

Homework 9

1. Find tangent spaces and tangent planes to the following surfaces at the given points:

(i) $S : z = x^2 + y^2$, $p = (1, 0, 1)$

(ii) $S : y = f(x)$, p - arbitrary, f is a smooth function $\mathbb{R} \rightarrow \mathbb{R}$.

(iii) S is obtained by revolving a curve $z = g(y)$ in the yz -plane about the z -axis, p - arbitrary.

2. Check if the following functions are smooth.

(i) $f : S^2 \rightarrow \mathbb{R}$ $(x, y, z) \mapsto xyz$

(ii) $f : S^2 \rightarrow \mathbb{R}$ $(x, y, z) \mapsto \sqrt{x^2 + y^2}$

(iii) $f : S \rightarrow \mathbb{R}^2$ where $S : z = x^2 + y^2$
 $(x, y, z) \mapsto (x, y)$

(iv) $f : S \rightarrow S$ where $S : x^2 + y^2 = 1$
 $(\cos \theta, \sin \theta, t) \mapsto (\cos 2\theta, \sin 2\theta, t)$

3. Show that the cylinder $x^2 + y^2 = 1$ is diffeomorphic to the hyperboloid $x^2 + y^2 - z^2 = 1$.