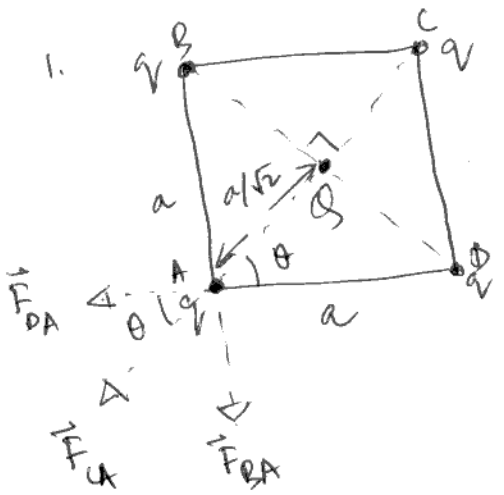
# Assignment 1

1\. 


Total force acting in the CA direction due to the 'q' charges.

$$|\vec{F}| = |\vec{F}_{DA}|\cos\theta + |\vec{F}_{BA}|\cos(90^\circ-\theta) + |\vec{F}_{CA}|$$

$$= \frac{q^2}{4\pi\epsilon_0 a^2}\cdot\frac{\cancel{a}/\sqrt{2}}{\cancel{a}} + \frac{q^2}{4\pi\epsilon_0 a^2}\cdot\frac{\cancel{a}/\sqrt{2}}{\cancel{a}} + \frac{q^2}{4\pi\epsilon_0(\sqrt{2}a)^2}$$

$$= \frac{q^2}{4\pi\epsilon_0 a^2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2}\right]$$

$$= \frac{q^2}{4\pi\epsilon_0 a^2}\,\frac{2\sqrt{2}+1}{2}.$$

This needs to be balanced by the force from 'Q' so that the sum of all forces on 'q' equals zero.

Force on q due to charge $Q = \dfrac{qQ}{4\pi\epsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}$.

$$\therefore\ \frac{q^2}{4\pi\epsilon_0 a^2}\,\frac{2\sqrt{2}+1}{2} + \frac{2qQ}{4\pi\epsilon_0 a^2} = 0.$$

$$\Rightarrow\ Q = -\left(\frac{2\sqrt{2}+1}{4}\right)q.$$

The equilibrium is not stable. To see this, you can displace 'Q' from the center. This will lead to further displacement & there will be no going back to the original configuration.

2. Building the sphere up layer by layer, we will assume that we can neglect self interaction of the "thin" layer that we are bringing in. (We will talk about this more in prob 5, Assignment 2).

Start with a sphere of charge 'q' already built. Now, to bring a thin layer of charge 'dq', the potential energy,

$$dU = \frac{q \cdot dq}{4\pi\epsilon_0 r} \quad ; \quad q = \frac{4}{3}\pi r^3 \rho.$$

($\rho$: charge density)

$$dq = 4\pi r^2 dr \rho.$$

$$\therefore dU = \frac{\frac{4}{3}\pi r^3 \rho \cdot \cancel{4}\pi r^2 dr \rho}{\cancel{4}\pi\epsilon_0 r}$$

$$= \frac{4\pi}{3\epsilon_0}\rho^2 \cdot r^4 dr.$$

$$\therefore U = \frac{4\pi}{3\epsilon_0}\rho^2 \int_0^a r^4 dr = \frac{4\pi}{3\epsilon_0}\rho^2 \frac{a^5}{5}.$$

Now, total charge in the sphere $= Q = \frac{4}{3}\pi a^3 \rho$.

$$\therefore \rho = \frac{3Q}{4\pi a^3}$$

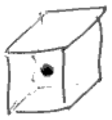
$$\therefore U = \frac{\cancel{4}\cancel{\pi}}{\cancel{3}\epsilon_0} \frac{\overset{3}{\cancel{9}} Q^2}{\underset{4}{\cancel{16}}\cancel{\pi^2} \underset{a}{\cancel{a^6}}} \frac{\cancel{a^5}}{5} = \frac{1}{4\pi\epsilon_0} \frac{3Q^2}{5a}$$

Can you come to the same answer by using,

$$U = \frac{\epsilon_0}{2}\int E^2 dv \, ?$$

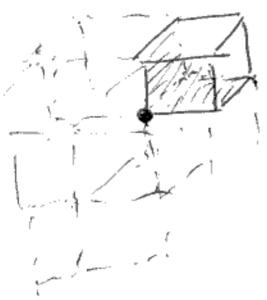
3. By Gauss' Law, flux from a point charge, $q$, is,

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0}.$$



Since the charge is placed at the center of the cube, by symmetry, the flux through all the faces is same. There are six faces of the cube.

$\therefore$ Flux through one face $= \frac{1}{6} \cdot \oint \vec{E} \cdot d\vec{a} = \frac{q}{6\epsilon_0}$.



When you shift the charge to a corner, try to construct a bigger cube with twice the edge length, such that the charge is at the center of this bigger cube. The bigger cube will be made of 8 small ones.

