MTH 101 - Symmetry
Assignment 1

1. Let $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$ and $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.
   (a) List the row vectors and column vectors of A.
   (b) Let A_i denote the i^{th} row vector. Then compute the product A_2X.
   (c) In the notation $A = (a_{ij})$, what are the entries a_{23}, a_{31}.
   (d) Compute AX and find all the solutions for $AX = 0$.

2. Let $A = (a_{ij})$ be an $m \times n$ matrix. List the row vectors and column vectors of A.

3. Let $M_n(\mathbb{R})$ be the set of all $n \times n$ matrices with entries from $\mathbb{R}$. Let $E^{\mathbf{r}}_{c\mathbf{r}} : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a function that scales the $\mathbf{r}^{th}$ row vector of the matrix by c and maps the other row vectors to themselves. Then given a matrix $A \in M_n(\mathbb{R})$, determine the matrix $E^{\mathbf{r}}_{c\mathbf{r}}(A)$.

4. Let $E^{\mathbf{k}}_{\mathbf{k}+c\mathbf{r}} : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be the function that replaces the $\mathbf{k}^{th}$ row vector of the matrix with the $\mathbf{k}^{th}$ row vector of the matrix plus c times the $\mathbf{r}^{th}$ row vector of the matrix and maps the other row vectors to themselves. Then given a matrix $A \in M_n(\mathbb{R})$, determine the matrix $E^{\mathbf{k}}_{\mathbf{k}+c\mathbf{r}}$.

5. Let $E^{\mathbf{r}}_{\mathbf{s}} : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be the function that interchanges the $\mathbf{r}^{th}$ row vector of the matrix with the $\mathbf{s}^{th}$ row vector of the matrix and maps the other row vectors to themselves. Then given a matrix $A \in M_n(\mathbb{R})$, determine the matrix $E^{\mathbf{r}}_{\mathbf{s}}(A)$.

6. Determine the matrices
$$E^{\mathbf{r}}_{c\mathbf{r}}(I_n), \quad E^{\mathbf{k}}_{\mathbf{k}+c\mathbf{r}}(I_n), \quad E^{\mathbf{r}}_{\mathbf{s}}(I_n),$$
   where I_n denotes the $n \times n$ identity matrix. (The matrices $E^{\mathbf{r}}_{c\mathbf{r}}(I_n)$, $E^{\mathbf{k}}_{\mathbf{k}+c\mathbf{r}}(I_n)$ and $E^{\mathbf{r}}_{\mathbf{s}}(I_n)$ are called **elementary matrices**.)

7. Let E be an elementary matrix and $e : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a function such that $e(I_n) = E$. Given a matrix $A \in M_n(\mathbb{R})$, check that $e(A) = EA$.

8. Let $D = (d_{ij})$ be an $n \times n$ diagonal matrix. Let A be a $n \times m$ matrix. Compute DA. Show that D can be written as the product of elementary matrices.

9. Describe explicitly all 2×2 row-reduced matrices.

Note: To determine an $m \times n$ matrix $A = (a_{ij})$, one has to explicitly determine the entries a_{ij}.