# Homework-4

($*$ means very important)

1. (i) Show that the curve $\alpha : [0,1] \to \mathbb{R}^2$,
$\alpha(t) = (t, \sqrt{1-t^2})$ is NOT smooth.

(ii) Do the same for the curve
$\beta : [0,1] \to \mathbb{R}^2$ given by $\beta(t) = (t, t^{1/3})$.

2. (i) Check that the book's definition of a surface is equivalent to that of ours.

(ii) Any open subset of a surface is a surface. In particular, any open subset of the XY-plane is a surface.

3. If $I, J \subseteq \mathbb{R}$ are intervals and $\varphi : I \to J$ is a diffeomorphism then $\varphi'(t) \neq 0$ $\forall t \in I$.

4.$^*$ Consider the following curves.
(i) Check if they are regular. (ii) Find their unit tangent vector and speed at any time $t$.

(a) $\alpha : (0,\infty) \to \mathbb{R}^2$ $\quad \alpha(t) = (t\cos t, t\sin t)$

(b) $\alpha : \mathbb{R} \to \mathbb{R}^2$ $\quad \alpha(t) = (t - \sin t, 1 - \cos t)$

(c) $\alpha : \mathbb{R} \to \mathbb{R}^2$ $\quad \alpha(t) = (e^{kt}\cos t, e^{kt}\sin t$

(d) $\alpha : \mathbb{R} \to \mathbb{R}^3$ $\quad \alpha(t) = (a\cos t, a\sin t, bt$
$a > 0,\ b > 0$

(e) $\alpha : (0,\infty) \to \mathbb{R}^3$ $\alpha(t) = (t^2, t^2+1, t^2+2)$ 2

Note: $a, b, K$ are constants.

Example: (d) $\alpha'(t) = (-a\sin t, a\cos t, b)$

$\Rightarrow$ Speed $= \|\alpha'(t)\| = \sqrt{a^2+b^2} \neq 0$

Hence $\alpha$ is regular also.

Then unit tangent vector at time $t =$

$$\frac{1}{\|\alpha'(t)\|} \cdot \alpha'(t) = \frac{1}{\sqrt{a^2+b^2}}(-a\sin t, a\cos t, b).$$

5.* In 4(c), 4(d), 4(e) Find arc length parametrizations for base points $t=0$, $t=0$ and $t=1$ respectively.

6. Suppose $\alpha : I \to \mathbb{R}^3$ is a curve with $\alpha''(t) = 0$. Show that $\alpha(I)$ is contained in a straight line.