

Homework - 5

1) Solve the exercises mentioned in class.

2) For a regular, smooth ^{plane} curve α show that

$$(i) \frac{d\vec{n}_s}{ds} = -\kappa_s \vec{T}$$

~~3)~~ (ii) Show that κ_s is a smooth function of s .

3) Suppose $\alpha: [a, b] \rightarrow \mathbb{R}^2$ is a smooth regular curve and $\beta: [a, b] \rightarrow \mathbb{R}^2$ be defined by $\beta(t) = \alpha(a+b-t)$. Show that β is regular. (i) Suppose their signed curvature functions are κ_1 and κ_2 respectively. Show that

$$\kappa_2(t) = -\kappa_1(a+b-t)$$

Interpret this geometrically.

4) Given an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 0$ find a parametrization. Using this find the points where the curvature is maximum and minimum respectively.

5) Draw the following curves and find curvature wherever defined.

$$(i) \rho = 2 + \cos\theta$$

$$(ii) \rho = 1 + 2\cos\theta$$

$$(iii) \rho = \sin 2\theta$$

6) Find curvature of the following curves at any point.

$$(i) y = e^x$$

$$(ii) y = \frac{1}{x}, x > 0$$

$$(iii) y = \log x$$