

Homework - 1

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1. (i) Check all the incomplete proofs discussed in class.

(ii) Solve all the problems mentioned in class.

2. Suppose $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ is continuous and at $(a, b, c) \in \mathbb{R}^3$. Show that

$g(t) = f(t, b, c)$ is continuous at $t = a$.

3. Check if the following functions are continuous:

(a) $f(x, y) = \frac{xy}{x^2 + y^2 + 1}$

(b) $f(x, y) = \begin{cases} \frac{x^2 y}{x^3 + y^3}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0) \end{cases}$

(c) $f(x, y) = \begin{cases} \frac{x^4 + y^4}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

4. Suppose $f: A \rightarrow \mathbb{R}$ and $|f|$ is continuous. Show that f is continuous where $|f|(a) = |f(a)| \forall a \in A$.

5. Let C be the parabola $x^2 = y^2$, (2)

i.e. $C = \{(x, y) : x = y^2\}$

Show that C is homeomorphic to \mathbb{R} .

6. Show that the unit disk
 $D = \{(x, y) : x^2 + y^2 < 1\}$ is homeomorphic to \mathbb{R}^2 .

7. Let A be square ~~set~~ in \mathbb{R}^2 with vertices $(0, 0)$, $(0, 1)$, $(1, 1)$, $(1, 0)$.
Show that A is homeomorphic to the circle $C = \{(x, y) : x^2 + y^2 = 1\}$.

8. Show by an example that a bijective continuous map is not necessarily a homeomorphism.