

Solutions to Homework 2

①

Recall: $A \subseteq X$ is ~~not to be~~ closed iff \forall sequence $\{p_n\}$ in A with $p_n \rightarrow p \in X \Rightarrow p \in A$.

(i), (ii): Note if $\{a_n\}$ is a sequence of real numbers then $a_n \leq a \forall n \Rightarrow \lim a_n \leq a$ provided the limit exists. Similarly, $a_n \geq a \forall n \Rightarrow \lim a_n \geq a$.

Ans. (i), (ii) The sets are closed.

(iii) Ans. Open. It is equivalent to showing that the ~~circle~~ $B = \{(x, y) : x^2 + y^2 = 1\}$ is closed. That is similar to (i), (ii).

Note: We also need to use the continuity of the functions involved $x^2 + y^2$, $x^3 + y^3$.

For (iii): If $p_n = (x_n, y_n)$ is a sequence in the set B and $p_n \rightarrow p = (a, b)$ then $x_n^3 + y_n^3 = 1 \forall n$. Since $x^3 + y^3$ is continuous $1 = x_n^3 + y_n^3 \rightarrow a^3 + b^3 \Rightarrow a^3 + b^3 = 1 \Rightarrow p \in B$.

(iv) We need to use e^x is continuous. Ans. Closed

(v) ~~set~~ We need to use that the component functions are continuous. Ans. closed.

Hint. Use sequential criteria for closed set. See the definition above.

(vi) Ans. This set is neither open nor closed.

Hint. Draw the curve. Show that no ball around $(0, 0)$ is in the set, whence it is not open. Also $(1, 1)$ is the limit of sequence in this set but it is not in the set.

Solutions to Homework 2

(2)

2) Ans.

(i), (ii), (iv): closed. Use the sequential criteria for closed set.

(iii) Ans. Open.(v) Ans. Neither open nor closed.

3. Use the definitions of open set and closed set.

4. The region inside the cylinder $x^2 + y^2 = 1/4$ is $U = \{ (x, y, z) : x^2 + y^2 < 1/4 \}$. This open.Hence, ~~answer is~~ $S \cap U = U$ is open in S .Check why U is open.

5. similar to (4).

6. S_2 is closed in \mathbb{R}^3 . Hence $S_2 \cap S_1$ is closed in S_1 .7. Ans. (i) $\frac{1-x^2}{(x+1)^2}$ Recall: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$ (ii) $2x^2 \cos x^2$ (iii) $\cos x e^{\sin x}$ (iv) $2(x+1)e^{(x+1)^2}$

8. This is an optional exercise. Give reference Apostol - Analysis

9. This is standard.

Reference: Apostol - Analysis.