PHY102 : Assignment 9

1. (Purcell 7.21) A solenoid of radius a_1 and length b_1 is located inside a longer solenoid of radius a_2 and length b_2. The total number of turns is N_1 on the inner coil, N_2 on the outer. Work out a formula for the mutual inductance M. (See figure in Purcell).

2. Compute the self inductance of a solenoid with N_1 turns, length l and crosssectional area A, with a current I_1 flowing through each turn. Now, an insulated coil of N_2 turns is wrapped around it. Calculate the mutual inductance M, assuming that all the flux from the solenoid passes through the outer coil. Relate the mutual inductance M to the self inductances L_1 and L_2 of the solenoid and the coil.

3. (Purcell 9.5) Here is a particular electromagnetic field in free space :

$$E_x = 0 \quad E_y = E_0 \sin(kx + \omega t) \quad E_z = 0$$
$$B_x = 0 \quad B_y = 0 \quad B_z = -\frac{E_0}{c}\sin(kx + \omega t)$$

   Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

4. (Purcell 9.7) Show that the electromagnetic field described by

$$\begin{aligned} \mathbf{E} &= E_0 \hat{\mathbf{z}} \cos kx \cos ky \cos \omega t \\ \mathbf{B} &= B_0(\hat{\mathbf{x}} \cos kx \sin ky - \hat{\mathbf{y}} \sin kx \cos ky) \sin \omega t \end{aligned}$$

   will satisfy Maxwell's equations in vacuum if $E_0 = \sqrt{2}cB_0$ and $\omega = \sqrt{2}ck$.

5. (Purcell section 9.7 and problem 9.12) Show that $\mathbf{E} \cdot \mathbf{B}$ and $E^2 - c^2B^2$ are invariant under Lorentz field transformations. A plane wave has $\mathbf{B}$ perpendicular to $\mathbf{E}$. What can you conclude ?

6. (Purcell 8.1) How large an inductance, in henrys, should be connected in series with a 120 volts, 60 watt light bulb if it is to operate normally when the combination is connected across a 240 volt, 60 Hz line ?

7. (Purcell 8.4) Consider the resistance R' connected in parallel, rather than in series, with the LC combination. Work out the equation, which applies to this circuit. Find also the conditions on the solution analogous to those that hold in the series RLC circuit. If a series RLC and a parallel $R'LC$ circuit have the same L, C and Q, how must R' be related to R ?