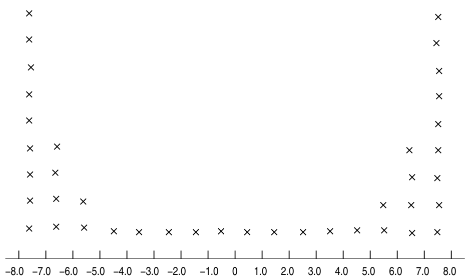


Figure 3.2: Binning data

- Place each data point into its respective bin, i.e., the reading 4.8 will be marked in the bin between 4.0 and 5.0.
- The data on the graph paper will look like Fig. 3.2, where every cross represents a measured value.

The corresponding probability distribution of values of V, denoted by $P(V)$, is given by

$$P(V)\Delta V \equiv \frac{n(V)}{N}, \tag{3.1}$$

where N represents the total number of times the voltage is measured. For a sine wave, the probability distribution $P(V)$ is given by

$$P(V) = \frac{1}{\pi\sqrt{(V_0^2 - V^2)}}, \tag{3.2}$$

where V_0 is the peak value of the voltage.

Plotting the waveform

From Eq. 3.1, the distribution function $n(V)$ can be written as

$$n(V) = NP(V)dV, \tag{3.3}$$

where we have changed ΔV to differential dV. The accumulated frequency of events up to a voltage V can then be obtained by integrating Eq. 3.3

$$N_V = \int_0^V NP(V)\mathrm{d}V = \frac{N}{\pi}\sin^{-1}\left(\frac{V}{V_0}\right), \tag{3.4}$$

Where V_0 is the measured peak value of the applied voltage. For discrete bins of size ΔV, the integration is replaced by sum over bins,

$$N_V = \sum n(V) = \sum NP(V)\Delta V.$$

In other words, by a cumulative process of adding the frequencies in bins, starting from the bin $V = 0$ to the bin $V = V$, we recover the sine wave. Inverting Eq. 3.3 we

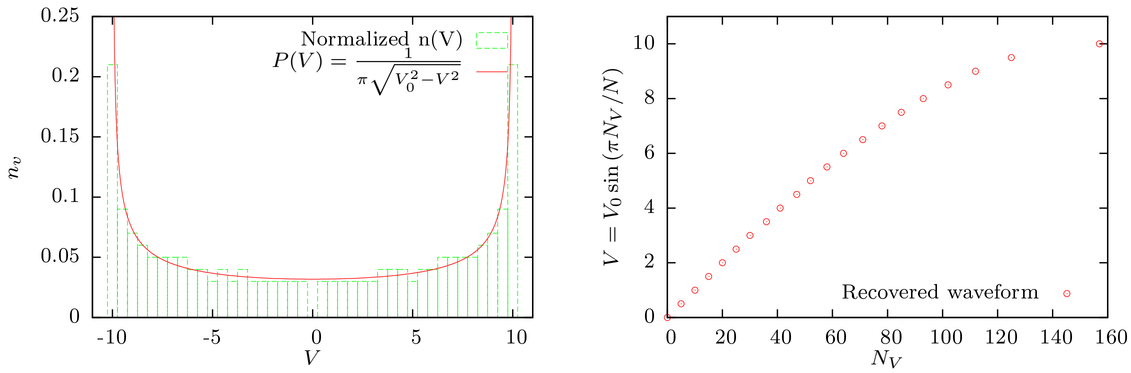


Figure 3.3: Left: Comparison of theoretical and experimental data. Right: The plot of N_V vs $V = V_0 \sin(\pi N_V/N)$ which represents one fourth of the recovered waveform.

get

$$V = V_0 \sin\left(\frac{\pi}{N} N_V\right), \tag{3.5}$$

with

$$N_V = \sum_{i=1}^{k} n_i(V) \tag{3.6}$$

where k represents the k_{th} voltage bin. The voltage points can be calculated from the following expression

$$V_k = V_0 \sin\left(\frac{\pi}{N}\sum_{i=1}^{k} n_i(V)\right). \tag{3.7}$$

Note that the data from which voltage points are recovered is random. Therefore, the time relation of these data points and hence the frequency of waveform cannot be determined.

Sample calculations

A sample set of data with peak value $V_0 = 10$ volts and a set of positive values (157 data points) is shown in Table 3.1. The total number of data points (by taking similar number of negative values also) are therefore $N = 2 \times 157 = 314$. A comparison of theoretical probability distribution and experimental observation is shown in the Left panel of Fig. 3.3. In the Right panel, we have shown the plot of N_V vs $V = V_0 \sin(\pi N_V/N)$ which represents a one fourth of the recovered sine waveform.

References

1. Arvind, P. S. Chandi, R. C. Singh, D. Indumathi and R. Shankar, Am. J. Phys. **72**, 76 (2004).

S.No	Bin	n(V)	N_V	$V = V_0 \sin\left(\frac{\pi N_v}{N}\right)$
1	0.0	0	0	0
2	0.0-0.5	5	5	0.5
3	0.5-1.0	5	10	1.0
4	1.0-1.5	5	15	1.49
5	1.5-2.0	5	20	1.99
6	2.0-2.5	5	25	2.47
7	2.5-3.0	5	30	2.96
8	3.0-3.5	6	36	3.52
9	3.5-4.0	5	41	3.99
10	4.0-4.5	6	47	4.53
11	4.5-5.0	5	52	4.97
12	5.0-5.5	6	58	5.48
13	5.5-6.0	6	64	5.97
14	6.0-6.5	7	71	6.52
15	6.5-7.0	7	78	7.03
16	7.0-7.5	7	85	7.51
17	7.5-8.0	8	93	8.02
18	8.0-8.5	9	102	8.52
19	8.5-9.0	10	112	9.00
20	9.0-9.5	13	125	9.49
21	9.5-10.0	32	157	10.00

Table 3.1: A sample table with set of positive voltage readings.

Experiment 4

Magnetic moment in the magnetic field

Mohammad Aslam and Soumyadip Halder

4.1 The Experiment

4.1.1 Aim

The aim of the experiment is to determine the torque due to a magnetic moment in a uniform magnetic field, as a function of

1. The strength of the magnetic field,
2. The angle between the magnetic field in the magnetic moment,
3. The strength of the magnetic moment.

4.1.2 Apparatus used

Pair of Helmholtz coils, conductors circular set, torsion dynamometer 0.01 N, coil holder, power supply var. 15 VAC/12 VDC/5 A, Support rod -PASS-, square, l 630 mm.

4.2 Theory

The Helmholtz coil provides uniform magnetic field at the centre. If a current carrying loop is placed at the centre, such a way that the loop contains non zero flux then there will be a torque on the loop. This torque will rotate the loop until balanced by the torque, generated from the metallic band of torsion dynamometer. The value of torque can be read from the reading of torsion dynamometer.

If a current I is passed through a closed loop of area A it produces a magnetic moment $\vec{m}$

$$\vec{m} = I\vec{A}, \tag{4.1}$$

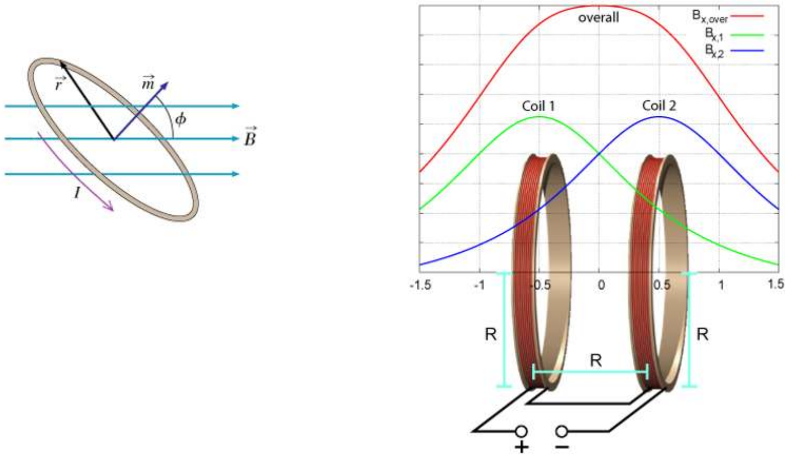


Figure 4.1: Pair of Helmholtz Coil

whose direction is along the direction of the area vector $\vec{A}$. If this current carrying loop is kept in a uniform magnetic field with flux density $\vec{B}$, it experiences the torque τ given by

$$\tau = \vec{m} \times \vec{B}. \tag{4.2}$$

If the angle between the area vector $\vec{A}$ and the magnetic field $\vec{B}$ is ϕ (as shown in Fig. 4.1), then

$$\tau = BIA \sin\phi. \tag{4.3}$$

The magnetic field B is given by $B = cI_H$, where c and I_H are the Helmholtz coil constant and the current through the Helmholtz coil, respectively. The torque is thus given by

$$\tau = cI_H IA \sin\phi. \tag{4.4}$$

The above equation shows that, the torque τ is directly proportional to

- current I_H in the Helmholtz coil,
- current I in the coil,
- Sine of angle between the area vector $\vec{A}$ and magnetic field $\vec{B}$.

