MTH202: Assignment 7

February 23, 2019

1. Show that the sum of two independent Bernoulli random variables with parameter p is a Binomial random variable with parameters $2, p$.

2. Let $X \sim Bin(n-1, p)$ and $Y \sim Ber(p)$. Show that $X + Y \sim Bin(n, p)$.

3. Let $X \sim Geom(p)$ and $Y \sim Geom(q)$ be two independent Geometric random variables. Find the probability mass function of $Z = X + Y$.

4. A fair die is rolled twice. Let X_1 and X_2 be the outcomes, and let $S = X_1 + X_2$ be the sum of these outcomes. What is the probability mass function of S?

5. In a physics experiment involving estimating the value of g, several independent trials are performed. The value obtained is deemed acceptable with error ≤ 0.2. Suppose that in each trial the probability that your estimated value is in $[9.6, 10]$ is p. You stop when you get five estimates within the permissible error. What is the probability that you achieve this in 8 trials?

6. In a small pond there are 50 fish, 10 of which have been tagged. A fisherman's catch consists of 7 fish (assume his catch is a random selection done without replacement). What is the probability that exactly 2 tagged fish are caught?

7. People arrive at a queue according to the following scheme: During each minute of time either 0 or 1 person arrives. The probability that 1 person arrives is p and that no person arrives is $q = 1 - p$. Answer the following questions:

   (a) Let C_r be the number of customers arriving in the first r minutes. What is $E[C_r]$?

   (b) Let W be the time (in minutes) until the first person arrives. What is $E[W]$?

   (c) Let W_r be the time (in minutes) until the r people arrive. What is $E[W_r]$?

8. Let X and Y be independent real-valued random variables and $C \in \mathbb{R}$ denote a constant. Show that:

(a) $Var(cX) = c^2 Var(X)$

(b) $Var(Y + c) = Var(Y)$

(c) $Var(X + Y) = Var(X) + Var(Y)$

9. An urn contains N balls, m of which are white. A ball drawn from the urn at each discrete time-step $t \geq 1$, independently each time. The colour of the drawn ball is noted and the ball is replaced in the urn. For $1 \leq i \leq n$, consider the following random variables.

$$X_i = \begin{cases} 1 & \text{ball drawn at } t = i \text{ is white} \\ 0 & \text{otherwise} \end{cases}$$

What is the probability mass function, expectation and variance of $Y = \sum_{i=1}^{n} X_i$? Compare the second moment and variance of the random variable Y with the second moment and variance of a Hypergeometric random variable with parameters m, N and n.

10. Give an example of two random variables with different mass functions but same expectation.

11. Given an example of a random variable with infinite expectation.

12. Give an example of a random variable with finite expectation but infinite variance.