

## Homework 6

- 1) If the curvature of a smooth & regular curve in the  $xy$ -plane is constant then show that ~~either~~ it is contained in a straight line or in a circle.
- 2) Find the <sup>unit speed</sup> curve  $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$  with signed curvature  ~~$\kappa(t) = \cos t$~~   $\kappa(t) = \cos t$ ,  $\forall t \in \mathbb{R}$ , where  $\alpha(0) = (0,0)$ ,  $\alpha'(0) = (1,0)$ .
- 3) Suppose  $A \in M(3, \mathbb{R})$ . Show that the following are equivalent:
- (i)  $AA^t = I$  ( $A^t =$  transpose of  $A$ .)
  - (ii)  $A \cdot v \cdot A \cdot w = v \cdot w \quad \forall v, w \in \mathbb{R}^3$
  - (iii)  $A \cdot v \cdot A \cdot v = v \cdot v \quad \forall v \in \mathbb{R}^3$
- 4) (i) Suppose  $AA^t = I$ ,  $\det A = 1$ . Show that  $A \cdot v \times A \cdot w = A(v \times w)$ .
- (ii) Use this to complete the proofs of the fundamental theorems for plane curves and space curves.
- 5) Compute  $\vec{T}, \vec{n}, \vec{b}, \kappa, \tau$  for the following curves at the given points:
- (i)  $\alpha(t) = (t, t^2, t^3)$  at  $t = 0$ .
  - (ii)  $\alpha(t) = (t \cos t, t \sin t, t)$  at  $t = 0$ .
6. Decide if the following curves are planar
- (i)  $\alpha(t) = (t, t^2, 1+t+t^2)$
  - (ii)  $\alpha(t) = (t, e^t, e^{2t})$