4.3 Procedure

The connections are shown in Fig. 4.2.

1. First connect the Helmholtz coils in the series so that the current flowing through the coils are same.
2. Check the conductor coil circuit. Make sure that all the connections are closed.
3. Set the zero position of the *torque reading*, imprinted on the torsional dynamonmeter by rotating the *force indication knob* [See Fig. 4.2 (point 8)].

1. Zero setting knob
2. Zero indication for lever arms
3. Suspension system
4. Lever arm
5. Protective tube for lever arms
6. Eddy current attenuation, shortens setting time
7. Rod for holding with standard support material
8. Force indication knob

Figure 4.2: Experimental set-up for determining the torque due to a magnetic moment in the magnetic field.

4. Set the *lever arms* [See Fig. 4.2 (point 4)] of the *suspension system* [See Fig. 4.2 (point 3)] to the zero position by rotating the *zero setting knob* [See Fig. 4.2 (point 1)]. The zero position for the *lever arms* [See Fig. 4.2 (point 4)] is imprinted as the *zero indication for lever arms* [See Fig. 4.2 (point 2)] on suspension system. Make sure that the metal band has the position for minimum torque.

In the whole experimental procedure the torque, τ, has to be measured with respect to the variables I_H, I, and ϕ. Out of these three variable, keep any of the two constant and make the table of the readings of τ for the different values of the third variable. Hence, three tables are to be formed. Plot three graphs and do the data fitting using least square method. Also obtain the Helmholtz coil constant c.

4.3.1 Precautions

The following precautions need to be taking while doing the experiment

1. The currunt should be varied very slowly.
2. The current in the Helmholtz coil must not exceed 3 Amperes and the current in the inductor coil must not exceed 4.5 Amperes.
3. Better result should be achieved for smaller angles of torsion.

Experiment 5

Biot Savart law

Leena Aggarwal and Shekhar Das

5.1 Introduction

In 1820, Hans Cristian Oersted, a Danish scientist, observed that a compass needle gets deflected when an electric current from a battery was switched on and off. He found that when an electric current flows through a wire, it produces a circular magnetic field. This discovery provided the first link between electricity and magnetism. In the same year Jean-Baptiste Biot and Félix Savart obtained an equation describing the magnetic field generated by an electric current which relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. This equation is now known as *Biot-Savart's law.*

5.2 The Experiment

5.2.1 Aim

The aim of this experiment is

1. To verify Biot-Savart's law by showing that magnetic field produced is directly proportional to the current passed in a coil.
2. To determine the variation of magnetic field with the distance from the center of the coil at a constant current in air.
3. To compare the magnetic fields at the center of coils with different diameters by passing same current and show that the field at the center is inversely proportional to the radius of the coil.

5.2.2 Apparatus used

Optical bench, set of circular loops with holders, power supply $0-30$ V DC, $0-20$ A, Gauss meter with axial probe, leads, saddle with micrometer. The apparatus is shown

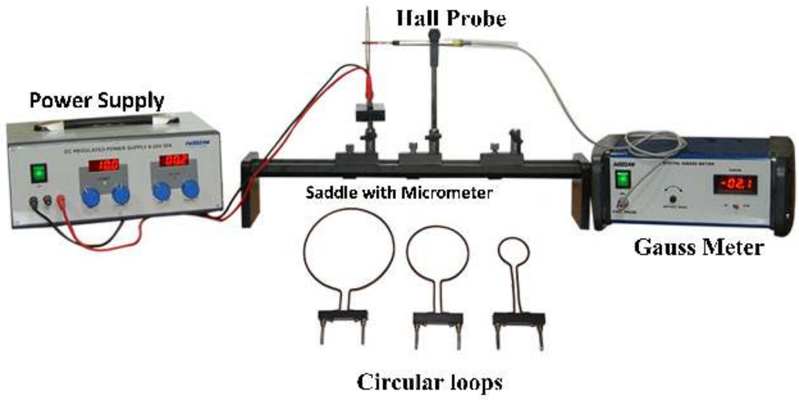


Figure 5.1: Apparatus used for the study of Biot-Savart's law.

in Fig. 5.1. The existence of magnetic field lines can be studied for a current carrying conductor of any shape. The conductor can be an infinite long wire, a circular loop or a cylindrical coil. In this experiment, we will be using circular loops of different diameter. A current is passed through the loop and the magnetic field is measured by using a digital gauss meter.

5.3 Theory

Consider a small element $\vec{ds}$ of a wire carrying a steady current I as shown in Fig. 5.2. The magnitude of magnetic field $d\vec{B}$ at point P due to this small current element should be proportional to the following factors:

$$d\vec{B} \propto \begin{cases} I\vec{ds} & \text{the strength of the current } I \text{ and the length of the element } \vec{ds}. \\ 1/r^2 & \text{inverse square of the distance from the center of the wire to point } P \\ \sin\theta & \text{angle between } \vec{ds} \text{ and } r. \end{cases}$$

Therefore,the magnetic field $d\vec{B}$ produced at point P by a small current element $\vec{ds}$ of a wire carrying current I is given by

$$d\vec{B} = \frac{\mu_0 I\vec{ds} \times \hat{r}}{4\pi r^3}, \tag{5.1}$$

where $\mu_0/4\pi = 10^{-7}$ TmA^{-1}.

The magnitude of the field is given by

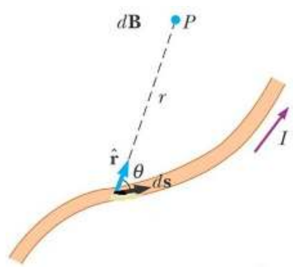


Figure 5.2: A current carrying wire

$$dB = \frac{\mu_0 I ds \sin\theta}{4\pi r^2}, \tag{5.2}$$

where θ is the angle between $\vec{ds}$, which indicates the direction of the current, and $\vec{r}$. The total field B due to the entire current distribution of wire is obtained by

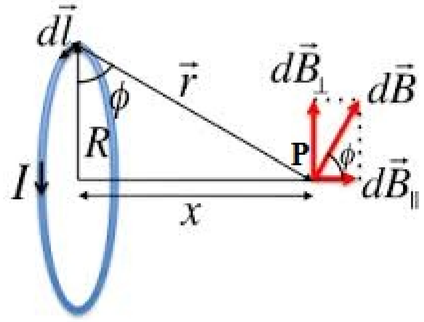


Figure 5.3: The element $Id\vec{l}$ of the circular loop of current sets up a field $\vec{B}$ at point P on the axis of the loop.

integrating over all current elements $Id\vec{s}$.

$$B = \int dB = \frac{\mu_0}{4\pi}\int \frac{Id\mathbf{s}\, sin\theta}{r^2}. \tag{5.3}$$

5.3.1 Biot-Savart‘s law for the circular current loop

Consider a circular loop or radius R carrying current I as shown in Fig. 5.3. Let us calculate the magnetic field $\vec{B}$ at point P on the axis of loop at a distance x from the center of the loop. The $d\vec{B}$ can be resolved into two components

(i) $d\vec{B}_{\parallel}$ along the axis of loop,

(ii) $d\vec{B}_{\perp}$, at right angle to the axis.

