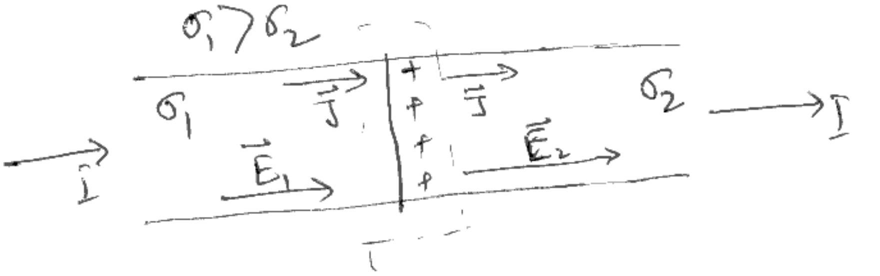
PHY102 : Quiz 1

~~1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]~~

Assignment 5 (Sols.)

1. 

- Normal component of $\vec{J}$ same on left & right of junction.

$$\therefore |\vec{E}_1| = \frac{J}{\sigma_1}, \quad |\vec{E}_2| = \frac{J}{\sigma_2} \quad \left(\because \sigma_1 > \sigma_2 \quad \therefore E_2 > E_1 \right)$$

By Gauss's law, the difference in the electric fields gives the surface charge density, σ' as,

$$E_2 - E_1 = \frac{\sigma'}{\epsilon_0}$$

$$\Rightarrow \frac{J}{\sigma_2} - \frac{J}{\sigma_1} = \frac{\sigma'}{\epsilon_0} \Rightarrow \sigma' = J\epsilon_0 \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right)$$

If A is the area of the interface, then,

$$Q = \sigma' A \quad \& \quad I = JA = \frac{JQ}{\sigma'}$$

$$\therefore Q = \frac{I\sigma'}{J} = \frac{I}{J} \cdot J\epsilon_0 \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right)$$

$$\therefore \boxed{Q = I\epsilon_0 \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right)}$$

~~2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$?~~ [2.5]

2. $I = \frac{V}{R} = V\left(\frac{A}{\rho L}\right)$ $\left(\because R = \frac{\rho L}{A},\ \rho : \text{resistivity}\right)$

$J = \frac{I}{A} = \frac{V\cancel{A}/\rho L}{\cancel{A}} = \frac{V}{\rho L}$.

Now, $J = n e v_d$ where n : $\frac{\text{no. of electrons}}{cm^3}$

v_d : drift velocity of electrons.

$\therefore n e v_d = \frac{V}{\rho L} \Rightarrow v_d = \frac{V}{\rho L n e}$.

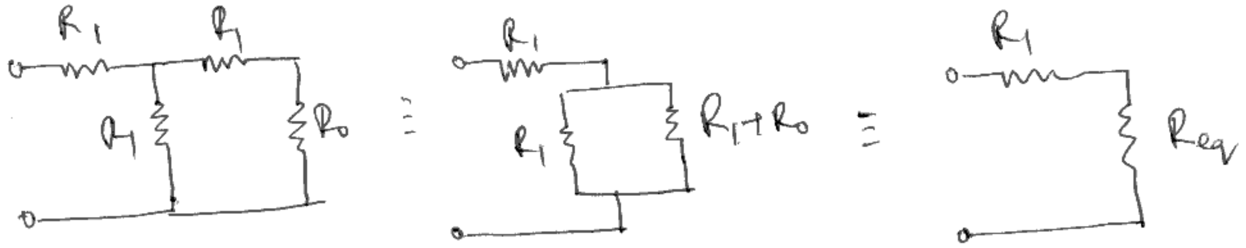
$\rho = 1.7 \times 10^{-6}$ ohm cm, $n = 8 \times 10^{22} / cm^3$

$e = 1.6 \times 10^{-19} C$, $V = 6$ Volt, $L = 1km = 10^5 cm$.

