## Assignment #8

**Ans [1]** (i) $S: x^2+y^2-z^2=1$

consider the surface patch of $S$

$\sigma_1 : U_1 \longrightarrow \mathbb{R}^3$ where $U_1 = \{ (u,v) \in \mathbb{R}^2 \mid u^2+v^2 > 1 \}$

$$\sigma_1^{+z}(u,v) = \left(u, v, \sqrt{u^2+v^2-1}\right)$$

$\sigma_2 : U_1 \longrightarrow \mathbb{R}^3$

$$\sigma_2^{-z}(u,v) = \left(u, v, -\sqrt{u^2+v^2-1}\right)$$

$\sigma_3 : U_2 \longrightarrow \mathbb{R}^3$ where $U_2 = \{ (u,v) \in \mathbb{R}^2 \mid u^2-v^2 < 1 \}$

$$\sigma_3(u,v) = \left(u, \sqrt{1+v^2-u^2}, v\right)$$

Now $\sigma_{1u} = \left(1, 0, \dfrac{u}{\sqrt{u^2+v^2-1}}\right)$, $\sigma_{1v} = \left(0, 1, \dfrac{v}{\sqrt{u^2+v^2-1}}\right)$

$$\sigma_{1u} \times \sigma_{1v} = \left( \frac{-u}{\sqrt{u^2+v^2-1}}, \frac{v}{\sqrt{u^2+v^2-1}}, 1 \right) \neq (0,0,0) \text{ for any } (u,v) \in \mathbb{R}^2.$$

similarily $\sigma_{2u} \times \sigma_{2v} \neq 0$ & $\sigma_{3u} \times \sigma_{3u} \neq 0$.

Thus $S$ is a regular surface.

$S: \{ (x,y,z) \in \mathbb{R}^3 \mid x^2+y^2-z^2-1=0 \}$

here $f(x,y,z) = x^2+y^2-z^2-1$ for level surface representation.

clearly $\nabla f = (f_x, f_y, f_z) = (2x, 2y, 2z) \neq 0$ for any $(x,y,z) \in S$.

so $S$ is a smooth surface also.

hence every regular surface patch of $S$ will be allowable.

(ii) $S: x^2+4y^2=4$

consider the surface patch of $S$

$\sigma_1 : U_1 \to \mathbb{R}^3$ $\quad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid 0<u<2\pi, v \in \mathbb{R} \}$

$\sigma_1(u,v) = (2\cos u, \sin u, v)$

$\sigma_2 : U_2 \to \mathbb{R}^3$ $\quad U_2 = \{ (u,v) \in \mathbb{R}^2 \mid -\frac{\pi}{2}<u<\frac{\pi}{2}, v \in \mathbb{R} \}$

$\sigma_2(u,v) = (2\cos u, \sin u, v)$

Now $\quad \sigma_{1u} = (-2\sin u, \cos u, 0)$, $\sigma_{1v} = (0,0,1)$

$\sigma_{1u} \times \sigma_{1v} = (\cos u, 2\sin u, 0) \neq (0,0,0)$ for any $u \in U_1$.

Similarily for $\sigma_{2u} \times \sigma_{2v} \neq (0,0,0)$ for any $u \in U_2$.

hence $S$ is a regular surface.

Now $\quad S: \{ (x,y,z) \in \mathbb{R}^3 \mid f(x,y,z) = x^2+4y^2-4=0 \}$.

$\nabla f = (f_x, f_y, f_z) = (2x, 8y, 0) \neq (0,0,0)$ for any $(x,y,z) \in S$.

Thus $S$ is a smooth surface. hence allowable surface patches are there. ($\sigma_1, \sigma_2$ are also allowable).

(iii) See it in the book. cylinder with axis as x-axis.

$S: y^2+z^2=1$

$\sigma_1(u,v) : U_1 \to \mathbb{R}^3$ $\quad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid 0<u<2\pi, v \in \mathbb{R} \}$

$\sigma_1(u,v) = (v, \cos u, \sin u)$

$\sigma_2(u,v) : U_2 \to \mathbb{R}^3$ $\quad U_2 = \{ (u,v) \in \mathbb{R}^2 \mid -\frac{\pi}{2}<u<\frac{\pi}{2}, v \in \mathbb{R} \}$.

$\sigma_2(u,v) = (v, \cos u, \sin u)$

check! Regularity.

$\nabla f = (0, 2y, 2z) \neq (0,0,0)$ for any point $(x,y,z) \in S$.

$S$ is smooth.

