PHY111: MECHANICS LABORATORY
LABORATORY MANUAL

(August - December 2014)

Department of Physical Sciences

IISER Mohali

Preface to 2014 version

In this version I have corrected mistakes pointed out by the students of MS13 batch.

R. Kapri
August 2014

Preface to 2012 version

The release of this version of the manual is delayed because I had to redo most of the things as I could not get the soft copies of the previously revised version. Thanks to Dr. Kavita Dorai for providing me LATEX files of the first draft of the manual written by her in 2008. That saved a lot of time. I was fortunate enough to find the copy of the manual used by Prof. H. S. Mani. In that copy he had marked the changes in the Kater's pendulum experiment, so I was able to incorporate those corrections in this version. Thanks to Dr. Ananth Venkatasan for providing me the hard copy of the manual he had used in 2010.

In this version the following changes are made: All the figures in the manual are redrawn. The chapter "Notes on error analysis" is revised. The section on the Gaussian distribution was expanded and the sections on error propagation and data fitting were rewritten. The chapters on "Compound pendulum" and "Kater's pendulum" are also revised. The chapter on "motion of gyroscope" were retyped. A new experiment "Moment of inertia of a flywheel" is also added.

The manual still needs revision. I request my colleagues and the students to read it carefully and point out mistakes to me so that it can be corrected in the newer version of the manual.

R. Kapri
November 2012

Preface to 2010 version

You can find good references on general experimental techniques in the library. The actual experiment you do also involves understanding the details of the apparatus available locally. Hence the purpose of the lab manual other than giving some background material is also to provide some details of the experimental setup at IISER Mohali.

When I was assigned this course in August 2010 I was fortunate that some of my colleagues took the initiative to write a lab manual for this course. Writing a lab manual can be a formidable task that may also get unappreciated. Hence I would like to mention the name of the Faculty and students who contributed to this manual.

The first version of this lab manual was written by Dr. Kavita Dorai when she taught the August 2008 session. Dr. Kavita sought some colleagues help to revise some chapters. The chapter on Kater's pendulum was rewritten by Prof. H. S. Mani. Dr. Rajeev Kapri revised the chapters on Maxwell's needle and Young's modulus by bending of beams. Dr. Pranaw Rungta rewrote the chapter on gyroscope. Two project students Mr. Abhishek Anand and Mr. Harsh Katyayan provided short project reports on the gyroscope that Dr. Runga found useful in writing the instructions for the gyroscope.

A lab manual should continually evolve both in improving the instructions and also the adding new experiments. Students can also contribute towards this. Anyone interested in setting up new experiments can approach me. Any contribution by students, like correction of any mistakes or adding new experiments will get acknowledged in this preface.

V. Ananth
August 2010

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Chapter 1

General Instructions

1.1 Objectives of the Lab Course

Laboratory work is at the core of physics and as scientists in the making, you should utilize your time in the physics lab to the fullest. In the 300 years since the time of Galileo and Newton, physics courses have come to rely on a formal approach, wherein the laws of physics are presented like mathematical axioms and various results are obtained by logical deduction. This is convenient to encapsulate centuries of insight into 12-14 weeks of study in a semester-long course. However, it de-emphasizes the fact that the laws of physics are descriptions of behavior observed in nature. Physics is an interpretation by people of their experience of the universe around them. If physics claims to be an objective description of nature/reality, it is because anyone who tries can validate the laws of physics by direct test.

This laboratory will give you the opportunity to explore various simple situations from which you can form your own assessment as to the validity of the laws of physics. The emphasis is not on discovery of novel phenomena but on an understanding of the accuracy of the physics description of basic aspects of mechanics. An important tool in this exploration is error analysis, which allows you to be quantitative as to the significance of your results, as well as lets you decide whether you have achieved results as accurate as can be expected with the apparatus available.

1.2 Grading

Your grade in this lab depends on the following parameters:

1. Grade for every lab session.
2. Lab Record of each experiment.
3. Final endsemester examination.

1.2.1 Grade for every laboratory session

You will be graded at the end of EVERY lab session. The grade given by the Lab Instructor is based on your demonstrated ability to think independently and perform the experiment with minimal instructions, your grasp of the basic physics underlying the experiment, and excitement/fervor shown in doing the experiment. You may discuss with the instructor any ideas you have to improve an existing experimental setup or work out a different method to arrive at the same result.

1.3 The Lab Record

Your Lab Records will remain in the lab and you are not permitted to take them away with you. ALL Physics Lab related activity such as performing the experiments, and writing up your work in the lab record has to be done in the lab. Start a table of contents (the name of the experiment and the date) on the first page of your lab record. After completing an experiment, you are required to maintain an **intelligent log** of the entire experiment. This NECESSARILY includes (but is not restricted to): A brief description of the experiment's motivation (try to predict quantitatively what you think will happen in the experiment), your tabulated observations, your results, DETAILED ERROR ANALYSIS, the sources of error you identified and how each contributed to the overall error in your measurement (quantify your results), and a brief paragraph describing what you found exciting (or boring) about the experiment, your suggestions to improve the experimental design(if you have any). Write in your notebooks in ink. If you make a mistake, cross it out with a single line. Knowing about your mistakes helps you gain experience and helps the grader understand what you did. Make a block diagram of the apparatus to help you remember what you connected where. Do not copy the theory from the lab manual but do include formulas that you will need to refer to later. Take brief notes as you work and write it down in the record notebook (do NOT waste time by working in rough and trying to transcribe it later in the notebook). Describe problems you have and what you do to fix them, so that you can avoid them in the future. Do write a brief description of the procedure you followed in doing the experiment and any safety measures you took (and deviations from the standard procedure outlined in the lab manual if any). Remember that while there is no need to reproduce stuff which is already in the lab manual, your record should have sufficient information for you to be able to recall the experiment that you did and the physics involved, three years from now.

1.3.1 Analysis & Conclusions

The record of your experiment should clearly summarize what you did:

- Was your prediction correct? If not, why not?
- Summarize any numerical results
- Do your results agree with theory? (It's OK if they don't. You should be able to convince the instructor as to why your experiment did not match the theory)
- Do your results seem reasonable to you?

1.3.2 If Your Experiment is a Total Disaster

- Don't panic. Talk to your instructor and get help.
- Write out your experience in the lab record. What measurements did you make to diagnose the problem?
- What did you do to try to fix it?
- What might have gone wrong?
- How could you test for this?

1.4 Plotting of graphs

All graphs you plot should be on the graph sheets included at the end of your record notebook (Do NOT tear them out). **Title each graph with the name of the experiment**. All error analysis and plots generated using **Gnuplot** (or other plotting software) should be printed out and included in your lab record. THE AXES OF EACH GRAPH MUST BE LABELED AND THE UNITS PROPERLY DISPLAYED.

1.5 Error Analysis

A detailed understanding of the errors that contributed to YOUR measurements is an ESSENTIAL part of the experiment and MUST be written up in your lab record. Make all your experimental observations directly in the lab record to save time. Do NOT attempt to do them in rough and then transcribe them later. If you decide to abandon a set of observations, write out your reasons for doing so and repeat the observations in a separate table on a separate page of the lab record.

1.6 Using your hands

As far as possible, this is a minimal instruction lab - to encourage you to develop confidence in your ability to handle instruments and to foster curiosity about the basic physics behind each experiment. You are hence encouraged to handle the experimental setups yourself and familiarise yourself with the working of each and every part of the setup. In the process of familiarisation, please observe all **safety precautions** and handle each experimental setup with as much care as you would if you had designed and built it yourself. Before embarking upon any experiment, read the lab manual carefully and ask the instructor/tutor in case you have any doubts. If something does not work, try and figure out the probable causes and ways to fix the problem before approaching the instructor/tutor for help.

1.7 Date of correction

The lab record for an experiment MUST be dated and MUST be completed either by or during the week after you complete the experiment and are allotted a new experiment. KEEP WELL WITHIN THIS DEADLINE. You will progressively lose 10% of the grade for every week of delay in maintaining your lab record.

1.8 Makeup of missing lab sessions

As far as possible, try not to miss scheduled lab sessions. Since you will be continuously evaluated throughout the semester, missing a lab will adversely affect your final grade. If you were unwell and simply had to miss a lab session, make it a point to get the instructor/tutor's permission to do the experiment on your own during a free period. These extra lab sessions will have to be fixed at the mutual convenience of the instructor/tutor, the lab supervisor and the student concerned.

Chapter 2

Notes on Error Analysis

2.1 Introduction

Error or uncertainty about a particular experimental measurement is the best estimate of the quantitative range within which you can trust your results. Any experimental measurement you make in the laboratory is meaningless unless quoted with an uncertainty/error. We are not talking about errors like misreading a scale or slipping a decimal point while taking a reading. Experimental uncertainties are a statement about the **resolution** of your measurement i.e. how far from the "true" value you are likely to be. There are two kinds of uncertainties associated with the measurement of an experimental quantity:

- **Random uncertainty**: associated with unpredictable variations in the experimental conditions. For example changes in room temperature, vibrations from nearby machinery, error in time period measurement when the experimenter does not start/stop the stopwatch at exactly the same point in the swing of the pendulum etc. So if a measurement is repeated a number of times with sufficient precision, a slightly different value of the measured quantity is obtained each time and if the experiment is free from bias these variations will be random and the measurements will group symmetrically about the "true" value.
- **Systematic uncertainty**: associated with inherent faults in measuring instrument or in measurement technique. This is an error that is consistent from measurement to measurement. For example, measuring length of a table with a tape that has a kink in it, a weak spring in a current meter, a calibration error in the measuring device, a clock that runs too fast etc. So if there is an experimental bias, the measurements will group around the wrong value and are said to contain a systematic error. If you always round down to the nearest tic mark on a meter stick while measuring length, you will make a systematic error of measuring a slightly shorter length.

Random uncertainties are easier to quantify and deal with. There is no general procedure for estimating the magnitude of systematic uncertainties.

2.2 Precision vs Accuracy

Random uncertainty decreases the **precision** of an experiment whereas **systematic** uncertainty decreases the **accuracy** of the experiment.

NOTE:- Systematic uncertainty does NOT mean that the uncertainty is repeatable. It means that the uncertainty has not been accounted for in the analysis.

Accuracy refers to the degree to which your value is correct within uncertainty. It is largely a matter of having the correct calibration of all reference measurements. If you used a uncalibrated meter stick that was shorter than the official length of a meter, you might measure the length of an object with great precision (lots of decimal places) but poor accuracy (what you think is a meter is not really a meter).

