

More on Languages

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Languages Continued

Σ^* = set of all strings over the alphabet Σ

A language L is a subset of Σ^*

ϵ is an empty string.

If $\omega \in \Sigma^*$, $|\omega|$ = length of the string.

Examples of languages

1. \emptyset
2. Σ^* > English is a language where the strings are all words in a dictionary and punctuation marks and the collection is grammatically correct > > C++ any string of characters the compiler accepts
3. $\Sigma = \{a,b\}$, $L = \{x \in \Sigma^* \text{ st } x \text{ begins with } a \ \& \ \text{ends with } b\}$
4. $\Sigma = \{0,1,\dots,9\}$, $L = \{x \mid x \text{ is the decimal representation of a prime}\}$

Equality of Languages

Languages are equal if the set of strings are equal.

Encoding (informal)

A bijective mapping from one set to another. - Pictures are colours encoded as numbers - Similarly movies

Ordering of a language

Σ is finite, hence can be ordered.

Σ^* can then be ordered by length first and then the lexicographic order on Σ

Existence of a string in a Language

Graph of a function

$$G(f) = \{x,y \mid y=f(x)\}$$

A string exists in the language if it exists in $G(f)$ where f is the defining function of the Language.

There are uncountable languages but a countable set of strings. So we cannot hope to describe all the languages using strings.

Deterministic Finite State Automaton

Solve the existence of string problem - Read the string one char at a time - Remember something, and react to the next char accordingly. - Do this again and again

Consider -

$$\Sigma = \{0,1\}$$

$$L = \{x \mid x \text{ consists of an even number of 1s}\}$$

Given a string, what do you do?

Go char by char, switching between an “even state” and an “odd state”

DEFN: Deterministic Finite State Automaton is a 5 tuple $(Q, \Sigma, \delta, q_0, F)$ where

- Q = A finite set called the set of states
- Σ = Alphabet
- δ = is a function $Q \times \Sigma \rightarrow Q$
- $q_0 \in Q$ and is called the initial state
- $F \subseteq Q$ and is the set of accepted states.