

Solutions to be given in class by tutors

1. What is the probability that the energy E_3 is observed in an experiment given that the system is prepared as the following state:

$$\Psi(x, t) = 0.2\psi_1 e^{-iE_1 t/\hbar} + 2i\psi_2 e^{-iE_2 t/\hbar} + 1i\psi_3 e^{-iE_3 t/\hbar} - 3\psi_4 e^{-iE_4 t/\hbar}$$

Here $\{\psi_i\}_{i=1}^4$ are eigenfunctions with corresponding eigenvalues of E_1, E_2, E_3 and E_4 . Remember that the probabilities should sum up to 1.

2. Let us say that a particle in an infinite box were to be described by a wavefunction:

$$f(x) = \left(\frac{30}{a^5}\right)^{1/2} x(a-x) \quad 0 \leq x \leq a$$

What is the probability of measuring the energy value $E_n = \frac{n^2 h^2}{8ma^2}$?

3. Is the operator $\hat{A} = i(\hat{x}^2 + 1)\frac{d}{dx} + i\hat{x}$ Hermitian?
4. For a system in a non-stationary state, given by $\Psi(x, t) = \sum_j c_j \psi_j e^{-iE_j t/\hbar}$ show that the average energy is independent of time but the expectation values of other properties vary with time.
5. Construct a 3 by 3 matrix representation of the following operators in a particle in an infinite box basis set.

- (a) \hat{x}
 (b) \hat{x}^2
 (c) \hat{H}
 (d) \hat{p}_x

The *matrix elements* of an operator \hat{A} in a basis $\{\psi_i\}$ are given by

$$a_{ij} = \int \psi_i^* \hat{A} \psi_j dx$$

Is there anything special that you notice about the matrix representation for the Hamiltonian operator? What about the other operators?

6. Match the following eigenfunctions in Column B to their operators in Column A.

- | | |
|---|--------------------------|
| (a) $(1-x^2)\frac{d^2}{dx^2} - x\frac{d}{dx}$ | (i) $4x^4 - 12x^2 + 3$ |
| (b) $\frac{d^2}{dx^2}$ | (ii) $5x^4$ |
| (c) $x\frac{d}{dx}$ | (iii) $e^{3x} - e^{-3x}$ |
| (d) $\frac{d^2}{dx^2} - 2x\frac{d}{dx}$ | (iv) $x^2 - 4x + 2$ |
| (e) $x\frac{d^2}{dx^2} + (1-x)\frac{d}{dx}$ | (v) $4x^3 - 3x$ |

7. For any dynamical quantity represented by operator \hat{A} , show that

$$\frac{d\langle \hat{A} \rangle}{dt} = \int \Psi^* \frac{i}{\hbar} [\hat{H}, \hat{A}] \Psi dx$$

Using this what is $\frac{d\langle \hat{p}_x \rangle}{dt}$ equivalent to? The Hamiltonian $\hat{H} = \hat{T} + \hat{V}(x)$.