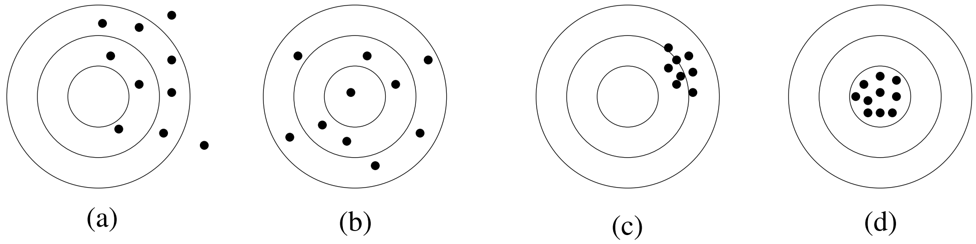


Figure 2.1: Random vs systematic errors.

Precision can be thought of as the number of meaningful digits to a measurement. A measurement of a length as being 1.023405 meters is more precise than a measurement of 1.02 meters.

As an illustration of the concepts of precision and accuracy, consider the analogy shown in Fig. 2.1. The measured quantity's true value lies at the center of all circles and the various dots represent the data points measured by the same apparatus.

- In the first experiment [Fig.2.1(a)], the data points show very different values and are scattered over the circles. In this case, the random as well as systematic errors are large and so the *measurement is neither precise, nor accurate.*
- In the second experiment [Fig.2.1(b)], the random errors are large but the systematic errors are small. The uncertainty in each measurement is large, so the *measurements are accurate but not precise.*
- In the third experiment [Fig. 2.1(c)], the values lie within an experimental uncertainty, that is, the random errors are small but since all the measurements are away from the center, the systematic errors are large. Therefore, the *measurements are precise but not accurate.*
- In the final experiment [Fig. 2.1(d)], the values lie both within an experimental uncertainty and the actual value, that is, the measured value is *precise and accurate.*

If we remove the circles from Fig. 2.1, we do not know the true value of the quantity being measured. In this situation, we can still assess the random errors (i.e., the precision of the measured quantity) easily but it is impossible to estimate systematic errors, i.e., we do not know if our measured quantity is accurate!

2.3 Three major sources of errors

2.3.1 Reading Error

Almost all direct measurements involve reading a scale (ruler, caliper, stopwatch, analog voltmeter, etc.) or a digital display (e.g., digital multimeter or digital clock). Sources of uncertainty depend on the equipment we use. One of the unavoidable sources of errors is a reading error. Reading Error refers to the uncertainties caused by the limitations of our measuring equipment and/or our own limitations at the time of measurement (for example, our reaction time while starting or stopping a stopwatch). This does not refer to any mistakes you may make while taking the measurements. Rather it refers to the uncertainty inherent to the instrument and your own ability to minimize this uncertainty. A reading error affects the precision of the experiment. The uncertainty associated with the reading of the scale and the need to interpolate between scale markings is relatively easy to estimate. For example, consider the millimeter (mm) markings on a ruler scale. For a person with a normal vision it is reasonable to say that the length could be read to the nearest millimeter at best. Therefore, a reasonable estimate of the uncertainty in this case would be $\Delta l = \pm 0.5$ mm which is half of the smallest division. A rule of thumb for evaluating the reading error on analogue readout is to use half of the smallest division (in case of a meter stick with millimeter divisions it is 0.5 mm), but only the observer can ultimately decide what is his/her limitation in error evaluation. Note that it is wrong to assume that the uncertainty is always half of the smallest division of the scale. For example, for a person with a poor vision the uncertainty while using the same ruler might be greater than one millimeter. If the scale markings are further apart (for example, meter stick with markings 1 cm apart), one might reasonably decide that the length could be read to one-fifth or one-fourth of the smallest division. It is an estimate of systematic differences between different scales of the multimeter. However it is the random error that determines the precision, and gives you an idea of the scatter that you might expect in your readings. Thus, the "$\pm$ digit" quoted by the manufacturer might be a better estimate of the random error. Though you should quote the systematic error at the end of your experiment when you are comparing your result with some "standard", it is better to use 1 digit for the random error in each reading. For example, if your reading is 3.48 mA, you should quote (3.48 ± 0.01) mA. It is usually difficult or impossible to reduce the inherent reading error in an instrument. In some cases (usually those in which the reading error of the instrument approximates a "random error distribution") it is possible to reduce the reading error by repeating measurements of exactly the same quantity and averaging them.

2.3.2 Random Error

Random Error refers to the spread in the values of a physical quantity from one measurement of the quantity to the next, caused by random fluctuations in the measured value. For example, in repeating measurements of the time taken for a ball to fall through a given height, the varying initial conditions, random fluctuations in air motion, the variation of your reaction time in starting and stopping a watch, etc., will lead to a significant spread in the times obtained. This type of error also affects the precision of the experiment.

2.3.3 Systematic Error & Instrument Calibration

Systematic Error refers to an error which is present for every measurement of a given quantity; it may be caused by a bias on the part of the experimenter, a miscalibrated or even faulty measuring instrument, etc. Systematic errors affect the accuracy of the experiment. After evaluating the reading error or the standard error, or both if necessary, we have to make sure that the scale of our measuring instrument is checked against an internationally established measuring standard. Such comparison is called calibration. In the real world, we frequently find that our measuring scale is in slight disagreement with the standard. For example, if you inspect such simple tools as rulers, you will find out that no two rulers are exactly the same. It is not uncommon to find a discrepancy of 1 mm or even more among meter sticks. The correct calibration of measuring instruments is obviously of great importance. However, in the first year laboratory, the instruments you will use are usually calibrated by the laboratory staff and ready to use (unless explicit lab instructions tell you otherwise). In addition to all the errors discussed above, there can be other sources of error that may pass unnoticed: variations in temperature, humidity or air pressure, etc. Such disturbances are more or less constant during our measurements (otherwise they would appear as random error when the measurement is repeated) and are generally referred to as the systematic errors. Systematic errors are very difficult to trace since we do not know where to look for them. It is important to learn to notice all the irregularities that could become the sources of systematic errors during our experimental work. Moreover, it is particularly important in data-taking always to record some information about the surrounding physical conditions. Such information may help us later on if we discover a serious discrepancy in our experimental results. As a rule, the place, date and time of measurements, and the type and serial numbers and specifications of the instruments which were used must be recorded. Estimate all your reading errors while you take your data and write them down with your data. Do the same for all manufacturers' error specifications. These usually cannot be guessed later on.

2.4 Mean & Standard Deviation

Mean

If the sources of error in a measurement (say measuring the length of a table) are random, the values of the length will vary randomly above and below the "true" value of the table length, and will not be biased/skewed toward the lower/higher values. The procedure to get the most precise value for the length is to take the average or arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + ... + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{2.1}$$

where N is the number of measurements and x_i is the value of one measurement. This definition of mean assumes that each measurement of x is independent and has the same experimental uncertainty.

Standard Deviation

Now that the mean ("best" value) is known, it is important to quantify how much the individual measurements are scattered about the mean or how "good" each individual measurement is. If the experiment is precise, all measurements will be very close to the mean value. So the extent of scatter about the mean is a measure of the precision and a way to quantify the **random uncertainty**.

For unbiased measurements (all data points have equal weights), the standard deviation σ is

$$\sigma = \sqrt{\frac{1}{(N-1)}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{2.2}$$

σ becomes larger if the data is more scattered about the mean.
NOTE:- Convince yourself at this stage that more scatter of data means a larger standard deviation and also that σ has the same units as x_i.

Most Probable Value:

For unbiased measurements, the standard deviation of the *mean of a set of measurements*, σ_m, is

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}} = \frac{\sigma}{\sqrt{N}}. \tag{2.3}$$

This is important since it states that the uncertainty in the **mean** of N measurements decreases as $1/\sqrt{N}$.
NOTE:- Convince yourself that σ_m is necessarily smaller than σ. Also think about the difference between σ and σ_m: σ is the standard deviation associated with individual data points whereas σ_m is the standard deviation of *the mean value of a set of data points*, that is, the uncertainty of a set of measurements made under identical conditions.
EXERCISE:- For a Gaussian distribution, convince yourself that the mean will be within the range $\bar{x}_i \pm \sigma_i$ 68% of the time, i.e., if another set of N measurements is made, the mean of this new set has a 68% likelihood of being within the range $\bar{x}_i \pm \sigma_i$.

Random errors and Gaussian distributions

In some measurements, there is a random element involved. Say that you measure the fraction of times that a coin lands face up. You might refuse to make the measurement, saying that you know the answer: its going to land face up exactly 50% of the time. What if you make two measurements? If you flip the coin twice, do you expect it to land face up once, and face down once, every time you flip it twice? Of course not! Since each flip of the coin is uncorrelated with the previous flip (the coin has no reason to remember how it landed last time), there is an intrinsic measurement error

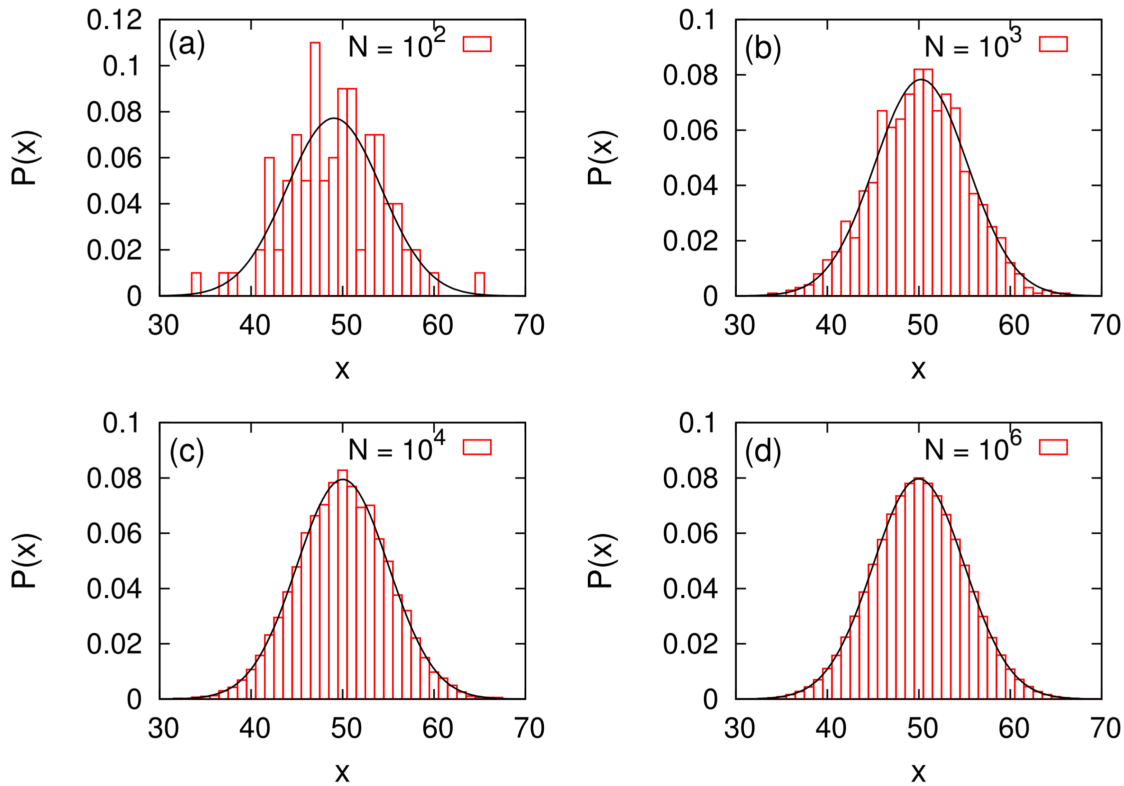


Figure 2.2: Probability distribution $P(x)$ of obtaining heads in a coin tossing experiment consisting of $N = 100$ tosses per trial. The histograms are the experimental data and the solid curve is the Gaussian fit, written in the brackets, to the data for (a) $M = 100$ trials ($\mu = 49.1 \pm 0.4$, $\sigma = 5.2 \pm 0.3$), (b) For $M = 10^3$ trials ($\mu = 50.3 \pm 0.1$, $\sigma = 5.09 \pm 0.08$) (c) For $M = 10^4$ trials ($\mu = 50.01 \pm 0.03$, $\sigma = 5.02 \pm 0.03$), (d) For $N = 10^6$ trials ($\mu = 49.996 \pm 0.003$, $\sigma = 5.005 \pm 0.002$).