Only $d\vec{B}_{\parallel}$ component contributes to the total magnetic field at point P because the component $d\vec{B}_{\parallel}$ for all current elements lie on the axis and add up directly, where as the component $d\vec{B}_{\perp}$ point in different directions perpendicular to the axis, and the sum of all $d\vec{B}_{\perp}$ for the complete loop is zero, from the symmetry consideration. Therefore, the vector integral over all $d\vec{B}$ is equal to an integral over the parallel components only. The magnitude of the field is given by

$$B = \int d\vec{B}_{\parallel}, \qquad \text{with} \qquad d\vec{B}_{\parallel} = dB\cos\phi = \frac{\mu_0 I ds\cos\phi}{4\pi r^2}, \tag{5.4}$$

where we have used Eq. 5.2. From Fig 5.3, we have

$$r = \sqrt{R^2 + x^2} \qquad \cos\phi = \frac{R}{r} = \frac{R}{\sqrt{R^2+x^2}}.$$

Putting these values in Eq. 5.4 we get

$$B \;=\; \int d\vec{B}_{\parallel} = \frac{\mu_0 I R}{4\pi(R^2+x^2)^{\frac{3}{2}}}\int ds$$

$$= \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}, \tag{5.5}$$

where $\int ds = 2\pi R$, is the circumference of the loop.

Special Cases

1. For magnetic field at the center of the loop, i.e., substituting $r = R$ in Eq. 5.5 we get
$$B = \frac{\mu_0 I}{4\pi R^2} \int ds = \frac{\mu_0 I}{2R}. \tag{5.6}$$

2. If the distance of point P is much larger than the radius of the loop (i.e., $x \gg R$). The magnetic field is given by
$$B = \frac{\mu_0 I R^2}{2x^3}. \tag{5.7}$$

5.4 Procedure

- Mount the circular conductor and hall probe holder on the optical bench.
- Mount the conductor loop on the holder.
- Mount the axial Hall probe in the holder for Hall probe. Align the Hall probe towards the center of the circular conductor.
- Adjust the zero of the digital Gauss meter. It must be zero when no current is passed through the conductor.
- Increase the current in steps of 2 A. Wait a minute after each increment to stabilize the magnetic field. Then note down the corresponding magnetic field value.
- Maximum current limit is 20 Amps for $0 - 3$V power supply.
- At $I = 20$A, move the Hall probe and measure the magnetic field as a function of x.
- Repeat the same steps for another circular loops with different diameter.

5.4.1 Experimental observations

Tabulate the observations as given below

1. Magnetic field B of a coil as a function of current I

Sr. no.	Circular conductor of dia 40 mm		Circular conductor of dia 80 mm		Circular conductor of dia 120 mm	
	I (A)	B (G)	I (A)	B (G)	I (A)	B (G)

2. Magnetic field B of a coil as a function of axial distance x

Sr. no.	Circular conductor of diameter 40 mm		Circular conductor of diameter 80 mm		Circular conductor of diameter 120 mm	
	x (cm)	B (G)	x (cm)	B (G)	x (cm)	B (G)

5.4.2 Results

The results are plotted in Fig. 5.4.

- Fig. 5.4(a) shows a linear dependence of magnetic field B on the current I flowing through the circular loop (i.e., $B \propto I$).
- Fig. 5.4(b) shows the variation of magnetic field with the axial distance x from the center of the loop. It also compares the magnetic field at the center of the coils of different diameters (40, 80, and 120 mm) when a current $I = 20$ A flows through them.

Further analysis: *Calculate the magnetic field B at the center of each coil using the data obtained, and show that B is inversely proportional to the radius of the coil.*

5.5 Precautions

- Set all the knobs of the current source at zero before turning on or off the system.
- Hall probe should align at the center of the circular loop.
- At zero current, Gauss meter should show zero value.
- Do not pass current for long time. It may cause connection problem due to over heating.
- Wait for few minutes to stabilize the magnetic field during each measurement.
- Repeat steps 1-13 times for each coil to check the reproducibility.

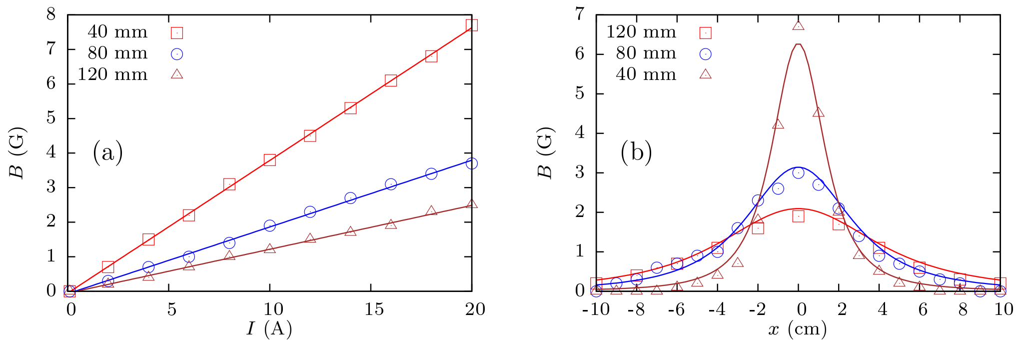


Figure 5.4: (a) Magnetic field B as a function of current I, and (b) Magnetic field B as a function of distance x from the center for circular loops of various diameters as indicated. The points are experimental observations and the solid lines are the best fit to the data by a straight line $y = mx + c$ and using Eq. (5.5) for (a) and (b), respectively.

Experiment 6

Studying electromagnetic induction using a magnet & a coil

Sudhanshu Shekhar Chaurasia and Jyotsana Ojha

6.1 The Experiment

6.1.1 Aim

The aim of the experiment is to help you understand the phenomenon of induced emf in a circuit caused by change of magnetic flux in the circuit. The experimental setup we have in our lab is designed by Prof. Babulal Saraf.

6.1.2 Basic setup

The setup consist of a magnet mounted on an arc of a semi-circle of radius say R_0 as shown in Fig. 6.1. The arc is rigid aluminium frame suspended at the center so that whole frame can oscillate freely in its plane. The position of weights on the diagonal arm can be altered in order to vary the period of oscillations. A coil has been wound around the arc so that the magnet can pass freely through the coil. The amplitude of swing can be read from the arc graduations. When the magnet moves through and out of the coil, the flux of the magnetic field through the coil changes, inducing an emf. The magnetic flux and the induced emf can be observed by connecting and oscilloscope to the coil.

6.2 Theory

The Faraday's second law tells us that the induced emf e is proportional to the rate of change of flux ϕ and the direction of this emf opposes the change in flux that produced it. Mathematically, this can be written as

$$e = -\frac{d\phi}{dt}. \tag{6.1}$$

The induced emf pulse e through the coil as a function of time t as observed using oscilloscope is plotted in Fig. 6.2.

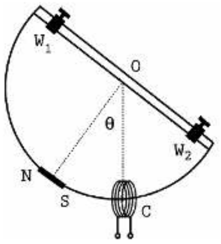


Figure 6.1: A magnet attached to an oscillating system passes through a coil periodically, generating a series of emf pulses.

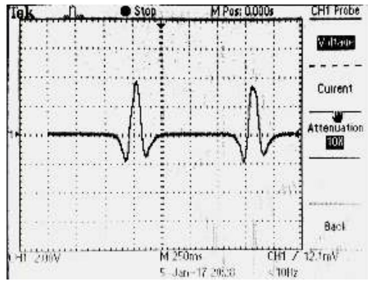


Figure 6.2: Plot of induced emf pulse e through the coil with time t.

6.2.1 Measuring induced emf

The basic idea is to charge a capacitor through a diode and measure the voltage developed across the capacitor. If the charging time RC (where R is the resistance used plus the coil resistance and the forward resistance of the diode) is larger than the time of generation of emf in the coil, the capacitor does not charge up to the peak value in a single swing but takes around 10 oscillations to do so. When the charging current ceases to flow in the galvanometer, the capacitor has been charged to the peak value of the emf.