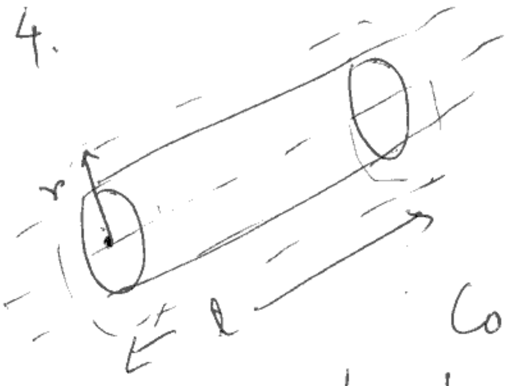
$\therefore$ Flux through the faces of the small cube

$= \frac{1}{8}$ of Flux through the faces of bigger cube $= \frac{1}{8}\left(\frac{q}{\epsilon_0}\right)$

Now, of the faces of the small cube, only 3 faces contribute to the flux when the charge $q$ is placed at the corner. The flux through the 3 faces that meet at charge (shaded ones) $= 0$.

$\therefore$ Flux through each of the 3 faces that contribute

$$= \frac{1}{8} \times \frac{1}{3}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{24}\frac{q}{\epsilon_0}.$$

4. 


The charge distribution has axial symmetry. The electric field is therefore radial.

Consider a cylinder of radius $r$ & length $l$ outside the charge distribution.

The flux through this surface $= E_r \cdot (2\pi r l)$.

$\therefore$ By Gauss' Law,
$$E_r \cdot 2\pi r l = \frac{\lambda \cdot l}{\epsilon_0}.$$

where $\lambda$ : charge / length.

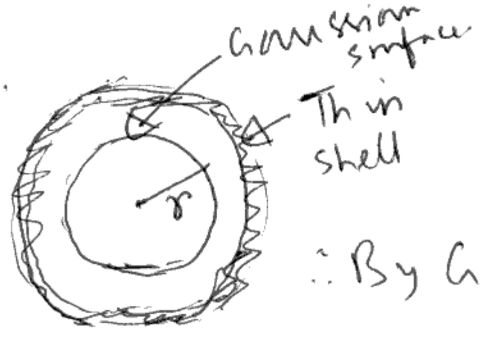
$$\therefore \quad E_r = \frac{\lambda}{2\pi\epsilon_0 r}.$$

To get the field inside the cylinder, we can similarly construct a Gaussian surface. But now, the charge enclosed is zero, since the cylinder is hollow.

$\therefore E_r = 0$ inside the hollow cylindrical symmetric charge distribution.

5. Charge distribution is spherically symmetric. $\therefore$ Electric field radially symmetric. We treat the cases $r < a$ & $r > a$ separately.

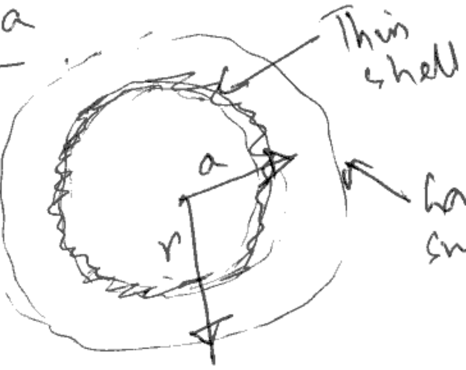
$r \leq a$.




Charge enclosed by Gaussian surface $= 0$. since all charge resides on the surface of shell.

$\therefore$ By Gauss' Law, $E = 0$, $r < a$.

$r \geq a$




Charge enclosed by Gaussian surface $= Q$.

$\therefore$ By Gauss' law,

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E_r \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E_r = \frac{Q}{4\pi\epsilon_0 r^2}$$

Same as field due to a pt charge $Q$.

$\therefore$ Field outside is the same as if all the charge were concentrated at the centre of the sphere.


Note the discontinuity in $E$ as we cross the surface $r = a$.

$$\Delta E = E(r \geq a) - E(r < a)$$

$$= \frac{Q}{4\pi\epsilon_0 a^2} - 0 = \frac{Q}{4\pi\epsilon_0 a^2}$$

$$= \frac{\sigma}{\epsilon_0} \quad \text{where, } \sigma = \frac{Q}{4\pi a^2}$$

6. From the previous sol$^n$, field inside $= 0$.

$\therefore$ Using, $U = \frac{\epsilon_0}{2} \int E^2 \, dv$, we have,

$$U = \frac{\epsilon_0}{2} \int_R^\infty \left( \frac{Q}{4\pi\epsilon_0 r^2} \right)^2 \cdot 4\pi r^2 \, dr = \frac{Q^2}{8\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} \, dr$$

$$= \frac{Q^2}{8\pi\epsilon_0 R}$$ (do the integration).

Note that this is smaller than the value you calculated for the solid sphere (Prob 2.)

Can you calculate this in the same way as we did for prob 2 ?