which we can approximate as being equal to the square root of the number of events $\sqrt{N}$. If we flip a coin $N = 100$ times, we would expect to have $\mu = 50$ heads. About 2/3 of the time we will find that the number of heads we get is within the range $50 - \sqrt{50} \approx 43$ and $50 + \sqrt{50} \approx 57$, and 1/3 outside this range. In the continuum limit, we expect to get something like a Gaussian (or the normal) distribution of obtaining heads x:

$$P(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \tag{2.4}$$

about a mean value μ, with standard deviation σ. These two quantities completely define the Gaussian (or the normal) distribution.

In Fig. 2.2(a) to 2.2(d), we have plotted the probability distribution of obtaining heads in a coin tossing experiment (consisting of $N = 100$ tosses per trial) when the experiment is repeated $M = 10^2, 10^3, 10^4$, and 10^6 times, respectively. The average values of heads, μ, and the standard deviation, σ, for each case is reported in the brackets. We can clearly see that μ approaches the value $N/2 = 50$ as the number of trials M increases.

2.5 Stating your results: Absolute & Relative Uncertainty

In general, the result of any measurement of physical quantity must include both the value itself (best value) and its error (uncertainty). The result is usually quoted in the form

$$x = x_{best} \pm \Delta x \tag{2.5}$$

where x_{best} is the best estimate of what we believe is a true value of the physical quantity and Δx is the estimate of absolute error (uncertainty). Note that depending on the type of the experiment the prevailing error could be random or reading error. In case the reading error and random error are comparable in value, both should be taken into account and treated as two independent errors. You will learn how to calculate Δx in this case in the "Propagation of Errors" section. The meaning of the uncertainty Δx is that the true value of x probably lies between $(x_{best}\Delta x)$ and $(x_{best} + \Delta x)$. It is certainly possible that the correct value lies slightly outside this range. Note that your measurement can be regarded as satisfactory even if the accepted value lies slightly outside the estimated range of the measured value.

Δx indicates the reliability of the measurement, but the quality of the measurement also depends on the value of x_{best}. For example, an uncertainty of 1 cm in a distance of 1 km would indicate an unusually precise measurement, whereas the same uncertainty of 1 cm in a distance of 10 cm would result in a crude estimate. Fractional uncertainty gives us an indication how reliable our experiment is. Fractional uncertainty is defined as $\Delta x/x_{best}$ where Δx is the absolute uncertainty. Fractional uncertainty can be also represented in percentile form $(\Delta x/x)100\%$. For example, the length $l = (0.50 \pm 0.01)$m has a best fractional uncertainty of $0.01/0.5 = 0.02$ and a percentage uncertainty of $0.02100 = 2\%$. Note that the fractional uncertainty is a dimensionless quantity. Fractional uncertainties of about 10% or so are usually characteristic of rather rough measurements. Fractional uncertainties of 1 or 2% indicate fairly accurate measurements. Fractional uncertainties much less than 1% are not easy to achieve, and are rare in an introductory physics laboratory.

Percentage disagreement: In some cases, you can compare the value of your experimental measurement with the standard value as

$$|\frac{x_{std} - x_{exp}}{x_{std}}| \times 100\% \tag{2.6}$$

If your percentage disagreement is more than ten percent, identify the reasons and explain why this is so in your report.

NOTE:- This percentage disagreement is to give you an idea of the accuracy of your experiment and in no case is to be used as a substitute for the detailed error analysis of your experiment.

2.6 Significant Figures

An uncertainty should not be stated with too much precision. The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty. For example, the answer 92.81 s with an uncertainty of 0.3 s should be rounded as (92.8 ± 0.3) s. If the uncertainty is 3 s, then the result is reported as (93 ± 3) s. However, the number of significant figures used in the calculation of the uncertainty should generally be kept with one more significant figure than the appropriate number of significant figures in order to reduce the inaccuracies introduced by rounding off numbers. After the calculations, the final answer should be rounded off to remove this extra figure.

- The uncertainty σ should have 1 digit or at most 2 digits (all uncertainty calculations are estimates; there is no such thing as exact uncertainty!). The result itself should be stated to the same precision as σ, for example, $10.25 \pm 0.15 sec$ or $10.3 \pm 0.2 sec$ but NOT $10.25 \pm 0.2 sec$.

- If σ is very large, you will lose significant digits. If the measurement is so bad that σ is larger than the value itself, you will have no significant digits but only know the order of magnitude!

2.6.1 Practical Hints

So far, we have found two different errors that affect the precision of a directly measured quantity: the reading error and the standard error. Which one is the actual error of precision in the quantity? For practical purposes you can use the following criterion. Take one reading of the quantity to be measured, and make your best estimate of the reading error. Then repeat the measurement a few times. If the spread in the values you obtain is about the same size as the reading error or less, use the reading error. If the spread in values is greater than the reading error, take three or four more, and calculate a standard error and use it as the error. In cases where you have both a reading error and a standard error, choose the larger of the two as "the" error. Be aware that if the dominant source of error is the reading error, taking multiple measurements will not improve the precision.

2.6.2 Mistakes and Misconceptions

In the introductory physics laboratory, it is almost always meaningless to specify the error to more than two significant digits; often one is enough. It is a mistake to write: $x = (56.7 \pm 0.914606)$ cm, or x = (56.74057 ± 0.9) cm. Instead, write: $x = (56.7 \pm 0.9)$ cm. You cannot increase either the accuracy or precision by extending the number of digits in your mean value beyond the decimal place occupied by the error. Keep in mind that the error, by its nature, denotes the uncertainty in the last one or two significant digits of the main number and therefore any additional digits obtained from multiplication or division should be rounded off at the meaningful position. So, first calculate your error; round it off to one significant figure; then quote the value of your measurement to the appropriate number of significant figures.

When quoting errors in a result do not use the flawed logic that "my result is x, the handbook gives a value for this quantity as y, thus the error in my result is $\pm(x-y)$". Your quoted error should be the result of your own analysis of your own experiment whereas $(x - y)$ relates to a comparison of your work to other people's work. (xy) represents the difference between your result and the accepted value. The discrepancy can be used to characterize the consistency between different sets of measurements, but has nothing to do with the estimate of error in your own experiment. If a result we produce differs significantly from the accepted value, we then are obligated to explain what has produced the difference. But in quoting our own result, we must provide the error of our own experiment.

2.7 Propagation of Errors

In the majority of experiments the quantity of interest is not measured directly, but must be calculated from other quantities. Such measurements are called indirect. The quantities measured directly are not exact and have errors associated with them. While we calculate the parameter of interest from the directly measured values, it is said that the errors of the direct measurements propagate. Errors can propagate in measurements. What happens to the final uncertainty in a measurement which depends on several variables, each with its own uncertainty? The answer is not obvious and two cases are possible: when the uncertainties in the individual variables are **independent** and when the individual uncertainties are **dependent**. In this lab, you will work with the assumption that the individual uncertainties are completely independent.

As an example, consider the following problem. Suppose we have measured the value of a quantity x with an uncertainty, which we denote Δx. In order to test a theoretical formula, suppose that we need to calculate y as function of x i.e., $y = f(x)$. We want to know the uncertainty in y due to the uncertainty in the value of x. This is equivalent to asking what will be the variation in y (call it Δy) as x varies from x to $(x+\Delta x)$? Mathematically, this variation is given by $\Delta y = f(x+\Delta x)-f(x)$. The answer comes from the differential calculus: if $y = f(x)$ and Δx is small, then

$$\Delta y \approx \frac{dy}{dx}\Delta x = \frac{df}{dx}\Delta x \tag{2.7}$$

This argument can be extended for the calculation of quantities that are functions of several different measured quantities. All you will need at this point are the results that you can find below for different types of functions. Note that we neglect the sign in the differential, since the sign of all errors may take on numerical values which are either positive or negative.

2.7.1 Propagation of Independent Errors

Suppose various quantities $x_1, \cdots, x_n, w_1, \cdots, w_n$ with uncertainties $\Delta x_1, \cdots, \Delta x_n, \Delta w_1, \cdots, \Delta w_n$ are used to calculate a quantity y. The uncertainties in $x_1, \cdots, x_n, w_1, \cdots, w_n$ *propagate* through the calculation to cause an uncertainty in y, provided all errors are independent and random, as follows:

Sums and Differences: If

$$y = x_1 + \cdots + x_n - (w_1 + \cdots + w_n),$$

then

$$\Delta y = \sqrt{(\Delta x_1)^2 + \cdots + (\Delta x_n)^2 + (\Delta w_1)^2 \cdots + (\Delta w_n)^2}. \tag{2.8}$$

Product and Quotients: If

$$y = \frac{x_1 \times \cdots \times x_n}{w_1 \times \cdots \times w_n},$$

then

$$\frac{\Delta y}{|y|} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \cdots + \left(\frac{\Delta x_n}{x_n}\right)^2 + \left(\frac{\Delta w_1}{w_1}\right)^2 + \cdots + \left(\frac{\Delta w_n}{w_n}\right)^2}. \tag{2.9}$$

Measured Quantity Times Exact Number: If A is known exactly and

$$y = Ax,$$

then

$$\Delta y = |A|\Delta x \qquad \text{or, equivalently,} \qquad \frac{\Delta y}{|y|} = \frac{\Delta x}{|x|}. \tag{2.10}$$

Uncertainty in a Power: If n is an exact number and

$$y = x^n,$$

then

$$\frac{\Delta y}{|y|} = |n|\frac{\Delta x}{|x|}. \tag{2.11}$$

Uncertainty in a Function of One Variable: If $y = f(x)$ is any function of x, then

$$\Delta y = \left|\frac{df}{dx}\right| \Delta x.$$

If $f(x)$ is a complicated function, then instead of differentiating $f(x)$, one can use an equivalent formula

$$\Delta y = |f(x_{best} + \Delta x) - f(x_{best})|. \tag{2.12}$$

General Formula for Error Propagation: If $u = f(x, y, z, \ldots)$ is a function of several variables with the independent variables $x, y, z, \ldots$ having independent and random uncertainties $\Delta x, \Delta y, \Delta z \ldots$. The uncertainty in u is then given by the formula

$$\Delta u = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2 + \cdots}, \tag{2.13}$$

where the partial derivatives are all evaluated at the best known values of $x, y, z, \ldots$.

