Quizzes

1. Consider a curve parametrized by some $\gamma : (-1, 1) \to \mathbb{R}^2$. For what values of t in the interval (-1.1) will $\dot{\gamma}(t)$ and $\ddot{\gamma}(t)$ be orthogonal if its arc length function, written in terms of the parametrization γ, is

   a) $s(t) = t$
   b) $s(t) = t^2$

2. Consider a curve parametrized by $\gamma : (-1, 1) \to \mathbb{R}^2$ which has the property that $\ddot{\gamma}(t_0) = c\dot{\gamma}(t_0)$, for some real number c and some $t_0 \in (-1, 1)$. What is the curvature of the curve at the point $\gamma(t_0)$? Justify your answer.

3. If all the tangent lines to a space curve pass through a fixed point c, then prove that the curve lies on a straight line.

4. a) Consider the function $F(x, y, z) = 2x^2 + y^2 - 3z^2 - 1$. Compute the partial derivatives $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, and $\frac{\partial F}{\partial z}$.

   b) For the same F as above, compute ∇F. Why is $S := \{(x, y, z) \mid F(x, y, z) = 0\}$ a surface?

5. Given a surface patch $\sigma(x, y) = (x, y, \sqrt{1 - x^2 - y^2})$, compute the standard unit normal at the point (0, 0, 1).

6. If for every point on a surface patch $\sigma : U \to S$ of a surface, S, the two principle curvatures are equal, then prove that the principle curvature is constant.