6.2.2 Measuring emf as a function of velocity

As the magnet starts far away from the coil, moves through it and recedes, the field through the coil changes from a small value, increases to maximum and then becomes small again. Moreover, the speed of the magnet is largest when it approaches the coil. The magnetic field thus changes quite slowly with time when the magnet is far away and rapidly as it approaches the coil. The flux varies similarly with time (since only a constant 'effective area' i.e the product of number of turns and area of the coil, relates ϕ and B).

The induced emf is proportional to $d\phi/dt$ and is negative when ϕ is increasing and positive when ϕ is decreasing. This variation of induced emf with time is plotted as a sequence of two "pulses". Consider the effect of these pulses on the charging circuit. The diode will conduct only during the positive pulse. At the first half-swing, the capacitor charges up to a potential e_1 given by $\frac{1}{RC}\int e(t)dt$. During the next half-swing, the diode will cutoff until the positive pulse reaches e_1 and then capacitor will charge up to a slightly a higher value say e_2 and so on, in a few oscillations, the capacitor will be charged up to the peak value e_0 of the voltage pulse. This will be indicated by the fact that the galvanometer stops showing any kicks.

The induced emf can be written down as

$$|e| = \frac{d\phi}{dt} = \frac{d\phi}{d\theta} \cdot \frac{d\theta}{dt}$$

The term $d\phi/d\theta$ depends on magnet and coil geometry. The second term $d\theta/dt$ is deduced from the oscillation equation

$$\theta = \theta_0 \sin \frac{2\pi t}{T}, \qquad \text{which gives} \qquad \frac{d\theta}{dt} = \frac{2\pi\theta_0}{T} \cos \frac{2\pi t}{T}.$$

The peak voltage e_0 corresponds to $(d\phi/dt)_{\max}$. Since the cosine term does not differ much from 1 for angles close to $2n\pi$,

$$|e_0| = \left(\frac{d\phi}{dt}\right)_{\max} \approx \left(\frac{d\phi}{d\theta}\right)_{\max} \frac{2\pi\theta_0}{T} \tag{6.2}$$

Repeat experiments for different swing amplitudes θ_0 and see if e_0 is proportional to θ_0. Slide the weights and change T and repeat for different values of T. Check of e_0 is proportional to $1/T$. Plot the observed emf e_0 against the maximum velocity $(\frac{2\pi\theta_0}{T} \times R_0)$.

6.2.3 Studying charge delivered due to induction

When the charging time (RC) of the capacitor is large compared with the pulse width, the charge collected in one positive pulse is

$$q_1 = \frac{1}{R} \int_0^t e(t)dt = -\frac{1}{R} \int d\phi$$

The positive pulse corresponds to ϕ changing from the maximum to zero which leads to

$$q_1 = \frac{\phi_{\max}}{R} \tag{6.3}$$

$$V_1 = \frac{\phi_{\max}}{RC} \tag{6.4}$$

The diode allows the capacitor to charge only for positive pulse. Arranging two sets of charging circuits so that one capacitor charges up on the positive pulse and the other on the negative pulse.

6.2.4 Studying electromagnetic damping

In the experiments considered so far, we have neglected the damping of oscillations. There are many reasons for damping, for example, the air resistance, the friction at the point of suspension etc. However, the most important and interesting source is the induced emf in the coil. The direction of the induced emf always opposes the change causing it (Lenz's law). In this experiment it is the motion of the magnet which induces the emf. Therefore, the induced current flows in the coil in such a way that it opposes the motion of the magnet. Since the velocity of the magnet changes after each oscillation and so is the energy dissipation. If E_n is energy of the system after n oscillations, then

$$\frac{E_n}{E_{n-1}} = a \qquad \text{which gives} \qquad \frac{E_n}{E_0} = a^n.$$

As the energy is proportional to the square of the amplitude, we have

$$\frac{\theta_n}{\theta_0} = \sqrt{\frac{E_n}{E_0}} = a^{n/2}. \tag{6.5}$$

6.3 Procedure

Measurement of emf

1. Design the circuit as shown in Fig. 6.3 (Left).
2. Fix the masses at a specific length, for that particular length note the value of peak voltage at different angles(θ_0). Repeat this for 4-5 different values of lengths.
3. When the magnet goes through the coil, magnetic flux changes and emf induced which causes the deflection in galvanometer and charging of the capacitor. When the capacitor is fully charged, the galvanometer stops showing deflection, then press the switch to know the peak voltage on the voltmeter.
4. Now keep pressing the button until the voltmeter shows zero.
5. For a particular time period of oscillation(T), plot peak voltage(e) vs angle (θ_0).
6. For a particular value of deflection angle(θ_0), plot peak voltage(e) vs. $1/T$.

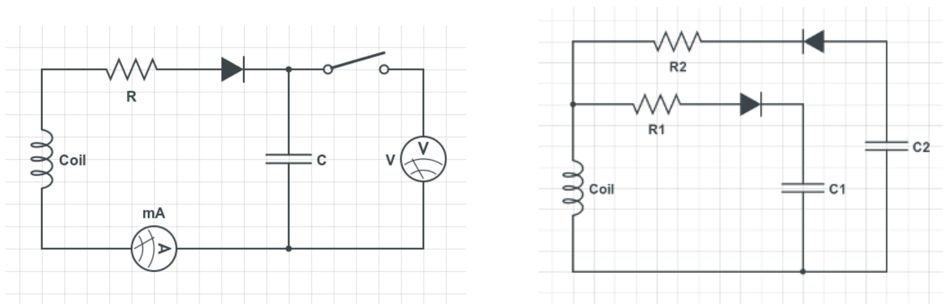


Figure 6.3: (Left) Schematic diagram of circuit. (Right) Schematic diagram of circuit for charge comparison due to both positive and negative induced emf pulses.

Charge delivered due to induction

1. Arrange things in the circuit such that RC is large and find the voltage acquired by the capacitor in different number of positive pulses. Check if the relation (6.4) for V holds and see if acquired voltage is proportional to the number of positive pulses. (What is the condition under which this will be valid ?).

2. The diode allows the capacitor to charge only for positive pulse. Arrange two sets of charging circuits as shown in Fig. 6.3(Right) so that one capacitor charges up on the positive pulse and the other on the negative pulse. Verify that the charges on the capacitors are nearly the same.

3. If you stop the oscillations (by hand) after a quarter oscillation (from the extreme position of magnet to its mean position), only one capacitor charges up. Try and find out if the sign of induced emf is as according to Faraday's law.

Electromagnetic damping

1. First keep the coil open circuited and plot $\log \theta_n$ as a function of n.

2. Do the same with a short circuited coil and also with a finite load such as $1k$ resistor. Finally try a big capacitor as a load. At each swing, the capacitor keeps charging up and the energy has to be supplied to build up this energy. Plot all the experimental data on the same graph and interpret the plots obtained.

6.4 Reference

1. A. Singh, Y. N. Mohapatra, and S. Kumar, Am. J. Phys. **70**, 424 (2002).

Experiment 7

Measurement of e/m by Millikan's method

Rajeev Kapri

7.1 The Experiment

This experiment is one of the most fundamental of the experiments in the undergraduate laboratory. It is based on different forces acting on an electrically charged oil drop moving in the homogeneous electric field of a parallel plate capacitor.

7.1.1 Aim

The aim of the experiment is to show that the electric charge exists as an integral multiples of the charge on a single electron which we represent by "e".

7.1.2 Apparatus

The basis setup is shown in Fig.

7.2 Theory

In 1851, George Gabriel Stokes derived an expression, now known as Stokes' law, for the frictional force – also called drag force – exerted on spherical objects with very small Reynolds numbers (i.e. very small particles) in a viscous fluid. Due to this drag force, the object attains a terminal velocity. Stokes found that for a spherical object of radius r moving through a fluid of viscosity η, the drag force is given by

$$F_d = 6\pi\eta r v, \tag{7.1}$$

where v is the terminal velocity.

