HW1 / Hints to Answers

2. $\varphi : t \mapsto (t, b, c)$ is continuous

$$g = f \circ \varphi .$$

3. (a) xy, x^2+y^2+1 are continuous &

x^2+y^2+1 is never zero.

Note: If f, g are continuous then f/g is continuous wherever $g \neq 0$

(b) f is not continuous

If $P_n = (x_n, m x_n) \to (0,0)$ then

$$f(P_n) = \frac{m}{1+m^3}.$$ For different sequences

$P_n \to (0,0)$ we have different limits

(c) Let $P_n = (x_n, y_n)$ any sequence, $P_n \to (0,0)$.

Write $x_n = r_n \cos\theta_n$, $y_n = r_n \sin\theta_n$.

θ_n need not be unique, we don't care.

$$f(P_n) = r_n^2(\cos^4\theta_n + \sin^4\theta_n).$$

Since $P_n \to (0,0)$, $r_n \to 0$. Note

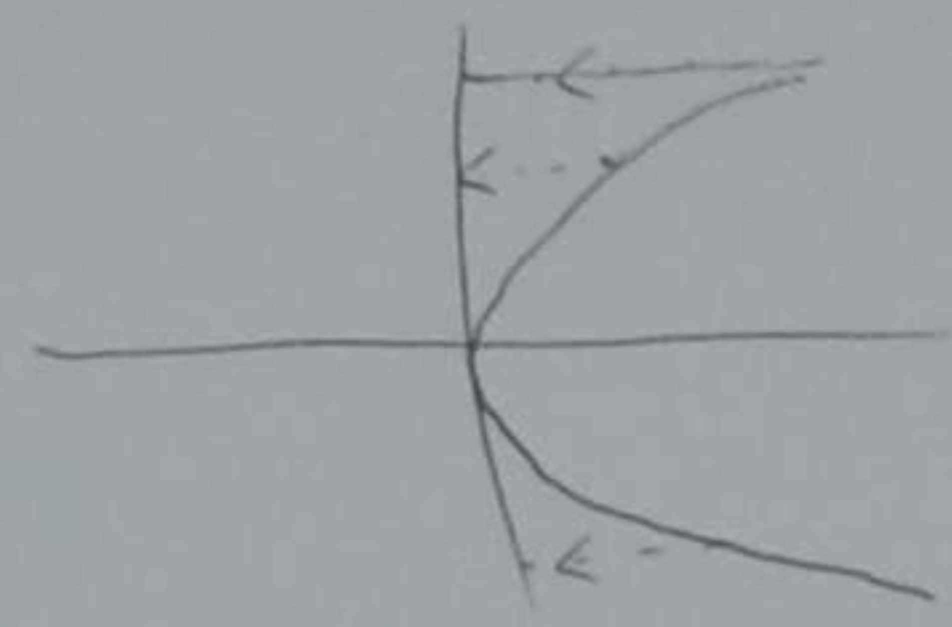
$|\cos^4\theta_n + \sin^4\theta_n| \leq 2$. Thus $f(P_n) \to 0$.

4. It is the composition of f and

$$\begin{aligned} \mathbb{R} &\to \mathbb{R}. \\ x &\mapsto |x| \end{aligned}$$

5. Define $f : C \to \mathbb{R}$

$$(x,y) \mapsto y$$



One has check
- f continuous
- f bijective
- f^{-1} continuous.

f is just projection on the y- ~~coordinate~~ axis.

$$f^{-1} : \quad y \longmapsto (y^2, y).$$

6. Let $f : \cancel{\mathbb{R}}\ [0,\infty) \to [0,1)$ be any homeomorphism

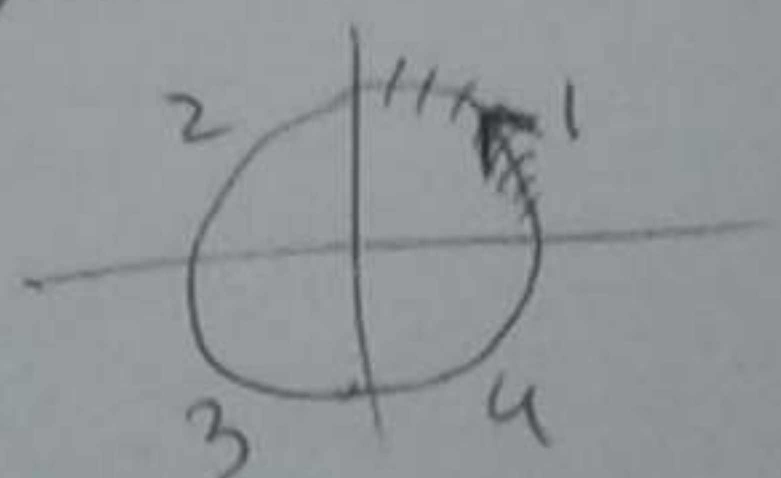
Then define $\varphi : \mathbb{R}^2 \to \mathbb{D}$

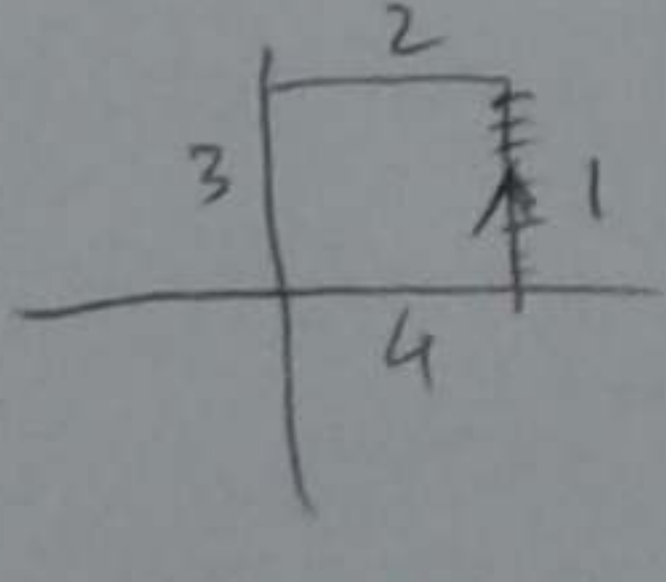
$$(x,y) \longmapsto f(r)\begin{cases} (x,y)/r & \text{if } (x,y) \neq (0,0) \\ 0 \end{cases}$$

where $r = \sqrt{x^2+y^2}$.

For f one could take $f(x) = \dfrac{e^x - 1}{e^x + 1}$

- Break the circle in 4 equal parts





Map each part onto homeomorphically onto the sides of the square.

- Consider $f : [0,1) \longrightarrow S^1 = \{(x,y) : x^2+y^2 = 1\}$

$$t \longmapsto (\cos 2\pi t, \sin 2\pi t)$$