Homework - 1

1. (i) Check all the incomplete proofs discussed in class.

   (ii) Solve all the problems mentioned in class.

2. Suppose $f: \mathbb{R}^3 \to \mathbb{R}$ is continuous at $(a,b,c) \in \mathbb{R}^3$. Show that

   $g(t) = f(t,b,c)$ is continuous at $t = a$.

3. Check if the following functions are continuous:

   (a) $f(x,y) = \dfrac{xy}{x^2+y^2+1}$

   (b) $f(x,y) = \begin{cases} \dfrac{x^2 y}{x^3+y^3}, & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0) \end{cases}$

   (c) $f(x,y) = \begin{cases} \dfrac{x^4+y^4}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

4. Suppose $f: A \to \mathbb{R}$ is continuous. Show that $|f|$ is continuous where $|f|(a) = |f(a)| \ \forall a \in A$.

5. Let C be the parabola $x = y^2$, i.e. $C = \{(x,y) : x = y^2\}$

Show that C is homeomorphic to $\mathbb{R}$.

6. Show that the unit disk $\mathbb{D} = \{(x,y) : x^2+y^2 < 1\}$ is homeomorphic to $\mathbb{R}^2$.

7. Let A be square in $\mathbb{R}^2$ with vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$. Show that A is homeomorphic to the circle $C = \{(x,y) : x^2+y^2 = 1\}$.

8. Show by an example that a bijective continuous map is not necessarily a homeomorphism.