NOTE:- This formula is based on a first-order Taylor series expansion of a function of many variables and is valid when the individual uncertainties Δx_i's are uncorrelated with each other and are small compared to the values of the quantities. The first-order Taylor series expansion of any function f

at x_0 is given by:

$$f(x - x_0) \approx f(x_0) + (x - x_0)\frac{d}{dx}f(x)|_{x=x_0}. \tag{2.14}$$

2.7.2 Exercises

Write out the error propagation formula (in terms of $\Delta f/f$) when the function $f(x, y)$ is of the form:

1. $f = x * y$
2. $f = x/y$
3. $f = x + y$
4. $f = x - y$
5. $f = x^m y^n$
6. $f = kx$ (k is constant)
7. $f = \ln_e x$
8. $f = \log_{10} x$
9. $f = e^x$

2.8 Fitting Data: Least Squares Regression

Frequently in the lab you will perform a series of measurements of a quantity y at different values of x. This gives a more accurate determination of a physical parameter rather than a single measurement. If you have a linear relationship $y = mx + b$, you can determine the uncertainty in the measured slope m and the intercept b.

A common method to find the best curve to fit a set of data points is the "method of least squares". If all the data points have nearly the same weight/error, one can try to arrange the curve so that as many points lie below the line as above. However, such a visual method is not quantitative.

The least-squares method of curve fitting can be described qualitatively as follows: Let the data set be represented by the functional form $f(x; a, b, \ldots)$ where $a, b, ..$ are adjustable parameters that can be varied to get the best fit curve. The function, f, can be a straight line ($f(x) = mx + b$ where the adjustable parameters are m and b) or a higher order polynomial or any other complicated function. For each data point (x_i, y_i), the value $y_i - f(x_i; a, b...)$ is computed and then the "chi-square" value χ^2 is calculated from the expression

$$\chi^2(a, b, \ldots) = \sum_i \frac{[y_i - f(x_i; a, b, \ldots)]^2}{\sigma_i^2}, \tag{2.15}$$

where σ_i is the uncertainty of each data point. The best fit is found by adjusting the parameters $a, b, \ldots$ until the minimum value of χ^2 is achieved. For N data points and n adjustable parameters, the "reduced chi-square" can be calculated from

$$\chi_\nu^2 = \frac{\chi^2}{\nu} = \frac{\chi^2}{N - n}, \tag{2.16}$$

where ν is the "degrees of freedom" in the problem. If the parameters are adjusted so that $\chi_\nu^2 \approx 1$, a "good fit" is achieved i.e. the difference between the fitted curve and the data is on an average, as big as the uncertainty in the data itself.

2.8.1 Fitting to a straight line

As an example of the least squares method, consider the problem of fitting of a set of N data points (x_i, y_i) to a straight line $f(x) \equiv y = mx + c$. It is assumed that the uncertainty σ_i associated with each measurement y_i is known, and the values of the dependent variable x_i's are exactly known. The chi-square merit function given by Eq.(2.15) for this case is

$$\chi^2(m, c) = \sum_{i=1}^{N} \frac{(y_i - mx_i - c)^2}{\sigma_i^2}. \tag{2.17}$$

To determine the parameters m and c, we need to minimize $\chi^2(m,c)$. At its minimum, the derivatives of $\chi^2(m,c)$ with respect to m and c vanishes:

$$\frac{\partial \chi^2}{\partial m} = -2\sum_{i=1}^{N} \frac{(y_i - mx_i - c)\,x_i}{\sigma_i^2} = 0, \tag{2.18a}$$

and

$$\frac{\partial \chi^2}{\partial c} = -2\sum_{i=1}^{N} \frac{y_i - mx_i - c}{\sigma_i^2} = 0. \tag{2.18b}$$

Define,

$$w_i \equiv \frac{1}{\sigma_i^2}; \qquad S \equiv \sum_{i=1}^{N} w_i; \qquad S_x \equiv \sum_{i=1}^{N} w_i x_i; \qquad S_y \equiv \sum_{i=1}^{N} w_i y_i; \tag{2.19}$$
$$S_{xx} \equiv \sum_{i=1}^{N} w_i x_i^2; \qquad S_{xy} \equiv \sum_{i=1}^{N} w_i x_i y_i.$$

With the above definition, the above equations can be rewritten as simultaneous equations for m and c:

$$cS + mS_x = S_y, \tag{2.20a}$$

and

$$cS_x + mS_{xx} = S_{xy}. \tag{2.20b}$$

The solution of these two equations in two unknowns is calculated as

$$m = \frac{SS_{xy} - S_x S_y}{\Delta}, \tag{2.21a}$$

and

$$c = \frac{S_{xx}S_y - S_x S_{xy}}{\Delta}, \tag{2.21b}$$

with

$$\Delta \equiv SS_{xx} - (S_x)^2. \tag{2.21c}$$

This gives the best fit values of the parameters m and c. The next task is the estimation of the probable uncertainties in the estimates of m and c, which is introduced by the measurement errors in the data. If the data are independent, then each contributes its own bit of uncertainty to the parameters. Recall from the propagation of error section [Equation (2.13)] that the standard deviation σ_f in the value of any function f will be

$$\sigma_f = \sqrt{\sum_{i=1}^{N} \sigma_i^2 \left(\frac{\partial f}{\partial y_i}\right)^2}. \tag{2.22}$$

For the straight line, the derivatives of m and c with respect to y_i can be directly evaluated from the solution:

$$\begin{aligned} \frac{\partial m}{\partial y_i} &= \frac{S_{xx} - S_x x_i}{\sigma_i^2 \Delta} \\ \frac{\partial c}{\partial y_i} &= \frac{S x_i - S_x}{\sigma_i^2 \Delta}. \end{aligned} \tag{2.23}$$

Substituting these in Eq. (2.22) and summing over the points we get the standard deviations

$$\sigma_m = \sqrt{\frac{S_{xx}}{\Delta}} \qquad \text{and} \qquad \sigma_c = \sqrt{\frac{S}{\Delta}}, \tag{2.24}$$

in the estimates of m and c respectively.

If we assume that the uncertainties in y have the same magnitude σ_y for all the data points, then the above equations remain valid with $w_i = 1/\sigma_y^2$. For this case, the above equations take the form

$$\begin{aligned} m &= \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta} \qquad (2.25)\\ c &= \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta} \\ \Delta &= N \sum x_i^2 - \left(\sum x_i\right)^2 . \end{aligned}$$

The standard deviation in m and c is given by

$$\sigma_c = \sigma_y \sqrt{\frac{\sum x_i^2}{\Delta}} \qquad \sigma_m = \sigma_y \sqrt{\frac{N}{\Delta}}, \tag{2.26}$$

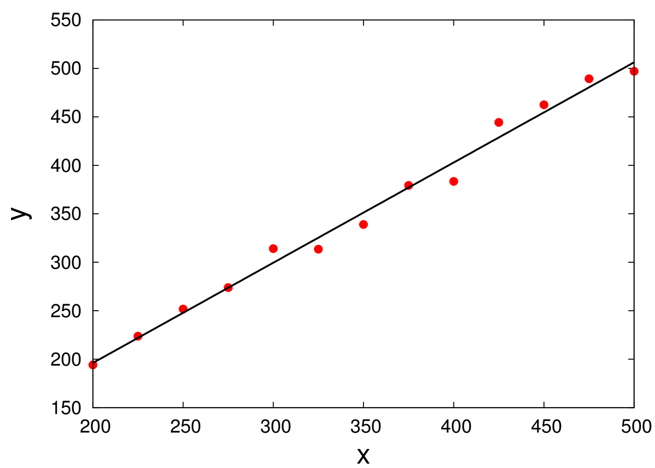
where, the uncertainty σ_y in the numbers $y_1, \ldots, y_N$ can be estimated by

$$\sigma_y = \sqrt{\frac{1}{(N-2)} \sum_{i=1}^{N} (y_i - m x_i - c)^2}, \tag{2.27}$$

assuming that the deviations $(y_i - m x_i - c)$ are normally distributed.

Example: Let us fit a straight line to a set of data (shown below) that is obtained by an arbitrary experiment. On the right hand side the data and the best straight line fit is plotted.

x	y
200.0	194.18
225.0	223.81
250.0	251.71
275.0	273.84
300.0	314.08
325.0	313.51
350.0	339.00
375.0	379.26
400.0	383.45
425.0	444.31
450.0	462.51
475.0	489.33
500.0	497.00



Step by step procedure to fit a straight line to a data set:

1. The data looks linear so we can try fitting a straight line $y = mx + c$ to it.
2. The above table does not mention the uncertainties of individual data points so we can assume that the uncertainties in y's have the same magnitude σ_y, which needs to be calculated.
3. To calculate m and c, we need to calculate the following sums

$$\sum x_i = 4550.0 \qquad \sum x_i^2 = 1706250.0 \qquad \sum y_i = 4565.99 \qquad \sum x_i y_i = 1715687.0$$

4. Using these values in Eq. (2.25), we get

$$\Delta = 1478750.0 \qquad m = 1.03 \qquad c = -10.59$$

5. Next we need to calculate the uncertainties in the constants m and c. We first calculate the uncertainty σ_y in y's by using Eq. (2.27). For the above set of data we get $\sigma_y = 11.19$. The uncertainties $\sigma_m = 0.03$ and $\sigma_c = 12.02$ in constant m and c respectively can then be calculated from Eq. (2.26).

Result: The slope m and the intercept c of the best fitted straight line to the above data is

$$m = 1.03 \pm 0.03 \qquad c = -10.59 \pm 12.02.$$

IMPORTANT NOTE:- You are expected to plot your data and do least squares analysis to find the best fit to your data and also estimate the goodness of fit. You may use **gnuplot** or other standard computer programs to find the best fit parameters and also the uncertainties in the parameters. Use the values of σ generated by the computer program in your analysis of error propagation in your experiment. LAB REPORTS WHICH DO NOT INCLUDE AN ANALYSIS OF ERRORS WILL NOT BE GIVEN A FULL GRADE.

