# NFA

Dhruva Sambrani

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## Formal Defn of an NFA

M = ( Q, $\Sigma$, $\delta$, q0, F )

$\delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow \mathcal{P}(Q)$

An NFA N accepts a string s = w1w2w3...wn iff $\exists$ a sequence of r0r1...rn $\in$ Q such that rn $\in$ F and ri+1 $\in$ $\delta$(ri, wi)

## Equivalence of NFA and DFA

Theorem: L is regular iff it can be recognized by an NFA.

Proof:

If L is recognisable by a NFA (let) N = (Q, $\Sigma$, $\delta$, q0, F)

Consider the DFA, D = (Q', $\Sigma$, $\delta$', q0', F') where -

- Q' = $\mathcal{P}$(Q)
- $\delta$' : Q' $\rightarrow$ Q' and
  - $\delta$'(A, c) $\rightarrow \bigcup$ E($\delta$(a,c)), where
    - a$\in$A
    - E(A) = the set of states connected to some q$\in$A by $\epsilon$. Trivially, some qs map to themselves via a $\epsilon$.
- q0' = E(q0)
- F' = {A'$\in$Q' | A$\cap$F $\neq \emptyset$}

If S is accepted by N iff $\exists$ r0...rn $\in$ Q r0=q0 and ri+1 $\in$ $\delta$(ri, wi)

$\implies$ ri+1 $\in$ Ri = E($\delta$(ri, wi)) and rn $\in$ Rn $\in$ F.