MTH-201 / Homework 3 (1)

~~Draw~~

* Draw figures wherever possible.

1. Find equations of a line passing through $(1,-1,2)$ and $(-1,1,3)$.

2. (i) If the equation of a line in $\mathbb{R}^2$ is $ax+by+c=0$ show that the line is perpendicular to (a,b), i.e. $a\vec{i}+b\vec{j}$.

   (ii) Similarly show that the plane $ax+by+cz+d=0$ is perpendicular to (a,b,c).

3. Check if the following lines L_1, L_2 are parallel.

   (i) $L_1: \left.\begin{array}{l} x = 2t+3 \\ y = 3t+4 \end{array}\right\}$ $\quad L_2: \left.\begin{array}{l} x = 4-t \\ y = 1-3t/2 \end{array}\right\}$

   (ii) $L_1: \left.\begin{array}{l} x = -t+1 \\ y = 2t-3 \\ z = -3t+4 \end{array}\right\}$ $\quad L_2: \begin{array}{l} x = t-2 \\ y = 1-t \\ z = 2t+3 \end{array}$

4. Two lines L_1, L_2 in $\mathbb{R}^3$ are called skew lines if they do not intersect and they are not parallel to each other.

   Consider the lines

   $L: \left.\begin{array}{l} x = t-1 \\ y = 2t-3 \\ z = 3t-4 \end{array}\right\}$ $\quad L_2: \begin{array}{l} x = t \\ y = -2t \\ z = at \end{array}$

Determine a so that L_1, L_2 are skew lines.

5. Find parametric equations of the straightline passing through $(1,2,3)$ and perpendicular to the plane $2x-3y-4z=5$.

6. Find parametrized curves whose images are the following curves:

$\boxed{\text{Draw these curves!}}$

(i) $\frac{x^2}{4}+\frac{y^2}{9}=1$

(ii) $x^2-4y^2=1$

(iii) $x^2+y^2+z^2=1$
$x=2y$

(iv) $x^2+y^2+z^2=1$
$z=a\sqrt{x^2+y^2}$, a is a constant.

7. Check that the following give surfaces in $\mathbb{R}^3$:

(i) $x^2+y^2+z^2=1$ (iv) $x^2+y^2=1$

(ii) $x^2-y^2+z^2=1$ (v) $z=x^2+y^2$

(iii) $y=x^2$

$\boxed{\text{Draw these surfaces!}}$

8. Draw the following curves:

(i) $\alpha: \mathbb{R}\to\mathbb{R}^3$ $\alpha(t)=(\cos t, \sin t, t)$

(ii) $\alpha: \mathbb{R}\to\mathbb{R}^2$ $\alpha(t)=e^t(\cos t, \sin t)$

Find the equations of the tangent lines to these curves at $\alpha(0)$, $\alpha(1)$.