Matrices Problem Sheet

Maths Workshop 2020

December 18, 2020

Problems

1.Find the order and evaluate the transpose of the following matrices :-

$$(i)\begin{bmatrix}1 & 2 & 3\\ 7 & 8 & 9\end{bmatrix} \qquad (ii)\begin{bmatrix}5\\ 2\\ 1\end{bmatrix} \qquad (iii)\begin{bmatrix}2 & 5 & 7\end{bmatrix}$$

2.Given matrices $A = \begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ and $B = \begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and the equation $(A+B)^2 = A^2 + B^2$. Evaluate the values of a and b.

3.If $P(x) = \begin{bmatrix}cos(x) & sin(x)\\ -sin(x) & cos(x)\end{bmatrix}$, then check that $P(x)P(y) = P(x+y) = P(y)P(x)$.

4.Let $A = \begin{bmatrix}1 & -1 & 0\\ 2 & -1 & 7\\ 3 & 0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1\\ 0\\ 2\end{bmatrix}$. Evaluate AB and verify that $(AB)^t = B^tA^t$.

5. Given $A = \begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$. Find the value of A^2 and A^3 and guess the value of A^n. Can you prove your guess using mathematical induction, or otherwise ?

6.Given matrix $A = \begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$ show that $A^n = \begin{bmatrix}a^n & 0\\ 0 & b^n\end{bmatrix}$ for some natural number n.