MTH202: Assignment 3

January 18, 2019

Conditional Probability: E, F be two events in Ω. If $P(F) > 0$, then

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

In general,

$$P(E_1 \cap E_2 \ldots \cap E_k) = P(E_1)P(E_2|E_1)P(E_3|E_2 \cap E_1) \ldots P(E_k|E_{k-1} \cap \ldots E_2 \cap E_1)$$

Bayes's formula: E, F be two events in Ω. Then,

$$P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)$$

<u>**Exercises**</u>

1. Consider the experiment of rolling two dice and the following events
   - $E_1 = \{$All outcomes such that the first roll is odd$\}$
   - $E_2 = \{$All outcomes such that the first roll is even$\}$
   - $E_3 = \{$All outcomes such that the first roll is a prime$\}$
   - $E_4 = \{$All outcomes such that the first roll is an odd prime$\}$

   Which of the following is true and why?

   (i) $P(E_4) \leq P(E_1)$.

   (ii) $P(E_1 \cap E_2) = 0$.

   (iii) $P(E_1) = P(E_2)$

   (iv) $P(E_4) \leq P(E_3)$

   (v) $P(E_3) \leq P(E_1)$

2. Prove that if $P(A) = P(B) = 1$, then $P(A \cap B) = 1$.

3. A, B be two events in Ω. Show that $P(A|B)P(B) = P(B|A)P(A)$.

4. A be an event in Ω. Show that $P(A|\Omega) = P(A)$.

5. E, F be two events in Ω such that $E \subset F$ and $P(E) \neq 0$. Show that $P(F|E) = 1$.

6. In question 1, what is $P(E_3|E_4)$.

7. E, F, G be events in Ω such that $E \cap F = \phi$ and $P(G) \neq 0$. Show that $P(E \cup F|G) = P(E|G) + P(F|G)$.

8. E, F be events in Ω such that $P(E), P(F) \neq 0$. Show that $P(E|F) \geq P(E)$ if and only if $P(F|E) \geq P(F)$.

9. A student has to sit for an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass, she needs to answer all three questions. What is the probability that the student will pass of she knowns the answers to 90 questions on the list?

10. Consider the set of students in IISER Mohali. We pick a student at random. Someone asserts that the probability that the student is from Hostel 8 is 0.3, and that the probability that it is a Male student is 0.6, and the probability that the student's name starts with Q is 0.05. Finally, the person also says that the probability that it is not a Male student and not from Hostel 8 and that the name starts with A is 0.7. Is it possible that all four estimates of probability are correct? If not, why not?

11. A laboratory blood test is 95 percent efficient in detecting a certain disease when it is, in fact, present. However the test also yields a false positive for 1 percent of healthy persons tested. If 0.5 percent of the population actually has the disease, what is the probability that a person has the disease given that the test result is positive?

12. Suppose that we have 3 cards that are identical in form except that both sides of the first card are coloured red, both sides of second card are coloured black and one side of third card is red while the other side is black. One card is randomly selected and put down on table. If the visible side of the chosen card is red, what is the probability that other side is black?