1

a)$-\sin(x)+C$
b)$\log(x)+C$
c)e^x+C
d)$\frac{x^4}{4}$
e)$\frac{a^x}{\log(a)}$
f)

$$\begin{aligned}
\text{Put}\quad ax+b &= t \\
\implies adx &= dt \\
\int \sin(ax+b)dx &= \int \sin(t)\frac{dt}{a} \\
&= -\frac{\cos(t)}{a}+C
\end{aligned}$$

2

a)

$$\begin{aligned}
\int_0^1 (3x^2+7x+2)dx &= \left[x^3+\frac{7x^2}{2}+2x\right]_0^1 \\
&= \frac{13}{2}
\end{aligned}$$

b)

$$\begin{aligned}
\int_0^{2\pi} \sin(x)dx &= \big[\cos(x)\big]_0^{2\pi} \\
&= 1-1=0
\end{aligned}$$

c)

$$\begin{aligned}
\text{Put}\quad 2x &= t \\
2dx &= dt \\
x\to 0 \implies t\to 0 \quad &, \qquad x\to 2 \implies t\to 4 \\
\int_0^2 e^{2x}dx &= \int_0^4 e^t\frac{dt}{2} \\
&= \frac{e^4-1}{2}
\end{aligned}$$

3

a)

$$\begin{aligned}
\text{Put}\quad x &= a\tan(\theta)\\
dx &= a\sec^2(\theta)d\theta\\
\int \frac{1}{x^2+a^2}dx &= \frac{1}{a^2+a^2\tan^2(\theta)}a\sec^2(\theta)d\theta\\
&= \frac{a\sec^2(\theta)}{a^2\sec^2(\theta)}d\theta\\
&= \frac{1}{a}d\theta\\
&= \frac{\theta}{a}+C\\
&= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C
\end{aligned}$$

b)

$$\begin{aligned}
\text{Put}\quad x &= \sqrt{6}\sin(\theta)\\
dx &= \sqrt{6}\cos(\theta)d\theta\\
\int \frac{1}{\sqrt{6-x^2}}dx &= \int \frac{1}{\sqrt{6-6\sin^2(\theta)}}\sqrt{6}\cos(\theta)d\theta\\
&= \int \frac{\sqrt{6}\cos(\theta)}{\sqrt{6}\cos(\theta)}d\theta\\
&= \int d\theta\\
&= \theta+C\\
&= \sin^{-1}\frac{x}{\sqrt{6}}+C
\end{aligned}$$

c)

$$\begin{aligned}
\text{Put}\quad (2x+3) &= 3\tan(\theta)\\
2dx &= 3\sec^2(\theta)d\theta\\
\int \frac{1}{(2x+3)^2+9}dx &= \int \frac{1}{9\tan^2(\theta)+9}\frac{3\sec^2(\theta)}{2}d\theta\\
&= \int \frac{\sec^2(\theta)}{6\sec^2(\theta)}d\theta\\
&= \frac{\theta}{6}+C\\
&= \frac{1}{6}\tan^{-1}\left(\frac{2x+3}{3}\right)+C
\end{aligned}$$

4

$$\begin{aligned}
\int \frac{1}{x^2-x+1}dx &= \int \frac{1}{x^2-2.(x).\frac{1}{2}+\frac{1}{4}-\frac{1}{4}+1}dx \\
&= \int \frac{1}{\left(x-\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)^2}
\end{aligned}$$

Now, you can put $\left(x-\frac{1}{2}\right)=\frac{\sqrt{3}}{2}\tan(\theta)$ and do what you did in the last questions.