References

1. Practical Physics, Third edition, by G. L. Squires, Cambridge University Press (1999).
2. An Introduction to Error Analysis, Second edition, by J. R. Taylor, University Science Books (1997).

Chapter 3

Determining ‘g’ using a Compound Pendulum

3.1 Aim

This experiment is designed to help you better understand angular motion. A compound pendulum is simply a rigid body with distributed mass which swings freely about some pivot point. This horizontal axis that the body is able to pivot about, does not coincide with the centre of gravity of the body.

During the course of this experiment, you will investigate the dependence of the period of oscillation of a physical pendulum on the distance between the point of suspension and the pendulum’s centre of mass, and use this relationship to determine g (the acceleration due to gravity at the earth’s surface).

3.2 Theory

Newton’s second law of motion $F = ma$ can be used to describe the motion of a simple pendulum but it cannot be easily used to describe the motion because the mass is distributed along the length of the pendulum and different points on the pendulum have different speeds. However, the angular velocity of all these points is same, therefore, one can use the relation $\tau = I\alpha$ (i.e., *torque* = *moment of inertia* × *angular acceleration*) to describe the motion of a compound pendulum.

In this lab, the compound pendulum is a steel bar of length L, which can be supported at different points (regular holes) along its length. The bar pivots about a fixed point which is at a distance r from the center of mass. The period T of the compound pendulum is given by

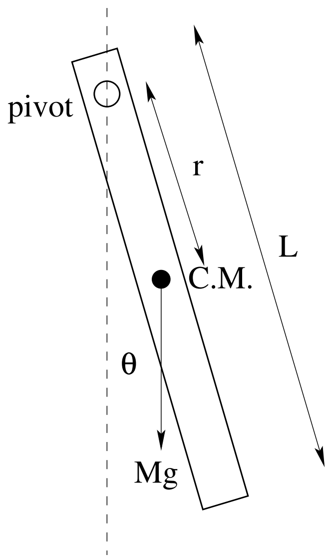
$$T = 2\pi\sqrt{\frac{I}{Mgr}} \tag{3.1}$$

where I is the moment of inertia about the pivot point, M is the total mass, r is the distance between the pivot point and the center of mass of the pendulum, and g is the acceleration due to gravity. The compound pendulum can be used to measure g accurately if the moment of inertia is known.

To understand Eq. 3.1 better, recall that the torque τ and the angular acceleration α are related by $\tau = I\alpha$ where the torque is due to the force of gravity Mg. Assuming that the full weight of the pendulum acts at the center of mass, the torque about the pivot point is computed from $\tau = Mgr\sin\theta$. Since $\alpha = \frac{d^2\theta}{dt^2}$, the equation of motion becomes (think about the significance of the minus sign)

$$Mgr\sin\theta = -I\frac{d^2\theta}{dt^2}. \tag{3.2}$$

For small angular displacements $\sin\theta \approx \theta$ and the equation of motion reduces to that of a harmonic



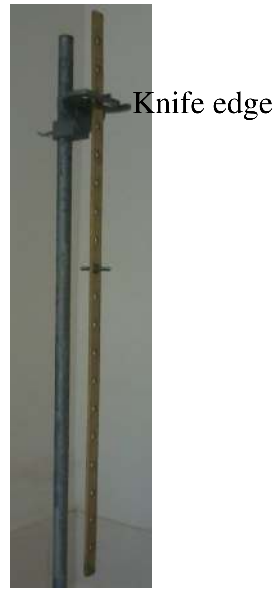


Figure 3.1: The compound pendulum

oscillator

$$\frac{d^2\theta}{dt^2} \approx -\frac{Mgr}{I}\theta \tag{3.3}$$

the solution to which is given by

$$\theta(t) = \theta_0 \sin(\omega t + \phi); \qquad \omega = \frac{2\pi}{T} = \sqrt{\frac{Mgr}{I}}. \tag{3.4}$$

Convince yourself that just as for a simple pendulum, the T of a compound pendulum is independent of the mass (depends only on the distribution of mass within the pendulum and not on total mass). Also note that if the r is small, the period of the compound pendulum can be made quite large. By contrast, the only way to get a long period with a simple pendulum is to make the string very long. The **compound pendulum** is also referred to synonymously as the **physical pendulum** or the **bar pendulum**.

The moment of inertia I about the pivot point is related to the moment of inertia about the center of mass I_{cm} by the parallel axis theorem

$$I = I_{cm} + Mr^2 \tag{3.5}$$

I_{cm} can be computed from the definition $I = \sum_i m_i r_i^2$ and for a uniform bar of length L, $I_{cm} = \frac{1}{12}ML^2$.

Let the moment of inertia about the pivot point be denoted I_r. Manipulate Eq. (3.1) and Eq. (3.5) and convince yourself that

$$T^2 = 4\pi^2 \frac{I_r}{Mgr} = \frac{4\pi^2}{gr}\left(\frac{I_{cm}}{M} + r^2\right). \tag{3.6}$$

For a uniform bar of length L this takes the form

$$T^2 r = \frac{4\pi^2}{g} r^2 + \frac{\pi^2 L^2}{3g}, \tag{3.7}$$

which is a linear equation in the variables $T^2 r$ and r^2. This implies that if the period T is measured for several different r, a plot between T^2r in the $y-$axis and r^2 on the $x-$axis will be a straight line with slope $4\pi^2/g$ and the intercept of $\frac{\pi^2 L^2}{3g}$. Such a graph is compelling confirmation of the theoretical calculations. Measuring the slope of this graph will give an estimate of the value of g.

3.3 Procedure

- Find the center of mass of the bar by balancing it on a knife-edge and put a mark there. This is necessary to be able to measure the distances of various points of suspension from the center of mass.
- The steel bar has several small holes used to insert and fit the small pivot rod. The distance r (referred to in the above equations) is the distance from the center of the steel bar to the pivot point (the inside edge of the hole). Make an accurate estimate of the distance r.
- Pass a knife-edge through a hole and secure to the bar by means of two nuts at the end of the edge. Suspend the bar, with the edge resting on the bracket. Check that it is the edge and not the rough top of the knife-edge that is resting and that the bar is free to oscillate.
- Use the bubble level to check that the pivot rod is level and that it is perpendicular to the support plates. Keep the amplitude of the swing small. (*Why should the amplitude be kept small*?). After the bar is set swinging, let it oscillate for several periods before starting your observations, to allow time for any sideways wobbling motion to damp out.
- Decide on how many oscillations to time by using error estimates in all the measurements. Set the bar oscillating with a small amplitude and time the oscillations. Decide on where you want to start the count. Take 5 readings for each hole (i.e., the same value of r). Start at one end of the bar and systematically take readings for determining T for each available value of r till you reach closest to the center of mass.
- For each r, measure the period T (for 20 oscillations) several times and get an averaged value. Briefly write out the procedure in your lab notebook, including a table of the number of measurements, the amplitude of the oscillations and the number of times each measurement was repeated. **Explore the effect of the varying the amplitude of the oscillation on your measurements**.
- Plot a graph of T versus r. Clearly show the shape of the curve and the location of any maxima or minima. How will you interpret this curve?
- Make a plot of T^2r versus r^2. Determine the slope m and the intercept b as well as their uncertainties δm and δb using the best fit to the line (do a least squares fitting on your data). From the values of m and b, you can make two independent determinations of $g : g_m = 4\pi^2/m$ and $g_b = \pi^2 L^2/3g$. Compute the uncertainties from δm and δb. What is your fractional uncertainty in g? Compare your measured g with the known value of g at Mohali. If g measured and the known g do not agree within the uncertainty δg, suggest possible sources of systematic error which could explain this discrepancy.
- Do you think this experiment is more accurate than the simple ‘g’ by free fall experiment. If so why? Write the answer down in your record.
- Measure the time period by suspending the bar using holes on the other side of the center of mass. Use r as negative for these holes. Plot T vs r for the entire range of r. Can you improve the accuracy of g using this full graph?

3.4 An alternative description

The period of the physical pendulum can be calculated from the equation of motion of the bar. If r is the distance between the point of suspension and the center of mass, the period of the pendulum is given by

$$T = 2\pi \left(\frac{k^2 + r^2}{gr} \right)^{1/2}, \tag{3.8}$$

where k is the radius of gyration of the bar about an axis passing through the centre of mass. *Note:- Read up on the radius of gyration, if you are not familiar with the concept. Write down an expression for the radius of gyration k in terms of the dimensions of the bar. Should the presence of holes in the bar be considered when calculating the theoretical value of k?.*

The behaviour of this physical pendulum of length r can be related to that of a simple pendulum of length r'. The values of r for which the pendulum has the same period of a simple pendulum of length r' are:

$$r = \frac{r' \pm (r'^2 - 4k^2)^{1/2}}{2} \tag{3.9}$$

As you can see from this equation, there are two values of r (call them r_1 and r_2) for which the period of the physical pendulum is the same as that of an equivalent simple pendulum.

There is also a value of r for which the physical pendulum has a minimum period. The minimum period may be found by minimizing Eq (3.8), i.e., by setting

$$\frac{dT}{dr} = 0.$$

Solving this equation gives the minimum period of the physical pendulum

$$T_{min} = 2\pi \left(\frac{2k}{g}\right)^{1/2} \tag{3.10}$$

3.5 Questions to Explore

Think about the following points and write down the answers in your record notebook.

- Plot the results and fit to obtain experimental values of k and g. Check that the relationship $r_1 r_2 = k^2$ for a fixed T holds for several values of r_1 and r_2.
- Compare the measured value of k to the value you calculate from the bar's dimensions. Find the minimum period T_{min}.
- Does Eqn 3.8 apply when a large amplitude is used or when damping is present (due to friction at the pivot or due to air resistance)?
- If Eqn 3.8 does not apply, will the value of g you find be higher or lower than the actual value? Justify your answer.
- Should the presence of holes in the bar be considered when calculating the theoretical value of k?

Chapter 4

Determining 'g' using a Kater's pendulum

4.1 Aim

In this experiment, you will learn how to use a Kater's pendulum to estimate "g" the acceleration due to gravity.

4.2 Theory

The Kater's pendulum is also called a **reversible pendulum**. In the 18th Century, most experiments used to find the acceleration due to gravity g, used pendulums and the familiar equation for the period of the pendulum T in terms of g

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{4.1}$$

where L is the length of the pendulum. However, a pendulum consisting of a small spherical bob on a string, requires accurately measuring the length from the support to the centre of mass of the bob, which was difficult to do. In 1815, Captain Henry Kater designed a reversible pendulum and a data analysis method which improved the precision of determination of g by a factor of 100!

