Vectors

Maths Workshop

Problems

1. Find a unit vector in the direction of $3\vec{i} + 12\vec{j} + 4\vec{k}$

2. Find the angle between the vectors $3\vec{i} + 4\vec{j}$ and $4\vec{i} + 4\vec{j} + 2\vec{k}$

3. Let $\vec{A} = 3\vec{i} + 2\vec{j}$ and $\vec{B} = 4\vec{i} - \vec{j}$. What is the projection of $\vec{A}$ along $\vec{B}$?

4. Given that $|\vec{A}| = 3$; $|\vec{B}| = 8$; $|\vec{A} \times \vec{B}| = 12$; what is the angle between the two vectors?

5. Let $\vec{A} = 3\vec{i} + 4\vec{j} - 4\vec{k}$. Find a vector in the x-y plane that is perpendicular to $\vec{A}$

6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then show that $\vec{a} \perp \vec{b}$

7.
(i) Prove that
$$(\vec{a} \times \vec{b}).\vec{c} = \begin{vmatrix} c1 & c2 & c3 \\ a1 & a2 & a3 \\ b1 & b2 & b3 \end{vmatrix}$$
(ii) Using the previous result prove that $[\vec{a}\ \vec{b}\ \vec{c}]$ is cyclic, i.e $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$. i.e., $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

(iii) Find $(2\vec{i} + 3\vec{j}).[(2\vec{i} + 3\vec{j}) \times (4\vec{j} + 7\vec{k})]$

8.
(i) Prove $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$
(ii) $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

Some Important formulae

(i) $$(\vec{a} \times \vec{b}).\vec{c} = \begin{vmatrix} c1 & c2 & c3 \\ a1 & a2 & a3 \\ b1 & b2 & b3 \end{vmatrix}$$
(ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$
(iii) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$
(iv) $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = (\vec{a}.\vec{c})(\vec{b}.\vec{d}) - (\vec{a}.\vec{d})(\vec{b}.\vec{c})$
(v) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = [\vec{a}\ \vec{c}\ \vec{d}]\vec{b} - [\vec{b}\ \vec{c}\ \vec{d}]\vec{a}$