Let r be the radius of the drop of oil of density ρ_o lying in the air of density ρ_a. If g is the acceleration due to gravity. There are two forces acting on the oil drop

1. The weight of the drop acting in the downward direction

$$W_d = \frac{4}{3}\pi r^3 \rho_o g. \tag{7.2}$$

2. The upward thrust

$$W_u = \frac{4}{3}\pi r^3 \rho_a g. \tag{7.3}$$

The resultant downward force experienced by the oil drop is given by

$$W = W_d - W_u = \frac{4}{3}\pi r^3(\rho_o - \rho_a)g. \tag{7.4}$$

If the drop attains a terminal velocity v then the downward force given by Eq. (7.4) exactly balances the Stokes force given by Eq. (7.1), that is

$$\frac{4}{3}\pi r^3(\rho_o - \rho_a)g = 6\pi\eta r v, \tag{7.5}$$

which gives the radius of the drop

$$r = \left(\frac{9\eta v}{2(\rho_o - \rho_a)g}\right)^{\frac{1}{2}}. \tag{7.6}$$

If Q be the charge on the drop and E is the electric field between the plates so that the drop begins to move upward with a uniform velocity u, then on equating the resultant upward force, $qE - \frac{4}{3}\pi r^3(\rho_o - \rho_a g)$, with the Stokes drag force we get

$$QE - \frac{4}{3}\pi r^3(\rho_o - \rho_a g) = 6\pi\eta r u.$$

On substituting the value of second term on the left hand side of above expression from Eq. (7.5), and the expression for the radius from Eq. (7.6) we get

$$\begin{aligned} Q &= \frac{6\pi\eta r(v+u)}{E} \\ &= \frac{6\pi\eta}{E}\left[\frac{9\eta v}{2(\rho_o - \rho_a)g}\right]^{\frac{1}{2}}(u+v) \\ &= (u+v)\frac{\sqrt{v}}{U}\eta^{3/2}\frac{18\pi d}{\sqrt{2(\rho_o - \rho_a)g}}. \end{aligned} \tag{7.7}$$

In the above expression we have substituted $E = U/d$, where U is the potential difference applied across the plates of the capacitor which are kept at d distance apart. Substituting the following

- The density of the oil used (olive oil) $\rho_o = 918$ kg/m^3.
- The density of air $\rho_a = 1.21$ kg/m^3.
- The acceleration due to gravity $g = 9.80$ m/s^2.

- The viscosity of air at room temperature and 1 atmosphere $\eta = 1.81 \times 10^{-5}$ N s/m^2.
- The separation of the parallel plates $d = 5 \times 10^3$ m.

parameters in Eq. 7.7 we get

$$Q = (v + u) \cdot \frac{\sqrt{v}}{U} \times 3.818 \times 10^{-10}\ \mathrm{C}, \tag{7.8}$$

which can be used to calculate the amount of charge on the oil drop.

7.3 Procedure

- Spray the oil drops into the oil drop box from the spraying hole by means of a sprayer.
- Select proper voltage (for example 200V) for the polar plates and drive out some unnecessary oil drops until only few drops, which are moving down slowly, remain in field of vision using a microscope.
- Select the drops with more or less the same radius (i.e., the same terminal velocity). Generally it is suitable to select the medium sized oil drops (why?). Use Eq. 7.6 to calculate the radius of the droplet.
- Measure the time t required for rising motion of the oil drop for a certain distance (for example 2mm). Since we are seeing the inverted image formed by the microscope, it will appear as if the drop is falling down.
- Move back the change-over switch to the middle gear and measure the time t required for falling motion (appears as rising motion) of the same oil drop as mentioned above for the same distance.
- For each drop measure
  1. the terminal velocity v at zero voltage,
  2. the rise velocity u at a definite voltage.
- After determining the velocities of the droplets, calculate the charge Q using Eq. 7.8.
- Represent the results in form of a histogram (number of measurements within a range of 10^{-20} C versus $Q/(10^{-20}$ C)) and extract a value of the electronic charge. The elementary electronic charge e is obtained by forming the largest common divisor from the different charge values.

Note: The main object of the experiment is to demonstrate the quantization of charge. Since some of the values of the physical constants above are approximate you should not worry too much if the value of e you obtain is outside the error range you predict from your measurement errors, (you will note the absence of quoted errors in these quantities!). You may neglect the buoyancy of air.

7.4 Precauations

- Keep the instrument in a semi dark room for performing the experiment.
- Level the instrument carefully with the help of level indicator.

Experiment 8

Capacitance of metal spheres

Rajeev Kapri

8.1 The Experiment

8.1.1 Aim

The aim of this experiment is to determine the capacitance of metal spheres of different diameters. The basic principle is to charge the metal spheres of different radii by means of a variable voltage. The induced charges on the conductor are determined with a measuring amplifier. The corresponding capacitances are deduced from voltage and charge values.

8.1.2 Apparatus

The apparatus consists of a high voltage (kV) power supply, metal spheres with different diameters, an amplifier, an auxiliary capacitor, a voltmeter, BNC connector and connecting cables. The experimental set-up to determine the capacitance of spherical conductors is shown in Fig. 8.1.

8.2 Theory

If a spherical conductor with capacitance C_{co} is connected to a charging voltage V_1 (in kV), the charge Q accumulated on the conductor is given by

$$Q = C_{co} V_1. \tag{8.1}$$

When this charged conductor is connected in parallel to an auxiliary capacitor of a known capacitance C_{ca}, the total capacitance of the circuit becomes $(C_{co} + C_{ca})$ and the same charge Q flows in it. If the voltmeter connected to the circuit measures a voltage V_2 (in volts), which were determined by means of a measuring amplifier having amplification factor A, we have

$$Q = (C_{co} + C_{ca}) V_2. \tag{8.2}$$

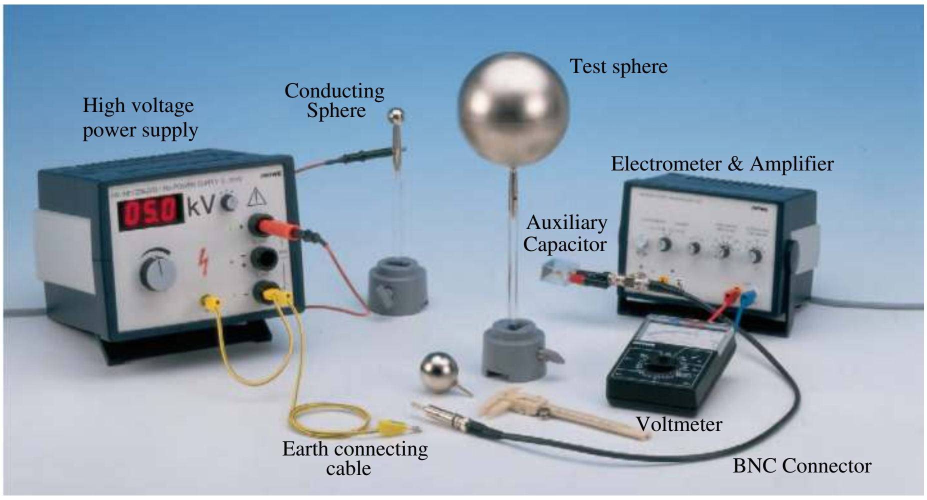


Figure 8.1: Experimental set-up to determine the capacitance of conduction spheres.

The capacitance of the conductor C_{co} is in pF, is much less than the capacitance of the auxiliary capacitor C_{ca} (10 nF), i.e., $C_{co} \ll C_{ca}$. Therefore, we can approximate Eq. 8.2 by

$$Q \approx C_{ca} V_2, \tag{8.3}$$

without introducing much error. Equating Eqs. 8.1 and 8.3, we get

$$\frac{V_2}{V_1} = \frac{C_{co}}{C_{ca}}. \tag{8.4}$$

Which means that if we plot V_2 as a function of V_1, we get a straight line whose slope will be C_{co}/C_{ca}, which can be obtained by data fitting. Since the value of C_{ca} is given, we can calculate the capacitance of the conductor.