A reversible pendulum is a pendulum that can be swung from either of two pivot points. When the mass distribution of the pendulum is adjusted so that the periods are the same from either pivot, the period of oscillation is the same as that of a simple pendulum with a length equal to the distance between the two pivot points. In other words if the time periods of oscillation about the knife edges K_1 and K_2 are equal to T, and if the distance between the knife edges is L, then the time period T for the reversible pendulum is also given by Eq. (4.1).

Kater's method uses many clever innovations: a solid swinging body, precision knife-edge suspension points on the pendulum, and reversing the pendulum and finding the period of swing about two different points. Two knife-edge pivot points and two adjustable masses are positioned on the rod so that the period of swing is the same from either side. *Just two accurate measurements are required:*

1. The period of the swing and,
2. One length (precisely measured between two knife edges which are firmly fixed on the pendulum itself).

The pendulum consists of a heavy metal bar with two knife edges permanently fixed on the ends of the pendulum bar and a large mass located near one end of the bar. A second smaller mass is adjustable in position. What you have to do is to adjust the position of the smaller mass

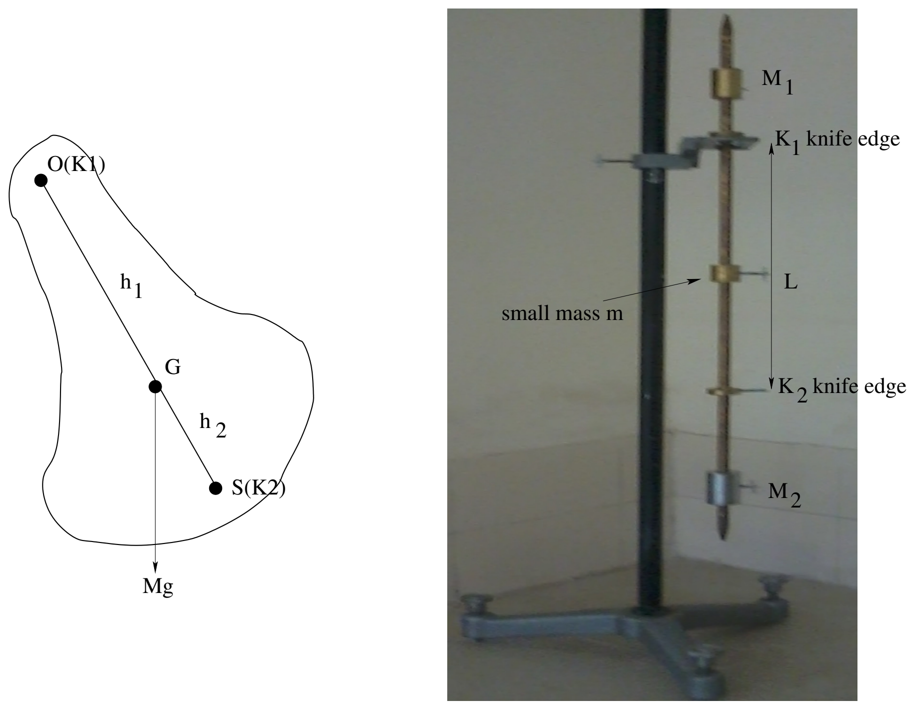


Figure 4.1: The Kater's pendulum

until the pendulum's period is the same for each of the suspension points. When this condition is achieved, the period is given by Eqn. 4.1 as mentioned earlier, where L is now the distance between the knife-edges. Notice that there is no single unique positioning of the masses for the equal period requirement to be met. Rather, keep one mass fixed to make the periods equal for a range of position of the other mass. The resulting period of the pendulum will be the same and will depend on the distance between the knife edges and on g. Alternatively, it might be easier for you to keep the smaller mass fixed and adjust the distance between the heavier masses.

At this point, also read the theory of the compound pendulum given in this manual. To recapitulate what was learnt in the case of the compound pendulum, consider a generalized compound pendulum with the pivot at K_1 and the center of mass at G. If this pendulum is moved from rest, a restoring torque will act on it and if the displacement angle θ is small, the period is given by

$$T = 2\pi\sqrt{\frac{I_{K_1}}{Mgh_1}} \tag{4.2}$$

where I_{K_1} is the moment of inertia about the axis K_1 and h_1 is the distance between K_1 and the center of mass G. Using the parallel axis theorem, the moment of inertia I_{K_1} is given by

$$I_{K_1} = I_G + Mh_1^2 \tag{4.3}$$

where I_G is the moment of inertia about an axis at the center of gravity so that

$$I_{K_1} = Mk^2 + Mh_1^2 \tag{4.4}$$

where k is the radius of gyration of the pendulum about an axis at the center of gravity G (recall that the radius of gyration is defined by $k^2 = I_G/M = \frac{1}{M}\int r^2 dm$), and the period about K_1 is

$$T = 2\pi\sqrt{\frac{1}{g}\left(h_1 + \frac{k^2}{h_1}\right)}. \tag{4.5}$$

If the second pivot is located at a point K_2 such that $GK_2 \equiv h_2 = k^2/h_1$, the period about K_2 is

$$\begin{aligned} T &= 2\pi\sqrt{\frac{1}{g}\left(\frac{k^2}{k^2/h_1}+\frac{k^2}{h_1}\right)} \\ &= 2\pi\sqrt{\frac{1}{g}\left(h_1+\frac{k^2}{h_1}\right)}, \end{aligned} \tag{4.6}$$

same as the period about K_1. Substituting $k^2 = h_1h_2$ in the above equation gives the time period of the Kater’s pendulum as

$$T = 2\pi\sqrt{\frac{h_1+h_2}{g}} = 2\pi\sqrt{\frac{L}{g}}, \tag{4.7}$$

same as a simple pendulum of length L, i.e., the distance between the knife edges K_1 and K_2.

4.3 Points to note

- The support should be absolutely unmoving. The knife edge must not “wander” in position on its support.
- The pendulum should swing in a plane, without wobbling about its own length axis.
- If the pendulum comes to a stop in less than 50 swings, it is losing energy somewhere. You know that you have support problems!

4.4 Procedure

- The first part of the experiment is to move one of the masses relative to the others through a wide range, to observe the change of period of the pendulum. Keep one mass fixed outside the knife edge and “slide” the other mass (placed initially between the knife edges) along in regular steps, noting the period for each position. Repeat this set of measurements by inverting the pendulum. Now plot both these periods (pendulum and reversed pendulum) as a function of the position of the sliding mass on the same graph. By inspection, you will see that the periods are equal in the region where the curves intersect.
- The time period about K_1 and K_2 by moving the small mass m to different positions be recorded, say for 5 oscillations for a fixed position of m. You will realize the positions of m at which the time periods about K_1 and K_2 are approximately equal.
- Now that the equal period point is approximately known, the next experiment is to determine this point more accurately. So move the sliding mass in smaller steps about the assumed equal period point and plot the periods (of the pendulum and reversed pendulum) again.
- To improve your accuracy you may repeat these measurements by changing the position of the first fixed mass and determining the equal period value again.
- To improve accuracy of the time make more observations or use a longer timing period. Establish the standard deviation of the time observations data.
- Remember that the accuracy of the final value of g depends only on the measurement of the time period and the distance between the knife edges. If the basic error in timing is δt, the error in period due to timing n cycles is $\delta t/n$. (The period measured for the Bureau of standards pendulum in Washington was 2.004454 seconds!).

The Kater’s pendulum became the standard for measurements of g for over a century. In 1936 the National Bureau of Standards determined the valued of “g” using Kater’s method to be $980.080 \pm 0.003 cm/sec^2$ in Washington. So by adding a second knife-edge pivot and two adjustable masses to the compound/physical pendulum in the previous experiment in this laboratory, the value of g can be determined to a precision of 0.2%. This is because while the value of the period of oscillation of any pendulum can be accurately found by timing a large number of oscillations and taking the average, it is not easy to accurately measure the length of the pendulum. For instance, it is hard to estimate where exactly the center of mass is.

4.5 Questions to Explore

Think about these issues and write down the answers in your record notebook.

- Once the position of the masses for equal periods is determined, the pendulum can be moved from place to place to survey the variation of g with location. Discuss the causes for the variation of g with latitude.
- Find the local value of g (at Chandigarh's latitude) and compare your experimental value with it.
- Show that the relative uncertainty in the result is given by

$$\frac{\delta g}{g} = \frac{\delta L}{L} + \frac{2\delta T}{T}$$

- How much error would be introduced if the pendulum swings had an amplitude of 10°?
- How much error would be introduced if the pendulum support moved sideways with an amplitude of 2% of the pendulum's length?

Chapter 5

Determining the modulus of rigidity using Maxwell's needle

5.1 Aim

The aim of this experiment is to find the modulus of rigidity, η, of copper wire by Maxwell's needle apparatus.

5.2 Apparatus

The basic setup consists of a Maxwell's needle, a copper wire of a suitable length and thickness, a fixed support, a telescope with attached scale, a stop-watch, and a screw gauge.

5.3 Theory

Maxwell's needle when used as torsional oscillator can be used to measure the modulus of rigidity of wires quite accurately. It consists of a long tube into which four short cylinders of equal lengths (each one-fourth of the long tube) can be slipped in. Out of four cylinders, two are hollow and the remaining two are solid. The long tube is suspended with the wire whose modulus of rigidity needs to be measured. The cylinders can be placed inside the long tube in two different ways so that the axis of oscillation passes through the suspended wire.

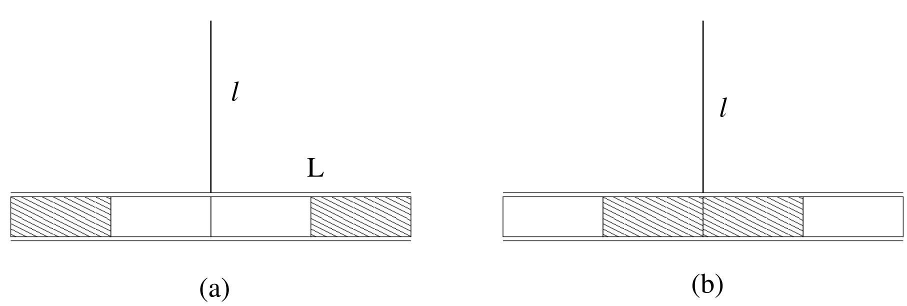


Figure 5.1: Two different configurations of the cylinders used for calculating modulus of rigidity η by Maxwell's needle apparatus. The solid cylinders are shown by shaded regions. Configuration with (a) solid cylinders in the outermost positions and (b) hollow cylinders in the outermost positions.