$\therefore v_d \simeq 2.8 \times 10^{-3}$ cm/s.

$\therefore t \simeq \frac{10^5 \text{ cm}}{2.8 \times 10^{-3} \text{ cm/s}} \simeq 3.6 \times 10^7 s \simeq 1$ yr !

3.



$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_1 + R_0}} = \frac{R_1(R_1 + R_0)}{R_1 + (R_1 + R_0)}$.

Finally, $R = R_1 + R_{eq} = R_1 + \frac{R_1(R_1 + R_0)}{R_1 + R_1 + R_0}$

We require, $R = R_0$.

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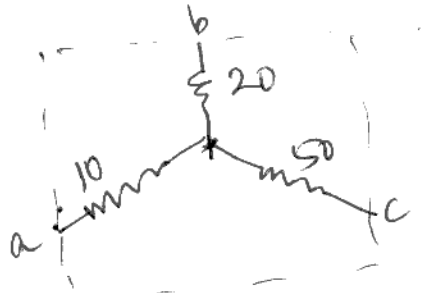
$$\therefore R_0 = R_1 + \frac{R_1(R_1+R_0)}{2R_1+R_0} = \frac{R_1(2R_1+R_0)+R_1(R_1+R_0)}{2R_1+R_0}.$$

$$\Rightarrow R_0(2R_1+R_0) = R_1(2R_1+R_0+R_1+R_0) = R_1(3R_1+2R_0)$$

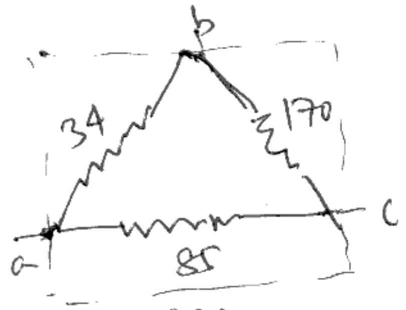
$$\Rightarrow \cancel{2R_0R_1} + R_0^2 = 3R_1^2 + \cancel{2R_0R_1} \Rightarrow R_0^2 = 3R_1^2$$

$$\therefore \boxed{R_1 = \frac{R_0}{\sqrt{3}}}$$

4.



(A)



(B)

For (A), resistance between any 2 terminals is basically the series combo of the 2 resistors which connect the terminals. The third resistance does not feature. Therefore, the resistance between terminals (a) & (b) would be,

$$R^{A}_{ab} = (10+20)\,\Omega = 30\,\Omega.$$

For (B), resistance between 2 terminals would involve a resistor in ‖l with 2 others being in series. For example, to find the resistance between terminals (a) & (b), $34\,\Omega$ is in ‖l to the

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

series combination of $170\,\Omega$ and $85\,\Omega$.

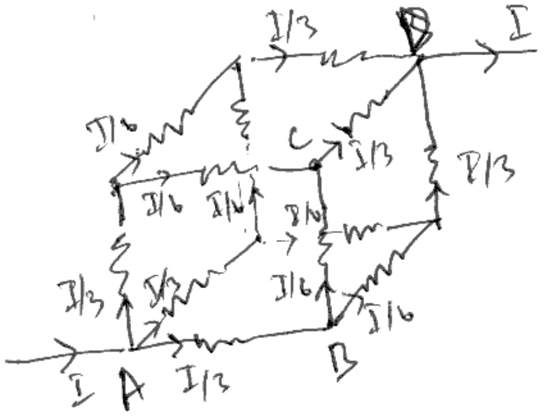
$$\therefore R^{B}_{ab} = \left(\frac{1}{\frac{1}{34} + \frac{1}{170+85}} \right) \Omega = \frac{1}{\frac{255+34}{34 \times 255}}$$

$$= \frac{\cancel{255}^{15} \times \cancel{34}^{2}}{\cancel{289}_{17}} = 30\,\Omega. = R^{A}_{AB}$$

You can similarly show for the other 2.