Theoretically, the capacitance C of a sphere of radius R is given by

$$C = 4\pi\epsilon_0 R, \tag{8.5}$$

where $\epsilon_0 = 8.86 \times 10^{-12}$ F$\cdot$m^{-1} (Farads per meter) is called the *vacuum permittivity, permittivity of free space* or *electric constant.*

8.3 Procedure

1. The two spheres are held on a barrel base and insulated against the latter. Separate them from each other by approximately 1 meter. Refer Fig. 8.1 for connections.

2. Connect the smaller sphere by means of the high voltage cord over the 10 MΩ protective resistor to the positive pole of the 10 kV output of the high voltage power supply. The negative pole is earthed.

3. Ground both the spheres. Let both the electrometer and power supply be in switched off position.

4. Turn on the power supply and let adjust the knob to set the desired voltage. Keep the test sphere grounded and the electrometer off while the smaller sphere charges for some time.

5. Remove the earthing cable from the test (bigger) sphere and the small sphere is briefly brought into contact with the test spheres to charge it. High voltage always must be reset to zero after charging.

6. Connect the test (bigger) sphere to the electrometer using the BNC connector provided.

7. Connect the voltmeter to the output of the electrometer and measure the voltage across the auxiliary capacitor with capacitance $C_{ca} = 10\text{nF}$ which is connected in parallel to the BNC connector.

8. Repeat steps 3 and above by increasing the charging voltage by 1 kV.

8.4 Results

- Obtain and plot the data of V_2 (in volts) as a function of V_1 (in kV) for metal spheres of various diameters.
- Fit the data to obtain the slope. Find the capacitance of the conductor by using Eq. 8.4 and the value of the slope.
- Compare the experimentally obtained conductance with the theoretical value given by Eq. 8.5.

8.5 Precautions

- The power supply and the electrometer have internal circuits that can lead to induction of charges and may affect the readings.
- Remember that you are working with a high voltage power supply. So be careful while operating it.

Experiment 9

Dielectric constant of different materials using parallel plate capacitor

Anzar Ali and Abhinay Vardhan

9.1 The Experiment

9.1.1 Aim

The aim of the experiment is

- To evaluate the capacitance of a parallel plate capacitor.
- To measure the dielectric constants of different materials.

9.1.2 Apparatus Used

High voltage power supply, plastic capacitor, plastic plate 283x283 mm, glass plate, $10M\Omega$ resistor, universal measuring amplifier, $0.22\mu F$ capacitor, voltmeter, connecting cords, BNC cables and adapter.

The experimental setup is shown if Fig. 9.1 (left) and the corresponding circuit diagram is shown in Fig. 9.1(right). The highly insulated capacitor plate is connected to the upper connector of the high voltage power supply over the 10 MV protective resistor. Both the middle connector of the high voltage power supply and the opposite capacitor plate are grounded over the 0.22μF capacitor.

9.2 Theory

The electric charge Q accumulated on the conductor which is connected to a power source is directly proportional to the potential difference V applied on it

$$Q = CV, \tag{9.1}$$

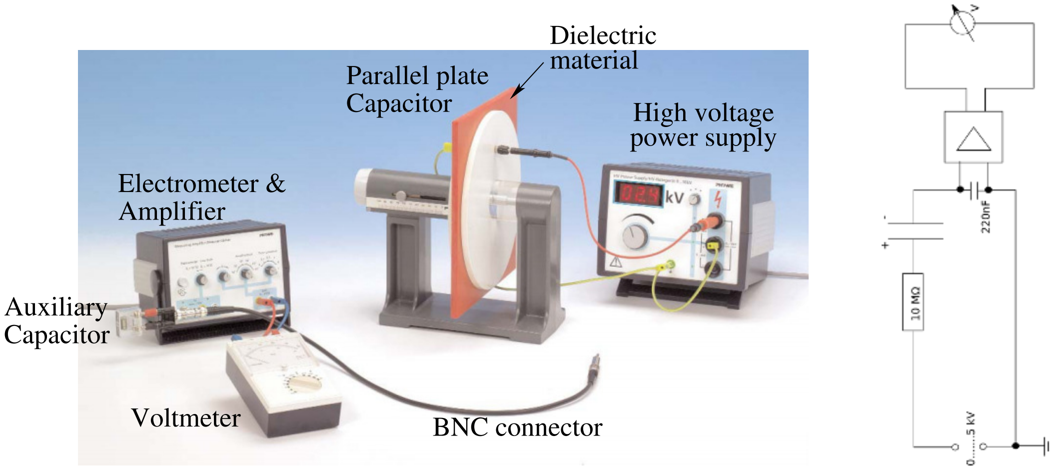


Figure 9.1: (Left) Experimental set-up to determine the dielectric constant of different materials. (Right) Circuit diagram for the experiment.

where C is a proportionality constant known as the capacitance of the conductor. It gives a measure of the ability of a body to store an electric charge. Higher the capacitance, more charge it can store. The capacitance can be increased by placing two conductors separated by a non conducting region which can either be a vacuum or an electrical insulator material known as a dielectric. Such an arrangement of conductors are called a capacitor. The capacitance C of such an arrangement depends on the size and shape of the conductors and the separation between them. The simplest arrangement is called a parallel plate capacitor in which a pair of flat conducting plates of surface area A separated by a distance d. The capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}, \tag{9.2}$$

where $\varepsilon_0 = 8.86 \times 10^{-12}$ F·m^{-1} (Farads per meter) or (C^2/Nm^2 in SI units) is called the *vacuum permittivity, permittivity of free space* or *electric constant*. The charge stored in the parallel plate capacitor when a potential difference of V is applied on its plates is given by

$$Q = \frac{\varepsilon_0 A}{d} V. \tag{9.3}$$

This relation allows us to determine the electric constant ε_0. If a material having dielectric constant ε is inserted between the plates, the charge stored in the capacitor becomes

$$Q = \frac{\varepsilon A}{d} V. \tag{9.4}$$

9.3 Procedure

There are mainly two parts of the experiment,

1. To observe the change in charge on the capacitors as we change the distance betwen them, and

2. To find the dielectric constant of different dielectric materials namely, i.e., air, plastic board and glass plate

First task is to ensure that no charge is present on both the plates by reading the potential across the $0.22\mu F$ capacitor, which should be zero. If not, ground the plates so that the charge on each of them becomes zero. This can be done by using the *push to zero* button on the universal measuring amplifier.

Variation as distance

1. Separate the two plates by a minimum distance $d = 0.5$ cm and fix the high voltage power supply at a particular value.
2. Increase or decrease the distance d between the plates and observe the value of potential across the $0.22\mu F$ capacitor.
3. The amount of charge Q stored in the capacitor can be calculated by using Eq. 9.3. Plot a graph of Q as a function of $1/d$. Fit the data to a straight line and obtain the value of ε_0 from the slope.

Dielectric Constant of different materials

The dielectric constant of various materials can be obtained using following steps:

1. Insert a plate made from the dielectric material and held it tightly between the parallel plates of the capacitor. The thickness of dielectric plate can be measured by the scale attached to capacitor plates.
2. Slowly increase the potential between the plates and read the potential across $0.22\mu F$ capacitor.
3. Convert the voltage into charge Q using Eq. 9.1 and plot a graph of charge Q as a function of potential V. The data is fitted to a straight line and the dielectric constant of the material can be calculated from the slope of the straight line.

9.4 Precaution

The following precautions have to be taken while doing the experiment:

1. Ground the plates before starting any part of the experiment to ensure no charge resides on the capacitor plates or on the dielectric material.
2. Try to stay as far as possible from the capacitor plates to avoid any fluctuations.
3. Wait for sometime after changing the distance or the potential between the plates for the charges to get stabilized.
4. Remember that you are working with a high voltage power supply. Therefore, never touch the inside (metallic part) of the capacitors.

Experiment 10

Experiment with an Induction Cooker

Arnob Mukherjee and Rajeev Kapri

10.1 The Experiment

10.1.1 Aim

To measure the efficiency of an induction cooker and observe the following physical phenomena using it

- Levitation of a copper ring.
- Cooker as a transformer.
- Observing damped oscillations.