(iv) $S: z = x^2+2y^2$

consider the surface patch of $S$

$\sigma_1 : U_1 \to \mathbb{R}^3 \qquad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R}, v \in \mathbb{R} \} = \mathbb{R}^2$

$\sigma(u,v) = (u, v, u^2+2v^2)$

Now $\sigma_u = (1, 0, 2u) \quad , \sigma_v = (0, 1, 4v)$

$\sigma_u \times \sigma_v = (-2u, -4v, +1) \neq (0,0,0)$ for any $(u,v) \in \mathbb{R}^2$.

So $S$ is a regular surface.

$S: \{ (x,y,z) \in \mathbb{R}^3 \mid f(x,y,z) = x^2+2y^2-z = 0 \}$.

$\nabla f = (2x, 4y, -1) \neq (0,0,0)$ for any $(x,y,z) \in S$.

Thus $S$ is smooth. hence $\sigma$ is allowable.

(v) $S: \quad y = \sin x$ consider the surface patch of $S$

$\sigma_1 : U_1 \to \mathbb{R}^3 \qquad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid 0 < u < 2\pi , v \in \mathbb{R} \}$

$\sigma_1(u,v) = (u, \sin u, v)$

$\sigma_2 : U_2 \to \mathbb{R}^3 \qquad U_2 = \{ (u,v) \in \mathbb{R}^2 \mid -\frac{\pi}{2} < u < \frac{\pi}{2}, v \in \mathbb{R} \}$

$\sigma_2(u,v) = (u, \sin u, v)$

Now

$\sigma_{1u} = (1, +\cos u, 0) \quad , \quad \sigma_{1v} = (0, 0, 1)$

$\sigma_{1u} \times \sigma_{1v} = (\cos u, -1, 0) \neq (0,0,0)$ for any $(u,v) \in U_1$.

Similarily $\sigma_{2u} \times \sigma_{2v} \neq 0$

Thus $S$ is a regular surface.

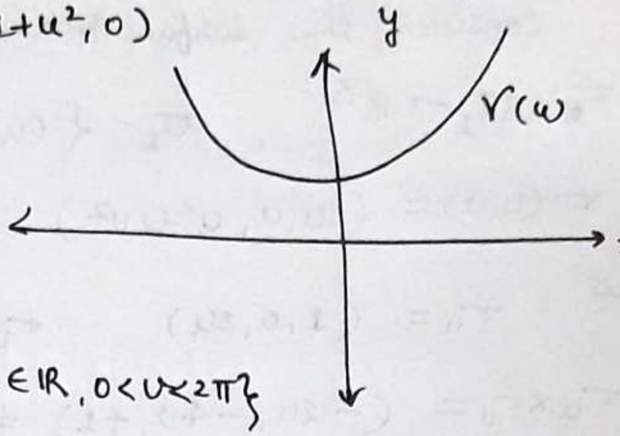
$S: \{ (x,y,z) \in \mathbb{R}^3 \mid f(x,y,z) = y - \sin x = 0 \}$

$\nabla f = (-\cos x, 1, 0) \neq (0,0,0)$ for any $(x,y,z) \in S$.

So $S$ is smooth. Thus $\sigma$ is allowable.

Ans [2] (i) $y = 1+x^2$ Now the Parametrization of given curve

$$\gamma(u) = (u, 1+u^2, 0)$$




Consider the angle $v$ from y-axis when rotating about x-axis.

Consider the surface patch

$\sigma_1 : U_1 \to \mathbb{R}^3$ $\quad U_1 = \{(u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R}, 0 < v < 2\pi\}$

$$\sigma_1(u,v) = (u, (1+u^2)\cos v, (1+u^2)\sin v)$$

$\sigma_2 : U_2 \to \mathbb{R}^3$ $\quad U_2 = \{(u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R}, -\frac{\pi}{2} < v < \pi/2\}$

$$\sigma_2(u,v) = (u, (1+u^2)\cos v, (1+u^2)\sin v)$$

Now

$$\sigma_{1u} = (1, 2u\cos v, 2u\sin v)$$

$$\sigma_{1v} = (0, -(1+u^2)\sin v, (1+u^2)\cos v)$$

$$\sigma_{1u} \times \sigma_{1v} = (2u(1+u^2), -(1+u^2)\cos v, -(1+u^2)\sin v) \neq (0,0,0)$$

