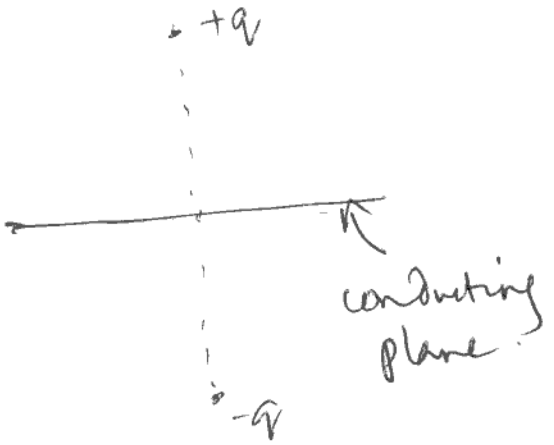
Assignment 04 (Sols.)

1.



At distance 'x' above the conducting plane, the force between plane and charge will be same as that between the charge and the image charge $-q$ a distance 'x' below the plane.

$$\therefore F = \frac{q^2}{4\pi\varepsilon_0 (2x)^2}$$

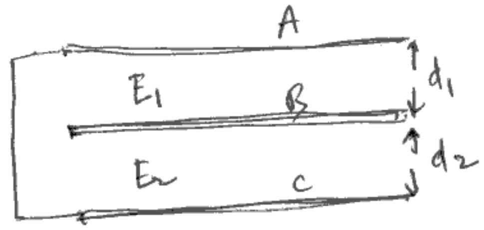
$$\therefore \text{Work done by 2nd student} = \int_h^\infty F\,dx = \frac{q^2}{4\pi\varepsilon_0}\cdot\frac{1}{4}\int_h^\infty \frac{dx}{x^2}$$

$$= \frac{q^2}{4\pi\varepsilon_0 (4h)}$$

→ correct answer.

For the 1st student, charges $+q$ and $-q$ are pulled apart symmetrically. So whatever work is done is the 'total' work. Work done in moving $+q$ is half of that total.

2.



Let's call the three conducting plates A, B and C.

Since A & C are connected by a wire, they are at the same potential. Therefore, if B is at some potential, then the potential difference between A & B and between B & C are same.

Now, the electric fields between the plates are given as E_1 & E_2. If σ_1 is the surface charge on the upper surface of B & σ_2 that in the lower surface then,

$$E_1 = \frac{\sigma_1}{\epsilon_0} \quad \& \quad E_2 = \frac{\sigma_2}{\epsilon_0}.$$

Now, potential difference between plates A & B and between B & C :

$$\phi = E_1 d_1 = E_2 d_2. \qquad (\because E = \phi/d)$$

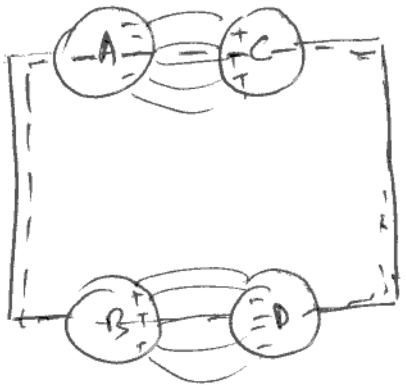
uniform fields!

$$\therefore \frac{\sigma_1}{\epsilon_0} d_1 = \frac{\sigma_2}{\epsilon_0} d_2 \Rightarrow \sigma_1 d_1 = \sigma_2 d_2.$$

Also, $\sigma = \sigma_1 + \sigma_2$.

Solving, $\sigma_1 = \dfrac{\sigma d_2}{d_1 + d_2}$, $\sigma_2 = \dfrac{\sigma d_1}{d_1 + d_2}$.

3.



Take a closed path through the conductors and the wires.

Now, $\oint \vec{E} \cdot d\vec{l} = 0$ for static electric field.

In this case, since A, B, C, D are conductors, as well as in the conducting wires, $\vec{E} = 0$.

However each gap region will contribute to the electric field and hence, the line integral is not equal to zero.

In other words, we cannot have a static charge distribution as in (b).

4. $V_1 = 100$ volts. $C_1 = 100\ pF = 100 \times 10^{-12} F = 10^{-10} F$

$\therefore Q = V_1 C_1 = 100 \times 10^{-10}\ C = 10^{-8}\ C$.

