IISER Mohali
MTH102: Analysis in One Variable
Homework No. 04
To be discussed during tutorial on February 05, 2016

- Please solve all the problems.
- **Tutorial Problems** will be discussed during tutorial sessions.
- If time permits, tutors may also discuss **extra problems** during tutorial sessions.

Tutorial Problems:

(1) Prove that if (s_n) is an unbounded non-decreasing sequence of real numbers, then $\lim s_n = +\infty$.
(2) Prove that if (s_n) is an unbounded non-increasing sequence of real numbers, then $\lim s_n = -\infty$.
(3) Let $s_1 = 1$ and $s_{n+1} = (\frac{n}{n+1})s_n^2$ for all $n \geq 1$.
   (a) Find the terms s_2, s_3, s_4 and s_5.
   (b) Prove that $\lim s_n$ exists.
   (c) Prove that $\lim s_n = 0$.
(4) Prove that every Cauchy sequence is bounded.
(5) Let $s_n = \sin(\frac{n\pi}{3})$ for each $n \geq 1$.
   (a) Compute $\limsup s_n$ and $\liminf s_n$. Deduce that $\lim s_n$ does not exist.
   (b) Determine a monotone subsequence of (s_n).
   (c) Determine a convergent subsequence of (s_n).
(6) Let $s_n = n\big(1 + (-1)^n\big)$ for each $n \geq 1$. Compute $\limsup s_n$ and $\liminf s_n$. What can you say about $\lim s_n$.
(7) Give an example of a Cauchy sequence of rational numbers which does not converge to a rational number.