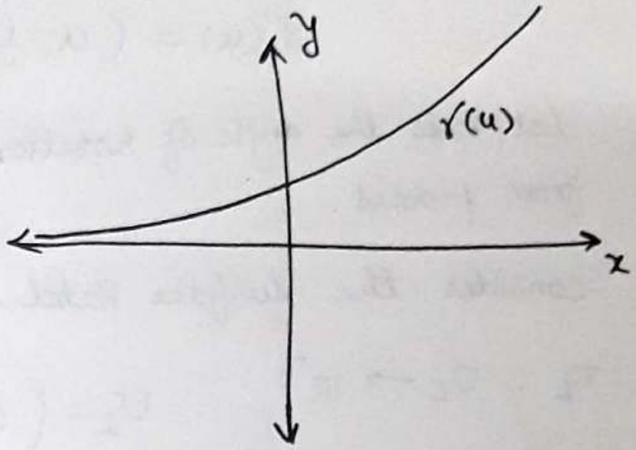
for any $(u,v) \in U_1$

similarily $\sigma_{2u} \times \sigma_{2v} \neq 0$.

Thus $S$ is a regular surface which obtained by rotating $\gamma(u)$ about the x-axis.

(ii) $y = e^x$ Now the parametrization of given curve.

$$\gamma(u) = (u, e^u, 0)$$




Consider the angle $v$, when rotating $\gamma(u)$ about x-axis. $v$ is angle from y-axis.

Consider the surface patch

$\sigma_1 : U_1 \to \mathbb{R}^3$ $\quad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R},\ 0 < v < 2\pi \}$

$\sigma_1(u,v) = (u,\ e^u \cos v,\ e^u \sin v)$

$\sigma_2 : U_2 \to \mathbb{R}^3$ $\quad U_2 = \{ (u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R},\ -\frac{\pi}{2} < v < \pi/2 \}$

$\sigma_2(u,v) = (u, e^u \cos v, e^u \sin v)$

Now

$$\sigma_{1u} = (1,\ e^u \cos v,\ e^u \sin v)$$

$$\sigma_{1v} = (0, -e^u \sin v,\ e^u \cos v)$$

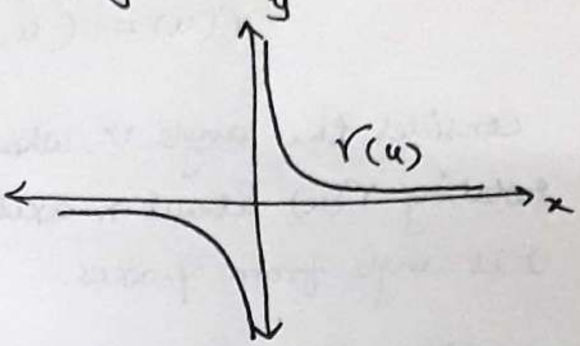
$\sigma_{1u} \times \sigma_{1v} = (e^{2u}, -e^u \cos v, -e^u \sin v) \neq (0,0,0)$ for any $(u,v) \in U_1$.

Similarily $\sigma_{2u} \times \sigma_{2v} \neq 0$.

Thus $S$ is a regular surface which obtained by rotating $\gamma(u)$ about the x-axis.

(iii) $y = \frac{1}{x}$ Now the Parametrization of given curve

$$r(u) = (u, \tfrac{1}{u}, 0)$$




Let $v$ is the angle of rotation of $r(u)$ from y-axis.

Consider the surface Patch

$\sigma_1 : U_1 \to \mathbb{R}^3$ $\quad U_1 = \{ (u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R} - \{0\},\ 0 < v < 2\pi \}$

$$\sigma_1(u,v) = ( u, \tfrac{1}{u}\cos v, \tfrac{1}{u}\sin v )$$

$\sigma_2 : U_2 \to \mathbb{R}^3$ $\quad U_2 = \{ (u,v) \in \mathbb{R}^2 \mid u \in \mathbb{R} - \{0\},\ -\frac{\pi}{2} < v < \frac{\pi}{2} \}$

$$\sigma_2(u,v) = ( u, \tfrac{1}{u}\cos v, \tfrac{1}{u}\sin u )$$

Now

$$\sigma_{1u} = ( 1, -\tfrac{1}{u^2}\cos v, -\tfrac{1}{u^2}\sin v )$$

$$\sigma_{1v} = \left(0, -\frac{\sin v}{u}, \frac{\cos v}{u}\right)$$

$$\sigma_{1u} \times \sigma_{1v} = \left( -\frac{1}{u^3}, -\frac{\cos v}{u}, -\frac{\sin v}{u} \right) \neq (0,0,0) \text{ for any } (u,v) \in U_1.$$

Similarily $\sigma_{2u} \times \sigma_{2v} \neq 0$.

Thus $S$ is a regular surface. which obtained by rotating $r(u)$ about x-axis.