5

a) Choose x as the first function.

$$\begin{aligned}
\int xe^x dx &= x\int e^x dx - \int \frac{d(x)}{dx}\left(\int e^x dx\right)dx \\
&= xe^x - \int 1.e^x dx \\
&= xe^x - e^x + C
\end{aligned}$$

b) First use the first identity given in the formulae section.

$$\begin{aligned}
\int x\sin(x)\cos(x)dx &= \int \frac{x}{2}sin(2x)dx \\
&= \frac{1}{2}\left[x\int \sin(2x)dx - \int \frac{d(x)}{dx}\left(\int \sin(2x)dx\right)dx\right] \\
&= \frac{1}{2}\left[\frac{-x\cos(2x)}{2} - \int \frac{-\cos(2x)}{2}\right] \\
&= \frac{1}{4}\left(-x\cos(2x)+\frac{\sin(2x)}{4}\right)+C
\end{aligned}$$

c) Use the second identity from the formulae section

$$\begin{aligned}
\int (x^2+4)\sin(mx)\cos(nx)dx &= \frac{1}{2}\int (x^2+4)[\sin((m+n)x)+\sin((m-n)x)]dx \\
&= \int [x^2\sin(m+n)x + x^2\sin(m-n)x + 4\sin(m+n)x + 4\sin(m-n)x]dx
\end{aligned}$$

You can evaluate the first 2 integrals using by parts and the other 2 are simple known integrals. Just to demonstrate-

$$\begin{aligned}
\int x^2\sin(m+n)xdx &= x^2\int sin(m+n)xdx - \int \frac{d(x^2)}{dx}(\sin((m+n)x)dx) \\
&= -x^2\frac{\cos(m+n)x}{(m+n)} + \int 2x\frac{\cos(m+n)x}{(m+n)}dx
\end{aligned}$$

You can find the second integral by using by-parts technique again.

6

So, ωt goes from 0 to 2π as t goes from 0 to T, where T is the time period. So, $\omega = \frac{2\pi}{T}$.

$$\begin{aligned}
\text{Average} &= \frac{\int_0^T E_0 \sin^2(\omega t)dt}{T-0} \\
\int_0^T \sin^2(\omega t)dt &= \int \frac{1-\cos(2\omega t)}{2}dt \\
&= \frac{T-0}{2} - \frac{\sin(2\omega t)}{4}\Big]_0^T \\
&= \frac{T}{2} - \frac{\sin(2\omega T)}{4} \\
&= \frac{T}{2} - \frac{\sin(4\pi)}{4} \\
&= \frac{T}{2} \\
\text{Average} &= \frac{E_0 T}{2T} \\
&= \frac{E_0}{2}
\end{aligned}$$

For the second part, the average of sine function over one cycle is zero.

7

$$\begin{aligned}
\text{Average} &= \frac{\int_0^T [E_0 \sin(m\omega t)\sin(n\omega t)]dt}{T-0}dt \\
&= \frac{\int_0^T [E_0 \cos\{(m-n)\omega t\} - E_0 \cos\{(m+n)\omega t\}]dt}{2T}
\end{aligned}$$

These are simple integrals which you can evaluate. If m=n, $\cos(m-n)\omega t$ is 0. In that case, the answer is $\frac{E_0}{2}$ otherwise it's 0.

8

Choose $\tan\left(\frac{1}{x}\right)$ as the first function.

$$\begin{aligned}
\text{Give Integral} &= \tan\left(\frac{1}{x}\right)\int 3x^2dx - \int \frac{d\tan\left(\frac{1}{x}\right)}{dx}\left(\int 3x^2dx\right)dx - \int x\sec^2\left(\frac{1}{x}\right) \\
&= x^3\tan\left(\frac{1}{x}\right) - \int \left(\sec^2\left(\frac{1}{x}\right)\right) \times -\frac{1}{x^2} \times x^3dx - \int x\sec^2\left(\frac{1}{x}\right) \\
&= x^3\tan\left(\frac{1}{x}\right) + \int x\sec^2\left(\frac{1}{x}\right) - \int x\sec^2\left(\frac{1}{x}\right) \\
&= x^3\tan\left(\frac{1}{x}\right) + C
\end{aligned}$$