

Homework - 2

①

1) Check if the following subsets of \mathbb{R}^2 are open or closed: Given reasons for your claims & draw the regions.

(i) $\{(x, y) : x^2 + y^2 \leq 2\}$

(ii) $\{(x, y) : 1 \leq x^2 + y^2 \leq 2\}$

(iii) $\{(x, y) : x^3 + y^3 \neq 1\}$

(iv) $\{(x, e^x) : x \in \mathbb{R}\}$

(v) $\{(x^2, \sin x) : x \in \mathbb{R}\}$

(vi) $\{(x^3, x^4) : |x| < 1\}$

2) Check if the following subsets of \mathbb{R}^3 are open or closed: Explain your answer and draw the regions.

(i) $\{(x, y, z) : z \geq 1\}$

(ii) $\{(x, y, z) : z = x + y\}$

(iii) $\{(x, y, z) : z > x^2 + y^2\}$

(iv) $\{(x, y, z) : x = y, z = 2y\}$

(v) $\{(x, y, z) :$

$$x^2 + y^2 + z^2 \leq 1,$$

$$z > \sqrt{x^2 + y^2}\}$$

3) Show that (i) intersection of any collection of closed sets is closed;
(ii) and union of any collection of open

sets is open.

(2)

4. Consider the portion U of the sphere $S: x^2 + y^2 + z^2 = 1$ inside the cylinder $x^2 + y^2 = 1/4$. Show that U is open in S . Draw the picture.

5. Let S_0 be the surface $z = x^2 + y^2$.

Consider the portion U of S strictly below the plane $z = 1$. Show that U is open in S .

6. Let S_1 be the surface $x^2 + y^2 + z^2 = 4$ and S_2 be the surface $z = x^2 + y^2$. Show that $S_1 \cap S_2$ is closed in S_1 .

7. Find $f'(x)$:

(i) $f(x) = \frac{x}{x^2+1}$ (ii) $f(x) = \sin x^2$

(iii) $f(x) = e^{\sin x}$ (iv) $f(x) = e^{(x+1)^2}$

8. (i) Show that $f: X \rightarrow Y$ is continuous iff $f^{-1}(U) \subseteq X$ is open \forall open $U \subseteq Y$

(ii) Similarly ~~for~~ statement for closed sets.

9. Prove that $f(x)$ differentiable at $x=c \Rightarrow$
($f: \mathbb{R} \rightarrow \mathbb{R}$) $f(x)$ continuous at $x=c$.