If a cylinder of length l and radius r is fixed at its upper end and twisted at its lower end through an angle θ, the molecules of the cylinder resist this twisting couple by an equal and opposite couple and the system behaves like a torsional oscillator. If η is the coefficient of rigidity, then the couple is given by $(\pi\eta r^4/2l)\theta$. Therefore the couple c required to produce a unit angular twist is given by

$$c = \frac{\pi\eta r^4}{2l}. \tag{5.1}$$

The needle is first suspended with the coper wire, whose modulus of rigidity is required, with solid cylinders in the outermost position (see Fig. 5.1(a)). If t_1 is the time period of oscillation and I_1 is the moment of inertia of the loaded tube about the suspended wire as an axis, then

$$t_1 = 2\pi\sqrt{\frac{I_1}{c}}. \tag{5.2}$$

The position of the solid and the hollow cylinders is then changed (see Fig. 5.1(b)). Let the time period now be t_2. If I_2 is the moment of inertia of the loaded tube in this position about the wire as an axis, then

$$t_2 = 2\pi\sqrt{\frac{I_2}{c}}. \tag{5.3}$$

From Eqs. (5.2) and (5.3), we have

$$c = 4\pi^2\frac{(I_1 - I_2)}{(t_1^2 - t_2^2)}. \tag{5.4}$$

Comparing Eq. (5.1) with Eq. (5.4) gives

$$\eta = \frac{8\pi l}{r^4}\frac{I_1 - I_2)}{(t_1^2 - t_2^2)}. \tag{5.5}$$

Let L be the half length of the long tube and M_h and M_s are the masses of the hollow and the solid cylinders respectively. The change in moment of inertia, $I_1 - I_2$, in interchanging the positions of the cylinders is $I_1 - I_2 = (M_s - M_h)L^2$. Substituting this in the above equation we get the required formula for calculating η

$$\eta = \frac{8\pi l}{r^4}\frac{(M_s - M_h)L^2}{(t_1^2 - t_2^2)}. \tag{5.6}$$

5.4 Procedure

- Put the hollow cylinders inwards and the solid cylinders outwards symmetrically in the tube.
- Place the telescope at a suitable distance. Adjust the scale till its image is visible in the mirror and the cross-wires are clearly visible. Observe the reading of the scale in the center of the image and adjust the vertical cross-wire so that it coincides with a cm division mark.
- Set the needle to vibrate in a horizontal plane and see that the amplitude is small and the needle does not oscillate up and down. Protect the apparatus from air current disturbances.
- Find the time for 20 vibrations of the needle. Place the solid cylinders inwards and the hollow cylinders outwards and repeat the and repeat the observations.
- Find the weights of the solid and hollow cylinders, measure the length of the Maxwell's needle and the diameter of the wire accurately keeping in mind that a small error in the measurement of diameter will be magnified four times.

5.5 Observations

- Record measurements of all the quantities needed in Eq. (5.6) with error bars.
- Calculate η with error bars.

5.6 Precautions

- The wire should not have any kinks.
- The needle should not vibrate up and down.
- The amplitude of vibrations should be small so that the wire is not twisted beyond the elastic limit.

Chapter 6

Finding Moment of Inertia using a Torsion Pendulum

6.1 Aim

This experiment is designed to give you a better grasp of the moment of inertia.

6.2 Theory

A torsional pendulum consists of a disk-like mass suspended from a thin wire. When the mass is twisted about the axis of the wire, the wire exerts a restoring torque on the mass. If twisted and released, the mass will oscillate back and forth executing simple harmonic motion. If the angle θ is small,

$$\tau = -\kappa\theta, \tag{6.1}$$

where κ is the torsion constant. If a mass with moment of inertia I is attached, the torque will give the mass an angular acceleration α, with $\tau = I\alpha$, leading to the equation of motion for the torsional pendulum

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}.\theta \tag{6.2}$$

The solution to this second order differential equation is

$$\begin{aligned} \theta(t) &= \theta_m \cos(\omega t + \phi) \\ \omega &= \sqrt{\frac{\kappa}{I}} \end{aligned} \tag{6.3}$$

where θ_m and ϕ are constants that depend on the initial position and angular velocity of the mass, and θ_m is the maximum angle. The period of the torsional pendulum is hence

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \tag{6.4}$$

If the wire is very stiff (large κ) the mass oscillates rapidly and the period T is short. If the mass is large i.e I is large, it oscillates slowly and T is long. The torsion constant can be determined from measurements of T if I is known or conversely, if κ is known, I can be determined.

6.3 Procedure

- With the simple shapes (ring and cylinders) given, find the moments of inertia of these objects by weighing them and measuring their dimensions. Measure the period of the torsion

Figure 6.1: The torsion pendulum apparatus.

pendulum with just the base plate and then with the cylinders and the ring which will give two different moments of inertia: ($I_0 + I_{ring}$ and $I_0 + I_{cyl}$) from which I_0 and κ may be found.

- Compute I for a solid disk of mass M and radius R (axis of rotation along symmetry axis)
- Find the moment of inertia of an irregular body.

6.4 Points to Ponder

- This simple apparatus and the principle behind the torsion pendulum is still being used (centuries after its first design) by modern physicists to perform increasingly sophisticated experiments on gravity!
- If the universe contains more than three spatial dimensions, as many physicists believe, our current laws of gravity should break down at small distances. Nothing seems more certain than the "fact" that there are three dimensions of space. But can we be sure that there are only three dimensions? Imagine a tightrope walker balancing on a cable high above the ground. To the tightrope walker the cable is effectively a 1D object, because he only needs one coordinate to specify his position as he walks back and forth. But an ant, for instance, sees the cable as a 2D object, because it can crawl along and also around the cable. Today, increasing numbers of physicists are seriously questioning whether we are like tightrope walkers, unaware of the true number of dimensions in space. New ideas from theoretical physics suggest that the best way to discover the actual dimensionality of space is to study how the gravitational attraction between two objects depends on the distance between them.
- Newton's theory, which assumes that the gravitational force acts instantaneously, remained essentially unchallenged for roughly two centuries until Einstein proposed the general theory of relativity in 1915. Einstein's radical new theory made gravity consistent with the two basic ideas of relativity: the world is 4D - the three directions of space combined with time - and no physical effect can travel faster than light. The theory of general relativity states that gravity is not a force in the usual sense but a consequence of the curvature of this space-time produced by mass or energy. However, in the limit of low velocities and weak gravitational fields,

Einstein's theory still predicts that the gravitational force between two point objects obeys an inverse-square law. General relativity has been tested with exquisite precision by astronomical observations, laboratory experiments and various spacecraft. Although Einstein's theory has passed all these tests so far, it is clear that quantum effects will cause general relativity to break down at distances comparable to the Planck length. However, the Planck length is so small that it has no discernible effect in any practical gravitational experiment. One of the outstanding challenges in physics is to finish what Newton started and achieve the ultimate "grand unification" - to unify gravity with the other three fundamental forces (the electromagnetic force, and the strong and weak nuclear forces) into a single quantum theory.

- It is amazing that, until a few years ago, gravity had not even been shown to exist for objects separated by less than about 1 mm. There were two reasons for this: first, gravity is intrinsically very weak compared with the electrostatic and magnetic forces; second, seismic, thermal and other background effects make the experiments very difficult. Torsion pendulums have been used for over 200 years to measure weak forces between macroscopic objects, and they are still the most sensitive tools for making such measurements. Early versions of the instrument were used to measure the density of the Earth (John Mitchell in 1750), the electrostatic force (Charles Augustin de Coulomb, 1785) and G (Henry Cavendish, 1798). Later, in 1890, a torsion pendulum was used to test the equivalence of gravitational mass (i.e. the m in $F = GMm/r^2$) and inertial mass (the m in F = ma). And today, modern versions of the torsion pendulum are being used in a variety of experiments, including high-precision measurements of G and tests of Lorentz symmetry. A torsion pendulum has also been used to verify that "dark matter" obeys the equivalence principle.

- Although modern torsion pendulums take many different forms that the physicist of 200 years ago would not recognize, the basic principles have remained essentially unchanged. In a traditional torsion pendulum the gravitational force between two test masses suspended on a fibre (the pendulum) and two fixed masses (the attractor) causes the fibre to twist by an amount that depends on the force. This twist is typically measured by reflecting a beam of light from a mirror on the pendulum. Ironically, the instrument is suited to gravitational measurements because the rotational motion of the pendulum about the torsion-fibre axis is not sensitive to the Earth's gravity. Moreover, it is insensitive to net forces acting on its centre of mass, which means that it can be substantially decoupled from external fluctuations, most of which are much larger than the effects of interest.

- Modern torsion pendulums are sensitive to torques as small as $10^{-18} Nm$. Since torque is defined as the product of a force and a length, and a typical length in a pendulum is about 1 cm, this is equivalent to a force sensitivity of about $10^{-16} N$. This remarkably small force is roughly equivalent to 1/100th of the weight of a single piece of a postage stamp that has been divided into a trillion equal pieces! To date, the torsion pendulum is the only instrument that is capable of precisely measuring the properties of the gravitational interaction at length scales below 1 mm.

Chapter 7

Moment of inertia of a flywheel

7.1 Aim

The aim of this experiment is to find the moment of inertia of a flywheel.

7.2 Apparatus

A fly wheel, a few different masses and a mass provided with a hook, a strong and thin string, stop watch, a meter rod, a vernier callipers.

7.3 Theory

To find the moment of inertia of a flywheel, a mass m is attached to the axle of the wheel by a chord which is wrapped several times round the axle. The length of the string is so adjusted that is gets detached from the axle just before the mass m touches the floor.

When a mass is allowed to fall it gains velocity. Therefore, its potential energy is partly converted into the kinetic energy and partly converted to the rotation of the flywheel. After the string has been detached from the wheel, the wheel continues to revolve for some time. Its angular velocity decreases because of friction and finally it comes to rest.

Let ω be the angular velocity imparted to the wheel at the moment the mass m is detached. If n_1 is the number of revolutions that the wheel makes in time t before coming to rest ($\omega_f = 0$), the average angular velocity is

$$\frac{\omega + \omega_f}{2} = \frac{2\pi n_1}{t}.$$

This gives the initial angular velocity $\omega = 4\pi n_1/t$. Applying the principle of conservation of energy at the time of detachment of the string, we have

Potential energy of mass m = Kinetic energy of mass m + Kinetic energy of flywheel + Work done against friction.

The system has the following energies:

- If h is the height from which the mass m has fallen, then the potential energy of mass is mgh
- If r is the radius of the axle then the linear velocity of mass m is $r\omega$. Therefore, the kinetic energy of mass m is $\frac{1}{2}mr^2\omega^2$.
- If I is the moment of inertia of the flywheel, then the kinetic energy of the flywheel is $\frac{1}{2}I\omega^2$.
- Let F be the energy per revolution used in overcoming the frictional force. If n is the number of revolutions the wheel makes during the descent of mass m, then the total energy used to overcome friction is nF.