10.2 Introduction

The main element of the induction cooker is the induction coil (an electromagnet) placed under the ceramic plate. It creates, together with the bottom of the pottery, an electromagnetic circuit which causes only the lower base of the utensil to be heated. It is necessary to use the utensils made from conductive, magnetized material as cast iron, enamel and other specialized materials with a flat bottom, which can be $12-30$ cm in diameter. Cooking is entirely safe as the surface of the plate remains cold during cooking.

10.3 Induction Cooker Description

Induction cooker consists of following parts:

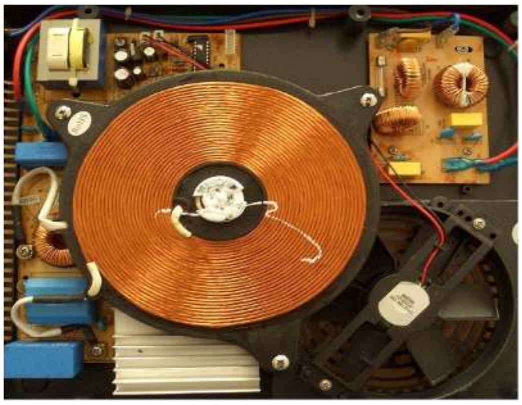


Figure 10.1: A high-frequency electromagnetic coil inside an induction cooker.

- Specially designed induction coil to generate electro magnetic field,
- a high frequency oscillator,
- power supply circuits,
- control and timer circuits and
- a ceramic cook top

The heart of an induction cooker is a high-frequency electromagnetic coil (see Fig. 10.1, which produces a strong high-frequency electromagnetic field. This coil is powered by a high-frequency oscillator (20 to 75 KHz). This coil is placed just beneath a shiny ceramic plate on to which we put cooking vessels. Whenever a magnetic material (iron, or alloys or iron) is placed over this coil high-frequency magnetic field passes through the pot which generates heat within metal due to eddy current and magnetic hysteresis. Eddy currents are induced in the magnetic material due to the change in the magnetic field. This current flows in a direction which opposes the main current and generates heat. Eddy current is directly proportional to the frequency. Hence a high-frequency current can generate more heat.

Another phenomenon is magnetic hysteresis. In simple words, it is the resistance of magnetic materials to the rapid change in magnetization. The inertia of magnetic particles in magnetic materials due to the changing magnetic field generate heat within the material. This heat is also directly proportional to the frequency of the magnetic field.

Here we can see that the cooking vessel itself is the heat generating element and the whole magnetic field is passing through it. Whenever you remove the vessel from the heater or stops the current flow through the coil, heat generation will be stopped.

10.4 Experimental Procedure

10.4.1 Efficiency of the cooker

To find the efficiency of the induction cooker

1. Take 1 liter (mass $m = 1$ kg) of water in the pot and record its initial temperature T_i.

Figure 10.2: Apparatus to show induction cooker as a transformer. (Left) A 6 V bulb connected in a holder with single coil. The number of turns in the coil increases if we use bulbs of (Middle) 24 V, and (Right) 230 V.

2. Cover the pot with a lid and heat the water for time t (say $t = 1$ minute) using the induction cooker kept at maximum power and calculate the amount of heat generated by the induction cooker. This is the input heat Q_{in}.

3. Record the final temperature T_f of the water. Take enough precaution so that only small amount of heat and water vapour gets released outside the system.

4. The heat used to raise the temperature of the water from T_i to T_f can be obtained *to a first approximation* using

$$Q_{out} = mC\left(T_f - T_i\right), \tag{10.1}$$

where $C = 4.187$ kJ/kg K is the specific heat of water.

5. The efficiency of the induction cooker can then be obtained by

$$\eta = \frac{Q_{out}}{Q_{in}} \times 100\%. \tag{10.2}$$

10.4.2 Levitation of copper ring

1. Take a copper wire (thickness 1 mm) which is coiled into a loop of diameter 10 cm and connected by a clamp.

2. Place a small metal pot filled with a little water (so not to overheat it) on the plate inside the copper circle.

3. Observe and explain what happens when the cooker is switched on.

   Note that this experiment should not last too long as the ring gets enormously heated which can melt the insulation

   An aluminium sheet can also be used in place of a copper wire and metal pot to observer the same phenomena.

10.4.3 Cooker as a transformer

1. Attach a bulb socket having a light bulb of 6 V (e.g. 0,1 A) with the copper ring (see Fig. 10.2(left)).

Figure 10.3: Induction cooker connected to an oscilloscope.

2. Place this coil on the plate of the cooker and place the pot above the plate (why?). Does the bulb glow?. How can you explain this.

3. Repeat the above arrangement with bulbs of different ratings, like 24 V , 230 V, and make them glow (see Figs. 10.2(middle) and 10.2(right)).

10.4.4 Observing Damped Oscillations

1. Connect one end of the single loop of the oscilloscope probe to the digital oscilloscope and approach another end to the plate [it is enough to connect the grounding conductor of the probe to the tip (see Fig. 10.3)].

2. Switch on the cooker. No need to place a pot onto the plate.

3. Observe the oscilloscope display. Can you explain it?

10.5 Precautions

1. Pay attention at any conductive rings on your hands if you handle any objects in the distance of some centimeters above the plate. Short influence of the electromagnetic field on your hands is not dangerous, but well conductive ring in appropriate position can be heated to high temperature in few seconds.

2. While performing the experiment, it is necessary to pay attention as we are working with dangerous induced voltage (even though the coil has only 40 turns)! We can only touch the bulb socket, not the wires which are not sufficiently insulated.

3. Placing the induction cooker during the experiments on a matallic table is not advisable.

4. Do not place empty utensils on a functioning induction stove since the utensil can heat up rapidly.

5. Do not place induction cook top near computers, laptops, etc. which are sensitive to magnetic field.

Experiment 11

Equipotential Lines

KETAN PATEL

11.1 Aim

To draw the lines of constant electric potential in water for different arrangements of electrodes.

11.2 Theory

Let's first understand the concept of electric potential. (Please go through this section before starting the experiment and try to understand as much as possible. Some the concepts discussed below will be introduced in your theory course of electromagnetism.)

11.2.1 From the Coulomb's law to the concept of potential

All of the electrostatics (study of forces between the static charges) follows from the Coulomb's law. It states that in vacuum the force on a point test charge Q due to a single point charge q which is at distance r away from the test charge is given by

$$\mathbf{F} = \frac{Q}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \ , \tag{11.1}$$

where $\hat{r}$ is a direction from the location of q to Q and ϵ_0 is a constant called permitivity of free space. If there exist more than one point charges $q_1, \ q_2, \ q_3, ..., \ q_n$ at the distances $r_1, \ r_2, \ r_3, ..., \ r_N$ away from the test charge Q, the total force is can be written as vector sum of forces created by each of q_i, *i.e.*

$$\mathbf{F} = \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\mathbf{r_i}} \ , \tag{11.2}$$

It is quite useful to rewrite the above formula as

$$\mathbf{F} = Q\mathbf{E} \quad \text{with} \quad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_\mathbf{i} \tag{11.3}$$

The $\mathbf{E}$ define above is called an Electric Field. It provides conceptual simplification of the Coulomb's law by providing test charge free interpretation of the force that is created by given charge distribution. *We highly encourage you to think more about the interpretation of* $\mathbf{E}$.

If there are many charges confined in the small region (let's say if Δq is the total charge confined in the region $\Delta x\ \Delta y\ \Delta z$ centered at the point $\mathbf{x}' \equiv (x', y', z')$) then the sum in eq. (11.3) is replaced by and integral and the electric field at some point $\mathbf{x} \equiv (x, y, z)$ is given by:

$$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{x}')}{r^2}\hat{\mathbf{r}}\ d^3x'\ , \tag{11.4}$$

where $\mathbf{r} \equiv r\hat{\mathbf{r}} = \mathbf{x} - \mathbf{x}'$, $\rho(\mathbf{x}') = \Delta q/(\Delta x\ \Delta y\ \Delta z)$ is a charge density at point at $\mathbf{x}'$ and $d^3x' = dx'\ dy'\ dz'$ is a volume element at $\mathbf{x}'$. Eq. (11.4) gives an electric field at a given point in space that is produced by charge distributions located everywhere else. (The electric field at a point nearby you has contributions also from the charges distributed in the entire universe!)

