MTH 201, Curves and surfaces

Practice problem set 6

1. Which of the following subsets of $\mathbb{R}^3$ are open in $\mathbb{R}^3$?

a) $\{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$
b) $\{(x, y, z) \mid x^2 + y^2 + z^2 < 1\}$
c) $\{(x, y, 0) \mid x^2 + y^2 < 1\}$
d) $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$
e) $\{(x, y, 0) \mid x^2 + y^2 \leq 1\}$
f) $\{(x, y, z) \mid 1 < x^2 + y^2 + z^2 < 2\}$
g) $\{(x, y, z) \mid 1 \leq x^2 + y^2 + z^2 < 2\}$
h) $\{(0, y, z) \mid 1 < y^2 + z^2 < 2\}$
i) $\{(x, y, z) \mid |x| < 1\}$
j) $\{(x, y, 0) \mid |x| < 1, |y| < 1\}$
k) A finite set

2. A function, $f : X \to Y$, from any subset X of $\mathbb{R}^m$ to any subset Y of $\mathbb{R}^n$ is said to be continuous at p if given any real number $\epsilon > 0$ (however small, but strictly positive), there is a real number $\delta > 0$, so that for any point x, where $\|x - p\| < \delta$, $\|f(x) - f(p)\| < \epsilon$. On which points of their domain are the following functions continuous?

a) $f : \mathbb{R}^2 \to \mathbb{R}^3$, $f(x, y) = (x^3, x + y, x)$
b) $f : D \to \mathbb{R}^3$, $f(x, y) = (x^3, x+y, 1/(x-6))$ where $D := \{(x, y) \mid x^2+y^2 < 1\}$
c) $f : D \to \mathbb{R}^3$, $f(x, y) = (x^3, x + y, 1/x))$ where $D := \{(x, y) \mid x^2 + y^2 = 1\}$
d) $D := \{(x, y) \mid x^2+y^2 < 1\}$ and $f : D \to \mathbb{R}^3$, where $f(x, y) = (x^3, x+y, 1/x))$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = (0, 0, 0)$.
e) Any function where the domain is finite