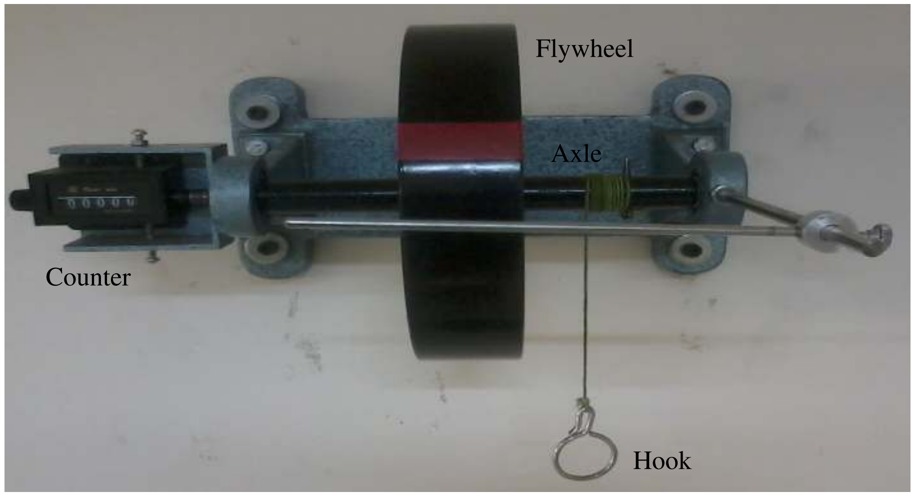


Figure 7.1: Fly wheel apparatus

The energy conservation leads to

$$mgh = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2 + nF. \tag{7.1}$$

We need to calculate I. The only other quantity that is unknown is F which can be calculated as follows: The kinetic energy possessed by the wheel is used up in overcoming friction. As the wheel comes to the rest after making n_1 revolutions, we have $n_1F = \frac{1}{2}I\omega^2$ which gives

$$F = \frac{1}{2n_1}I\omega^2. \tag{7.2}$$

Substituting the value of F in Eq. 7.1, we get

$$mgh = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2\left(1 + \frac{n}{n_1}\right),$$

which gives,

$$I = \frac{2mgh - mr^2\omega^2}{\omega^2\left(1 + \frac{n}{n_1}\right)}. \tag{7.3}$$

7.4 Precautions

1. There should be least possible friction in the flywheel.
2. The length of the string should be less than the height of the axle of the flywheel from the floor.
3. The loop slipped over the pin should be loose enough to be detached easily.
4. The string should be thin and should be wound evenly.
5. The stop watch should be started just when the string is detached.

Chapter 8

Determining Young's modulus using bending of a beam

8.1 Aim

This experiment is setup to help you determine the Young's modulus of the material of a beam of rectangular cross section by measuring the non-uniform bending of the beam supported at both ends and loaded in the middle.

8.2 Basic Setup

The apparatus includes the beam of rectangular cross section (meter scale) made up of brass, two knife-edge supports, a hook for the load, slotted weights, vernier calipers and screw gauge.

8.3 Theory

Young's modulus is a measure of the stiffness of an isotropic elastic material. It is defined as the ratio of the stress over the strain in the range of stress in which Hooke's Law holds. This can be experimentally determined from the slope of a stress-strain curve created during tensile tests conducted on a sample of the material.

If a load W is applied on the free end of the cantilever of length l it gets deflected from its axis. If I is the geometrical moment of inertia of the cantilever and Y the Young's modulus of the material, the total depression y produced on the cantilever is given by

$$y = \frac{Wl^3}{3YI}. \tag{8.1}$$

For a beam of rectangular cross-section, the moment of inertia is $I = bd^3/12$, where b is the breadth and d the thickness of the beam.

If a bar is supported at two knife edges A and B, l metre apart in a horizontal plane so that equal lengths of the bar project beyond the knife edges and a weight W is suspended at the middle point C, then it acts as a double cantilever. The middle part of the bar is practically horizontal. It is, therefore, equivalent to two inverted cantilevers fixed at the middle point C and loaded at A and B with a load $W/2$ acting upward. Therefore, the depression at C is given by using Eq. (8.1)

$$y = \frac{W/2(l/2)^2}{3YI} = \frac{Wl^3}{4Ybd^3}. \tag{8.2}$$

Once the depression and the applied load $W = mg$ (where g is the acceleration due to the gravity)

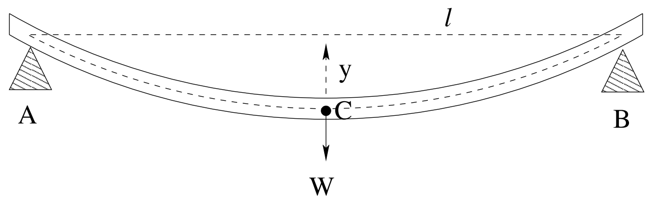


Figure 8.1: Bending of a beam in the presence of a load at the center.

is known, the Young's modulus can be calculated as

$$Y = \frac{mgl^3}{4ybd^3}. \tag{8.3}$$

8.4 Procedure

- Support the beam on the knife-edge symmetrically so that about three-fourths the length of the beam lies between the supports.
- Attach a hook at the mid-point of the beam to carry load.
- Add a dead load of 50 gm and observe the reading on the microscope.
- Increase the load in steps of 50 gm up to a load say 300 gm and record the corresponding readings.
- Repeat the experiment by unloading in steps of 50 gm. Calculate the mean depression for each load.
- Plot a graph between loads M on the x - axis and the corresponding depressions of the center of the beam y along the y - axis. The Young's modulus of the material of the beam is found out by fitting the graph and Eq. (8.3).

8.5 Precautions

- The knife-edges should be at equal distances from the centre of the bar. The stirrup should be parallel to the knife-edges and placed exactly at the centre to get a symmetrical loading of the bar.
- The thickness d should be measured very accurately at a number of points along the whole length of the bar. The thickness is a small quantity and since its cube is to be used in the calculations, a small error in its measurement can cause a large error in the result.

Chapter 9

Determining Young's modulus using Searle's apparatus

9.1 Aim

The aim of this experiment is to determine Young's modulus of elasticity of a metal wire using Searle's apparatus.

9.2 Theory

Remember Hooke's law, which states that Stress/Strain = Constant, the constant being the modulus of elasticity. For linear stress, the ratio of tensile stress to longitudinal strain is called the Young's modulus of elasticity.

Consider attaching a load to a coiled spring – this gives rise to two different effects, neither of which is a simple stretching process. If we imagine a weight W suspended from a point on the vertical axis of the coil, its effect is to produce a torque WR about any point at a distance R on the approximately horizontal axis of the spring. One effect of this is to twist the wire about on its own axis, and the descent of the weight is primarily a consequence of this twisting process. The second effect is that the coils of the spring will tighten or loosen a little, so that the spring as a whole twists about the vertical axis. This involves a bending of the coils - a change in their curvature. The final result is expressible as a proportionality (the spring constant k) between the applied load and the distance through which the load moves. Under conditions of static equilibrium,

1. For a given material made up into wires of a given cross-sectional area, the extension $\Delta\ell$ under a given force is proportional to the original length ℓ_0. The dimensionless ratio is called the strain.
2. For rods of a given material with different cross-sectional areas, the same strain $\Delta\ell/\ell_0$ is caused by applying forces proportional to the cross-sectional areas. The ratio $\Delta P/A$ is called the stress and has units of force per unit area. ΔP is the force applied.
3. Provided the strain is very small (less than about 0.1% of the normal length ℓ_0, the relation between stress and strain is linear in accordance with Hooke's law and the value of this constant is called the Young's modulus of elasticity.

If the wire is of length L and cross-section A and is stretched by an amount ℓ by a force F acting along its length, the Young's modulus is given by

$$Y = \frac{F}{A} \times \frac{L}{\ell} \tag{9.1}$$

where $F = mg$ and $A = \pi r^2$.

Figure 9.1: The Searle's apparatus

Notice that Young's modulus represents a stress corresponding to 100% elongation, a condition that is never approached in the actual stretching of a wire. Failure occurs at stresses two or three orders of magnitude less than this, i.e., at strain values of between 0.1 and 1%. There is no possibility of obtaining by direct stretching of a wire, the kind of large fractional change of length that one can so readily achieve with a coiled spring. So the wire can be thought of as a very hard spring, and the oscillations besides being of quite high frequency must be of very small amplitude if the strength limit of the material is not to be exceeded.

The macroscopic elastic property described by Young's modulus is analyzable in terms of the microscopic interactions between atoms in the material. Clearly, if the overall length of a wire increases by 1%, the individual interatomic spacings along that direction also increase by 1% and one can in principle relate the elastic modulus to atomic properties as described by the potential-energy curve of the interatomic forces. However, in this experiment we are concerned only with the macroscopic description.

9.3 The Basic Setup

The setup consists of Searle's apparatus suspended by two long steel wires, a screw gauge, slotted weights and a hanger. Searle's apparatus consists of two metal frames hinged together so that they can have only vertical relative motion. A spirit level is supported at one end on a cross bar frame whose other end rests on the tip of a micrometer screw which moves vertically. The micrometer screw has a disc with equal divisions along its circumference enabling movement of the screw and enabling measurement of the movement of the spirit level. If there is any relative motion between the two frames, the spirit level no longer remains horizontal and the bubble is displaced from its equilibrium position.

To bring the bubble back to its original position the screw has to be moved up or down and the distance through which the screw moves gives the relative motion between the two frames. The frames are suspended by two identical wires, one is the experimental wire and the other is a reference wire. The hook attached to the frame of the reference wire has a constant weight to keep the wire taut. Slotted weights are added to the hanger attached to the experimental wire, to apply the stretching force.

9.4 Procedure

- Suspend weights from the hooks of both frames so that the wires are stretched and free from kinks. Attach a constant weight on the reference wire to keep it taut.
- Measure the length of the experimental wire. Find the pitch and least count of the screw gauge and find the diameter of the wire.
- Gradually increase the load on the experimental wire and observe the reading on the micrometer at each stage after leveling the instrument with the spirit level.
- Unload the wire by removing the weights in steps and again take the readings of the micrometer.
- Plot a graph between the load and the extension and find the slope. Using the value of the slope, find the Young's modulus. Estimate the sources of error in your experiment.

9.5 Values of Y

The standard (approximate) values of Young's modulus Y (in units of $GPa(kN/mm^2)$ and N/m^2) for different materials is tabulated below:

Material	$Y(GPa)$	$Y(10^{10}N/m^2)$
Aluminium	69	6
Brass/Bronze	100-125	9
Steel	200	20
Copper		12
Tungsten	400-410	
Diamond	1220	