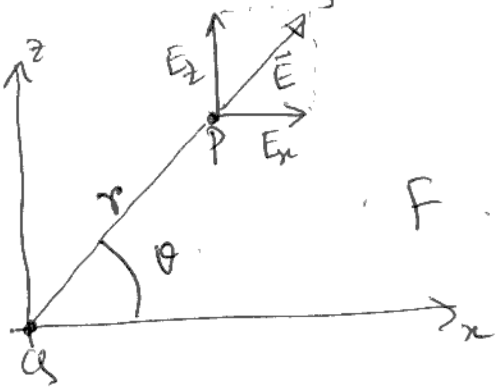
PHY102 : ~~Quiz~~ 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

Field of a pt charge moving with constant velocity.

Take a pt. charge Q at rest at the origin in the frame F.



Electric field is directed radially outwards & at pt P, we have,

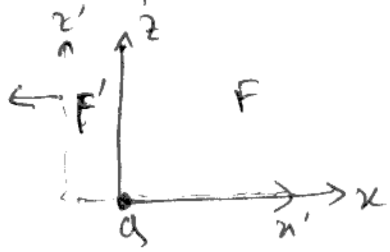
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+z^2)^{3/2}}$$

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{Qz}{(x^2+z^2)^{3/2}}$$

since, $r = (x^2+z^2)^{1/2}$ & $\cos\theta = \frac{x}{\sqrt{x^2+z^2}}$, $\sin\theta = \frac{z}{\sqrt{x^2+z^2}}$

Consider now a frame F' which is moving in the $-$ve x direction with speed v, w.r.t frame F.

$\therefore$ To an observer at rest in F', the charge Q is moving in the $+$ve x direction with speed v.



The Lorentz transformations that we had written in class were for a frame F' which was moving in the $+$ve x direction w.r.t frame F.

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5$cm ? [2.5]

Therefore, in this case, where frame F' is moving in -ve x direction w.r.t. frame F, these transformations become,

$$x = \gamma(x' - vt'), \quad y = y', \quad z = z', \quad t = \gamma\left(t' - \frac{v}{c^2}x'\right)$$

Note that we are ~~assum~~ going to assume that origin of two frames coincide at time zero according to observers in both frames.

Now, transformations for electric fields are,

$$E_z' = \gamma E_z \quad \& \quad E_x' = E_x .$$

$\therefore$ For the instant $t' = 0$, when $x = \gamma x'$, we have,

$$E_x' = E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{Q(\gamma x')}{[(\gamma x')^2+z'^2]^{3/2}}$$

$$E_z' = \gamma E_z = \frac{1}{4\pi\epsilon_0}\frac{\gamma Q z}{(x^2+z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{\gamma Q z'}{[(\gamma x')^2+z'^2]^{3/2}}$$

$$\therefore \frac{E_z'}{E_x'} = \frac{z'}{x'} = \tan\theta'$$

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$\therefore$ Vector, $\bar{E}'$ makes same angle with x' axis as does the radius vector $\bar{r}'$!

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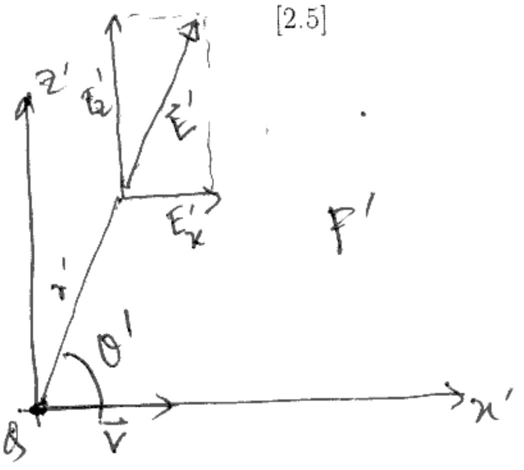

$\therefore$ $\vec{E}'$ points radially outward along a line drawn from the ~~ist~~ instantaneous position of Q.

(For implications of this read Purcell, sec. 5.6 & 5.7).

What about the strength of this electric field $\vec{E}'$?

$$E'^2 = E_x'^2 + E_z'^2 = \frac{1}{(4\pi\epsilon_0)^2}\cdot\frac{\gamma^2 Q^2 (x'^2+z'^2)}{\left[(\gamma x')^2 + z'^2\right]^3}$$

$$= \frac{1}{(4\pi\epsilon_0)^2}\cdot\frac{\gamma^2 Q^2 (x'^2+z'^2)}{\left[\frac{x'^2}{1-\beta^2} + z'^2\right]^3} \qquad \left(\because \gamma^2 = \frac{1}{1-\beta^2}\right)$$

$$= \frac{1}{(4\pi\epsilon_0)^2}\cdot\frac{\gamma^2 Q^2 (x'^2+z'^2)}{\gamma^6\left[x'^2 + z'^2 - z'^2\beta^2\right]^3}$$

$$= \frac{1}{(4\pi\epsilon_0)^2}\cdot\frac{1}{\gamma^4}\cdot\frac{Q^2 (x'^2+z'^2)}{(x'^2+z'^2)^3\left[1-\beta^2\frac{z'^2}{x'^2+z'^2}\right]^3}$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{Q^2 (1-\beta^2)^2}{(x'^2+z'^2)^2\left[1-\beta^2\frac{z'^2}{x'^2+z'^2}\right]^3}$$

Since, $\sin\theta' = \frac{z'}{(x'^2+z'^2)^{1/2}}$ & $r' = (x'^2+z'^2)^{1/2}$

2. ~~Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5$cm ?~~ [2.5]

$$\therefore E'^2 = \frac{1}{(4\pi\epsilon_0)^2} \cdot \frac{Q^2(1-\beta^2)^2}{r'^4\left(1-\beta^2\sin^2\theta'\right)^3}$$

$$\Rightarrow \boxed{E' = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{r'^2} \cdot \frac{1-\beta^2}{\left(1-\beta^2\sin^2\theta'\right)^{3/2}}}$$

For low speeds, $\beta \simeq 0 \Rightarrow E' \simeq \frac{1}{4\pi\epsilon_0}\frac{Q}{r'^2}$. (expected)

For high enough speeds, because of the $\sin\theta'$ factor, the field is stronger at right angles to motion. ($\theta' = \pi/2$) than in the direction of motion ($\theta' \simeq 0$)!

$\therefore$ Field lines concentrated in a pancake $\perp^r$ to direction of motion. — not spherically symmetric! Symmetric about a plane $\perp^r$ to direction of motion of charge.