After charging battery is disconnected & the capacitor is connected in $\parallel^{l}$ to another capacitor of capacitance C_2 then, the total charge remain same. If V_2 is the final voltage, then,

$$Q = V_2(C_1 + C_2) \quad (\parallel^{l} : \text{capacitance add up}).$$

$V_2 = 30$ volts.

$$\therefore 10^{-8} = 30.(10^{-10} + C_2).$$

$$C_2 = \frac{10^{-8}}{30} - 10^{-10} = 10^{-10}\left(\frac{100}{30} - 1\right).$$

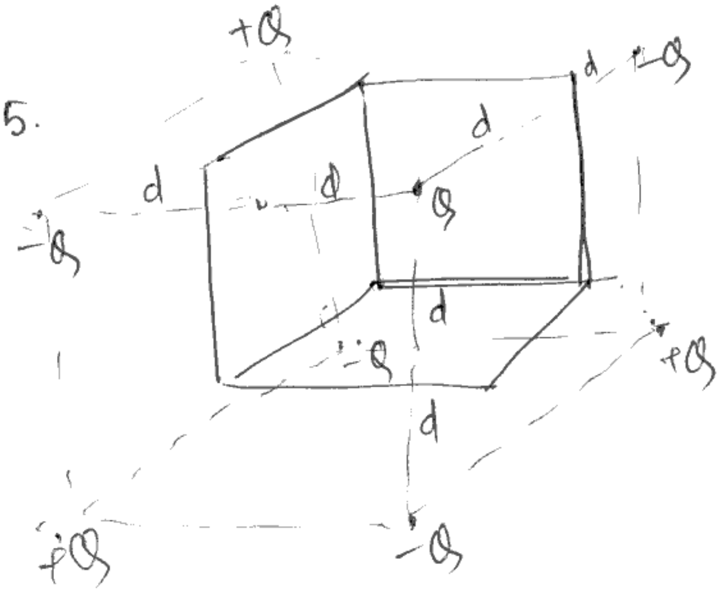
$$= \frac{7}{3} \times 10^{-10}\ F = \frac{7}{3} C_1$$

$$E_i = \frac{Q^2}{2C_1} = \frac{1}{2} Q V_1 \ ; \ E_f = \frac{1}{2} Q V_2.$$

$$\therefore \text{Energy lost} = E_i - E_f = \frac{1}{2} Q (V_1 - V_2).$$

$$= \frac{1}{2} \cdot 10^{-8} \cdot (100 - 30) = 35 \times 10^{-8}\ J.$$

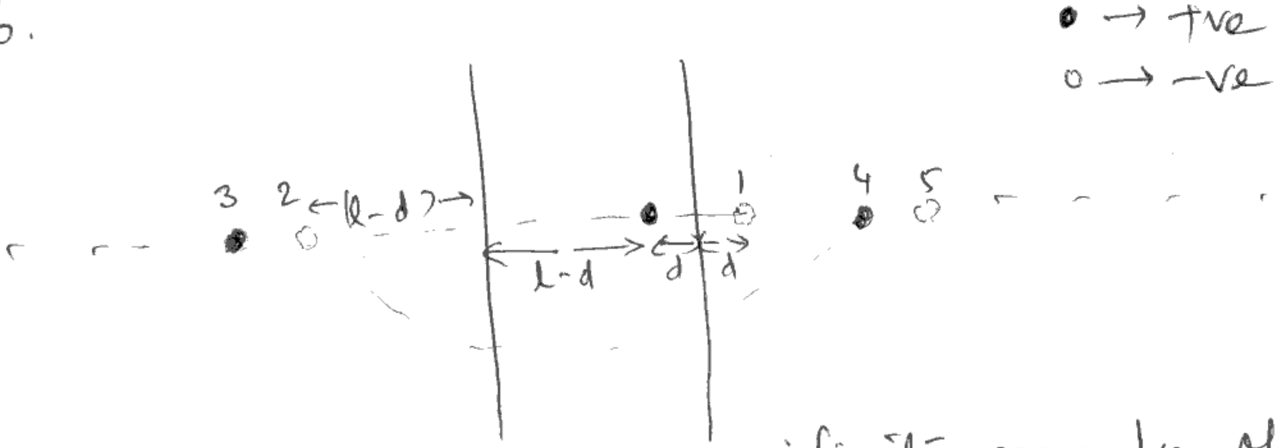
5.



7 image charges forming 7 corners of the cube of side 2d.

Calculate force.

6.



We will need an infinite number of image charges as shown.

7. Field outside the outer shell = 0.

∴ Potential at outer shell = potential at infinity.

∴ When outer shell is grounded, charge will not move.

If inner shell is grounded, then,

potential diff between inner and outer shells

= potential diff between outer shell & infinity

If Q_f is final charge on inner shell, then,

electric field between shells $= \frac{Q_f}{4\pi\epsilon_0 r^2}\hat{r}$

∴ potential diff between inner and outer shells

$$= -\frac{Q_f}{4\pi\epsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2} = -\frac{Q_f}{4\pi\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Potential diff between outer shell & infinity.

$$= \frac{(-Q+Q_f)}{4\pi\epsilon_0}\int_{R_2}^{\infty}\frac{dr}{r^2} = \frac{(-Q+Q_f)}{4\pi\epsilon_0}\cdot\frac{1}{R_2}$$

$$\therefore \frac{-Q_s+Q_F}{4\pi\epsilon_0} \cdot \frac{1}{R_2} = -Q_F\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow Q_F = \frac{R_1}{R_2} Q_s.$$