These are the only 2 possible configurations.

5.



Resistors at the edges of the cube all have same value. R_o.

A current I entering at A will get equally divided in the three sides and so on.

Now take any path say ABCD & for that path,

$$V_{AD} = \underbrace{\frac{I}{3} R_o}_{AB} + \underbrace{\frac{I}{6} R_o}_{BC} + \underbrace{\frac{I}{3} R_o}_{CD.} = \frac{5}{6} R_o I$$

$$\therefore \boxed{R_{AD} = \frac{V_{AD}}{I} = \frac{5}{6} R_o}$$

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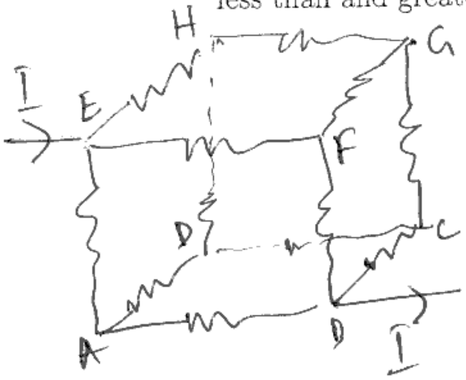

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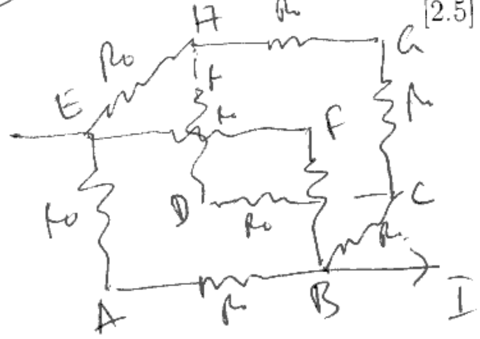
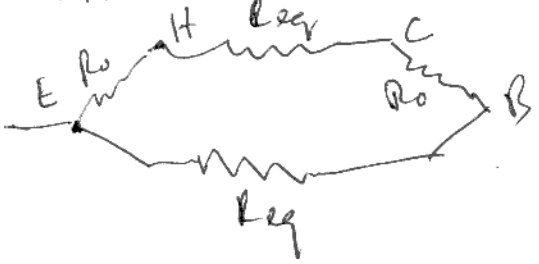
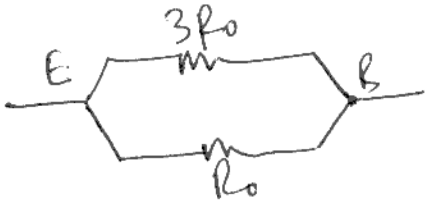


By symmetry, A & F & D & G are equivalent. Current in the branches AD & FG must be zero.



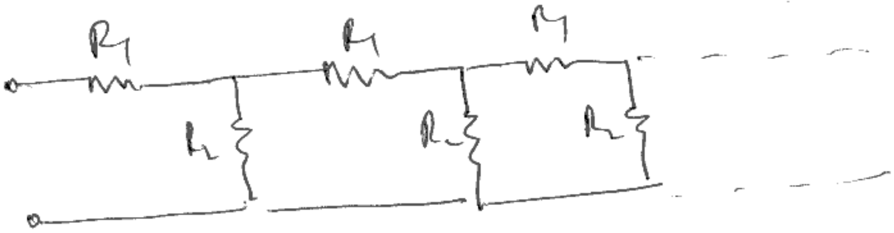
$$\frac{1}{R_{eq}} = \frac{1}{R_0 + R_0} + \frac{1}{R_0 + R_0}$$
$$= \frac{1}{R_0}$$
$$\therefore R_{eq} = R_0.$$

$\equiv$



$$\therefore R_{EB} = \frac{1}{\frac{1}{3R_0} + \frac{1}{R_0}} = \frac{3}{4} R_0.$$

6.



