PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

• First, let us look back at the calculation of the force in frames F & F'.

We first considered the body to be initially at rest i.e. momentum $p = 0$ at $t = 0$. This is in the F frame which we will call the "rest frame". It is confusing a bit since a force acts on the body causing it to accelerate — therefore, the body is not going to be at rest for $t > 0$! Anyway, with a force acting in the x-direction, we showed that, in the "lab frame" F' (an observer in this frame sees the "rest frame" moving with velocity v). the forces components $\parallel^{l}$ & $\perp^{r}$ to frames' relative motion are given as,

$$F_x' = F_x \quad \& \quad F_y' = \frac{F_y}{\gamma}.$$

∴ The component of force $\perp^{r}$ to the frames' relative motion in the lab frame is smaller by $\frac{1}{\gamma}$ than that in the rest frame.

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5$cm ? [2.5]

- Now look at the problem we discussed today in the class.

We had 2 sheets of charge densities $+\sigma$ and $-\sigma$. In the frame F, the sheets are stationary. We determined the electric field in the frame F′ which moves towards left w.r.t. F with velocity $\vec{v}$. We found.

$$E_z' = \gamma E_z$$

Does this contradict with the force relations we derived, knowing that charge is relativistically invariant ?

The answer is "absolutely not". !

To see this, note that in order to calculate the force, given the electric field, we have to bring in a test charge, q. To find the force ~~in F′~~ on q in F′, we will be moving with the particle. In this "particle frame" the particle will be, at least momentarily, at rest. The sheets with the charged densities

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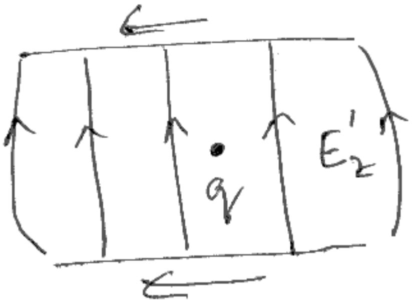
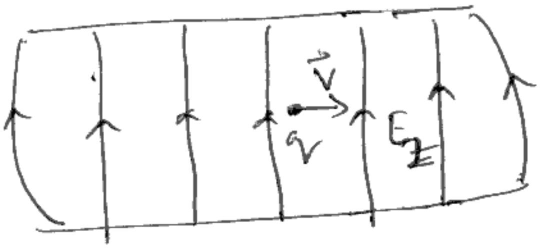
σ & σ' are moving. F is now the "lab frame" where the charge q is moving with velocity $\vec{v}$.





"Particle frame F'" "Lab frame F".

(Note that the plates are deliberately shrinked in F' to remind length contraction :).

$$\therefore E_z' = \gamma E_z \quad \Rightarrow \quad F_z = \frac{dp_z}{dt} = \frac{1}{\gamma}\frac{dp_z'}{dt'} = \frac{1}{\gamma}F_z'$$

$$\therefore F_z' = \gamma F_z .$$