

**IISER Mohali**  
**MTH102: Analysis in One Variable**  
**Homework Sheet 10**  
**To be discussed during tutorial on April 08, 2016**

- Please solve all the problems.
- **Tutorial problems** will be discussed during tutorials.
- If time permits, tutors may also discuss **extra problems** during tutorials.

**Tutorial Problems:**

- (1) Let  $f : (-1, 1) \rightarrow \mathbb{R}$  be given by  $f(x) = \frac{1}{1-x}$ . Determine the Taylor series expansion of  $f$  about 0.
- (2) Let  $f : [0, a] \rightarrow \mathbb{R}$  be given by  $f(x) = x^3$ . Calculate the upper and the lower Riemann integrals of  $f$ . Is  $f$  Riemann integrable on the interval  $[0, a]$ ?
- (3) Consider the function  $f : [0, a] \rightarrow \mathbb{R}$  given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Calculate the upper and the lower Riemann integrals of  $f$ .
- (b) Is  $f$  Riemann integrable on the interval  $[0, a]$ ?
- (4) Let  $f, g : [a, b] \rightarrow \mathbb{R}$  be two functions such that  $f$  is Riemann integrable on  $[a, b]$  and  $f(x) = g(x)$  for all but finitely many  $x \in [a, b]$ . Show that  $g$  is also Riemann integrable and  $\int_a^b f(x)dx = \int_a^b g(x)dx$ .  
Hint: Use induction on the number of points where the two functions differ.
- (5) Give an example of a function  $f : [0, 1] \rightarrow \mathbb{R}$  which is not Riemann integrable, but for which the function  $|f|$  is Riemann integrable.  
Hint: Start with a non-Riemann integrable function you know.

**Extra Problems:**

- (1) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be the function given by  $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ . Determine the Taylor series expansion of  $f$  about 0.
- (2) Consider the function  $f : [0, a] \rightarrow \mathbb{R}$  given by

$$f(x) = \begin{cases} \frac{x}{5} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Calculate the upper and the lower Riemann integrals of  $f$ .
- (b) Is  $f$  integrable on the interval  $[0, a]$ ?