

MTH202: Assignment 9

March 8, 2019

1. Suppose the lifetime of a component in a machine is distributed as $Exp(1)$. Then,
 - Find the lifetime L which is typical component is 60 percent certain to exceed.
 - If 5 components are sold, find the probability that at least 1 of them will have a lifetime less than L .

2. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f_X(x) = Cx$$

where, C is a constant.

- (a) Find C .
 - (b) $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.
3. The lifetime (in days) of a machine is given by a random variable with density function:

$$f(x) = \begin{cases} 0 & \text{for } x \leq 100 \\ \frac{50}{x^2} & \text{for } x > 100 \end{cases}$$

There are 5 such machines in a factory. What is the probability that exactly 2 machines will breakdown within the first 150 days?

4. Consider a random variable $X \sim Unif([0, 1])$. Compute the following:
 - (a) $P(X < 3)$.
 - (b) $P(X > 3)$.
 - (c) $P(3 < X < 8)$.
 - (d) $E[4X^2 - 2X]$.
 - (e) $E[e^X]$.
5. Let $\phi(z)$ denote $P(Z \leq z)$ for a standard normal random variable Z .
 - Let $X \sim \mathcal{N}(2, 4)$ and $Y = 3 - 2X$. Find $P(X > 1)$, $P(2 < Y < 1)$, $P(X > 2|Y < 1)$ in terms of ϕ .

- Let $X \sim \mathcal{N}(0, 2)$. Compute $E[|X|]$.
6. Let $X \sim \text{Exp}(\lambda)$. What is the distribution function of X ?
 7. Let $U \sim \text{Unif}([0, 1])$ and $X = -\ln(1 - U)$. Show that $X \sim \text{Exp}(1)$.
 8. Suppose the number of customers arriving at a store obeys a Poisson distribution with an average of λ customers per unit time. That is, if Y is the number of customers arriving in an interval of length t , then $Y \sim \text{Poi}(\lambda t)$. Suppose that the store opens at time $t = 0$. Let X be the arrival time of the first customer. Show that $X \sim \text{Exp}(\lambda)$.
 9. Test scores on OWLs at Hogwarts are normally distributed with mean 250 and variance 900. Only the top 5 percent students will qualify to become an Auror. What is the minimum score Harry must get in order to qualify?
 10. Suppose the number of miles a car can run before the battery wears out is exponential distributed with mean 10,000 kms. The owner of the car wants to take 5000 kms trip. What is the probability that she will be able to complete the trip without having to replace the battery?