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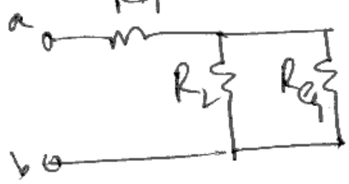


Let R_{eq} be the equivalent resistance. For the infinite ckt, we can replace everything on the right of the first 2 elements by R_{eq}. ~~and~~ ~~still get the equation~~

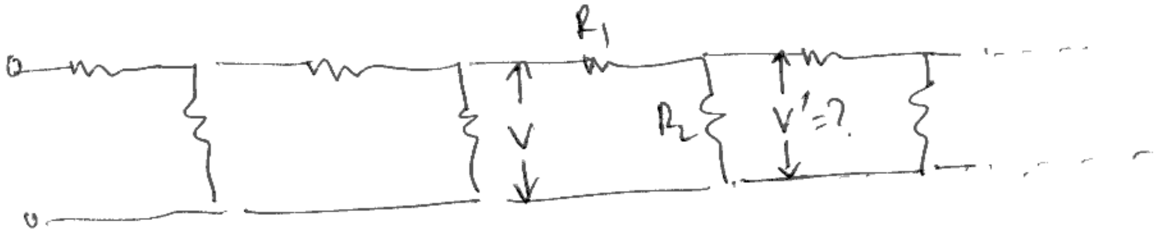
But, $R_{ab} = R_{eq}$ $\therefore$ $R_{eq} = R_1 + \dfrac{R_{eq} \cdot R_2}{R_{eq} + R_2}$.

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

$$R_{eq}^2 + \cancel{R_{eq}R_2} = R_{eq}R_1 + R_1R_2 + \cancel{R_{eq}R_2}.$$

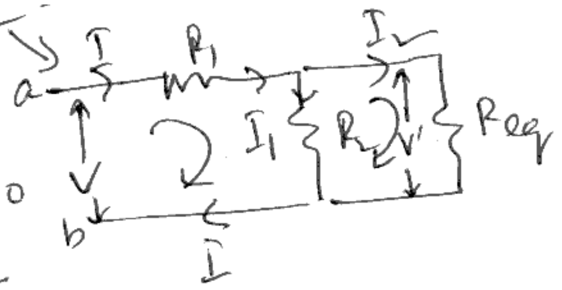
$$\Rightarrow \quad R_{eq}^2 - R_{eq}R_1 - R_1R_2 = 0.$$

$$R_{eq} = \frac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2}$$ (Not the -ve root Why?)



$\equiv$

Consider the loop region



$V' = I_2 R_{eq}$. & $I_2 R_{eq} - I_1 R_2 = 0$
$\therefore V' = I_1 R_2$

Also, $V = IR_1 + I_1 R_2 = IR_1 + V'$.

$$\Rightarrow \quad V' = V - IR_1$$

But, $V = IR_{eq}$. since equivalent resistance between pts a & b is still R_{eq}

$$\therefore \quad V' = V - \frac{VR_1}{R_{eq}} = V\left(\frac{R_{eq} - R_1}{R_{eq}}\right).$$

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

If $\frac{V'}{V} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{R_{eq} - R_1}{R_{eq}} \Rightarrow R_{eq} = 2R_{eq} - 2R_1$

$\therefore R_{eq} = 2R_1$.

Now, $R_{eq} = \dfrac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2}$

$\therefore R_1 + \sqrt{R_1^2 + 4R_1R_2} = 4R_1 \Rightarrow R_1^2 + 4R_1R_2 = 9R_1^2$

$\Rightarrow {}^{2}\cancel{8}R_1^2 = \cancel{4}R_1R_2 \Rightarrow \boxed{R_2 = 2R_1}$.

To terminate ladder, just ~~connect~~ replace rest of ladder at any pt by the equivalent resistance R_{eq}.