Homework - 2

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1) Check if the following subsets of $\mathbb{R}^2$ are open or closed: Given reasons for your claims & draw the regions.

(i) $\{(x,y) : x^2+y^2 \leq 2\}$

(ii) $\{(x,y) : 1 \leq x^2+y^2 \leq 2\}$

(iii) $\{(x,y) : x^3+y^3 \neq 1\}$

(iv) $\{(x,e^x) : x \in \mathbb{R}\}$

(v) $\{(x^2, \sin x) : x \in \mathbb{R}\}$

(v) $\{(x^3, x^4) : |x| < 1\}$

2) Check if the following subsets of $\mathbb{R}^3$ are open or closed: Explain your answer and draw the regions.

(i) $\{(x,y,z) : z \geq 1\}$

(ii) $\{(x,y,z) : z = x+y\}$

(iii) $\{(x,y,z) : z > x^2+y^2\}$

(iv) $\{(x,y,z) : x = y,\ z = 2y\}$

(v) $\{(x,y,z) : x^2+y^2+z^2 \leq 1,\ z > \sqrt{x^2+y^2}\}$

3) Show that (i) intersection of any collection of closed sets is closed;

(ii) and union of any collection of open sets is open.

4. Consider the portion $\mathcal{U}$ of the sphere $S: x^2+y^2+z^2=1$ inside the cylinder $x^2+y^2=1/4$. Show that $\mathcal{U}$ is open in S. Draw the picture.

5. Let S be the surface $z=x^2+y^2$.

Consider the portion $\mathcal{U}$ of S strictly below the plane $z=1$. Show that $\mathcal{U}$ is open in S.

6. Let S_1 be the surface $x^2+y^2+z^2=4$ and S_2 be the surface $z=x^2+y^2$. Show that $S_1 \cap S_2$ is closed in S_1.

7. Find $f'(x)$:

(i) $f(x) = \dfrac{x}{x^2+1}$ (ii) $f(x) = \sin x^2$

(iii) $f(x) = e^{\sin x}$ (iv) $f(x) = e^{(x+1)^2}$.

8. (i) Show that $f: X \to Y$ is continuous iff $f^{-1}(\mathcal{U}) \subseteq X$ is open $\forall$ open $\mathcal{U} \subseteq Y$

(ii) Similarly ~~for~~ statement for closed sets.

. Prove that $f(x)$ differentiable at $x=c \Rightarrow$ $f(x)$ continuous at $x=c$.
$(f: \mathbb{R} \to \mathbb{R})$