One can further simplify eq. (11.4). To achieve this, we use

$$\frac{\hat{\mathbf{r}}}{r^2} = -\nabla\left(\frac{1}{r}\right)\ , \tag{11.5}$$

where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$. The advantage of the above equation is that it represents the vector quantity (in the LHS) with a gradient of a scalar quantity (in the RHS). [Show that the relation in eq. (11.5) holds if you haven't already done it once in your life.]

Using eq. (11.5) in eq. (11.4), one can write (note that the integration is over primed coordinates so it is possible to write)

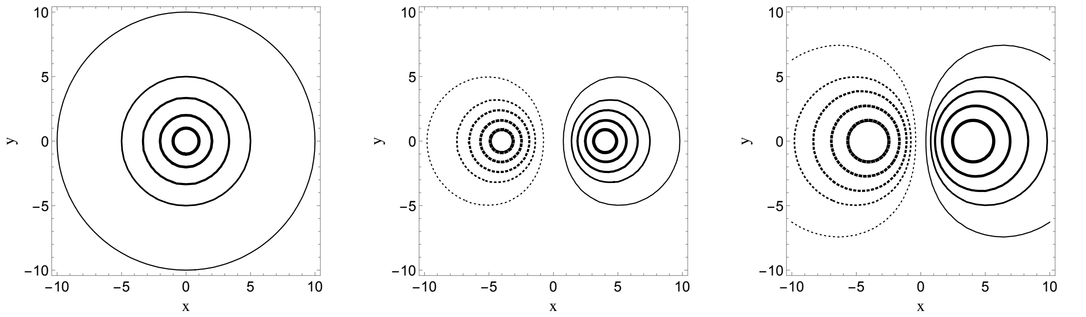
$$\mathbf{E}(\mathbf{x}) = -\nabla\left(\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{x}')}{r}\ d^3x'\right) \equiv -\nabla\phi(\mathbf{x})\ . \tag{11.6}$$

where

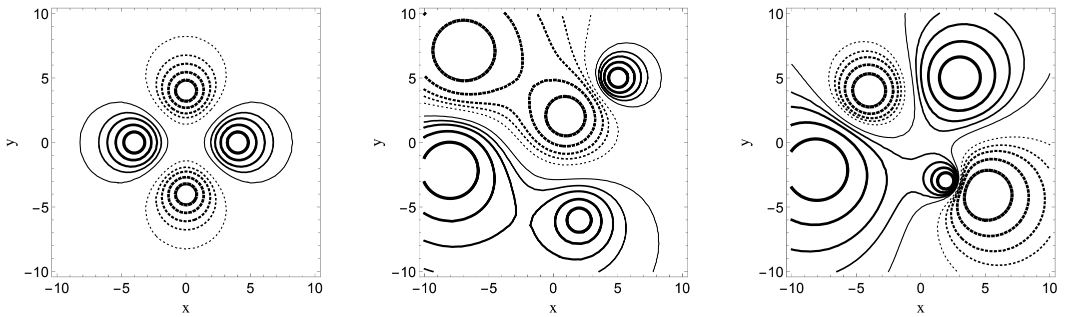
$$\phi(\mathbf{x}) \equiv \phi(x, y, z) = \left(\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{x}')}{r}\ d^3x'\right)\ . \tag{11.7}$$

is called **electric scalar potential**. Because of its simple scalar nature, $\phi(\mathbf{x})$ can easily be evaluated for given charge distribution. One can derive electric field and force acting on a test charge in the presence of this field very easily from $\phi(\mathbf{x})$.

[**Note:** Using eq. (11.6), you can show (a) $\nabla \times \mathbf{E} = 0$ and (b) The $\phi(\mathbf{x})$ and $\phi(\mathbf{x}) + c$ lead to the same $\mathbf{E}(\mathbf{x})$ if c is a constant function in space.]



(a) Charge configuration 1 (b) Charge configuration 2 (c) Charge configuration 3



(d) Charge configuration 4 (e) Charge configuration 5 (f) Charge configuration 6

Figure 11.1: Equipotential lines obtained using eq. (11.7) for different configurations of point charges.

Using eq. (11.7), we draw equipotential lines for some simple point charge configurations. Consider the point charges in two dimensions:

1. $+q$ at $(0, 0)$.
2. $+q$ at $(4, 0)$ and $-q$ at $(-4, 0)$.
3. $+2q$ at $(4, 0)$ and $-2q$ at $(-4, 0)$.
4. $+q$ at $(4, 0)$, $(-4, 0)$ and $-q$ at $(0, 4)$, $(0, -4)$.
5. $+q$ at $(5, 5)$, $(2, -6)$, $+3q$ at $(-8, -2)$, $-2q$ at $(1, 2)$ and $-3q$ at $(-7, 7)$.
6. Try to guess this charge configuration from Figure.

The equipotential lines for the above charge configurations are shown in respective figures below. In all the figures, the lines, in descending order of thickness, show the $\phi = \pm 1, \ \pm 0.5, \ \pm 0.3, \ \pm 0.2, \ \pm 0.1$ in units of $q/(4\pi\epsilon_0)$. The continuous lines correspond to positive ϕ while dashed lines correspond to the negative values of ϕ.

- Compare the different equipotenrial lines in a given figure. Make also comparison between the equipotential lines presenting same potential in two different figures.

- How does the strength of potential decrease in each configuration when moving away fro the charges?

11.3 Experiment

As described in the above, the equipotential surface is an imaginary surface described by the points at which the electric scalar potential possesses the same value. We would be performing our experiment on a two dimensional surface (on a graph paper), therefore we would talk about equipotential lines and not surface.

The equipotential lines are collections for points $\{(x_1, y_1),\ (x_2, y_2), ...,\ (x_n, y_n)\}$ in two dimensions for which the potential satisfies the condition $\phi(x_1, y_1) = \phi(x_2, y_2) = ..., = \phi(x_n, y_n)$.

Since it is experimentally difficult to probe the potential in vacuum or in air, we would use water as a medium.

11.3.1 Procedure

You are given a D.C. power source, a voltmeter, different kind of electrodes (bars and discs), a metal ring, a transparent container for water and a test probe.

1. Take two graph papers and place two bar electrodes with maximum distance between them on the graph paper. Mark their position identically on both the graph papers.

2. Place a clear transparent container **on** one of the graph paper. Place the bar electrodes on already marked positions.

3. **Don't trust your instruments.** Take a battery of known output voltage and check if the voltmeter is calibrated correctly. If yes, using that voltmeter check the *actual* output DC voltage of power source given to you. Note down this actual supplied voltage.

4. Make connections as shown in the Fig. **??**. Connect one electrode with +ve and other with -ve output of power source. Connect the +ve of voltmeter with test probe and -ve of voltmeter with -ve of power source.

5. Fill the container with normal water until the electrodes get half submerged into the water. Place a test probe such that it touches the water.

6. Switch on the power source. Starting at some point, note down the potential. Move your test probe in such a way that the potential remain constant. Note down this positions and mark them on the other graph paper. Connects the points representing the same value of potential.

7. Start at another point and do the same. Repeat this until you get a clear picture of how potential is distributed over the entire surface for a given arrangement of electrodes.

8. **Use imagination**: Draw equipotential lines for **at least four** different configurations of electrodes. Use different electrodes, put them in the way you wish (not necessarily in some symmetric pattern) and draw the equipotential lines. You can use also more than 2 electrodes kept at different potentials. Also, place the metal ring in the water and check the potential inside and outside of it.

11.3.2 After performing the experiment

- Observe carefully all the figures of equipotential lines that you have obtained for different electrodes configuration. Try to interpret them.
- Make comparisons between theoretical equipotential lines drawn in Fig. 1 with the equipotential lines you obtain from the experiment.
- Go again through the theory section. Can you tell how the electric field will look like in each case? Is it possible to derive electric field from the potential configuration that you have drawn?
- What is your proposal for measuring equipotential surfaces in three dimensions?