Surprise!...Test...

1. Write down the harmonic oscillator wave functions for the quantum numbers $n = 0$, $n = 1$, $n = 5$ for a potential $V = \frac{1}{2}x^2$. Sketch the classical probability densities for each of the states together with the quantum mechanical probability densities.

2. The 3 dimensional isotropic harmonic oscillator has the potential energy function:

$$V(x, y, z) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + \frac{1}{2}k_z z^2$$

   Here k_x,k_y and k_z are the respective force constants.

   (a) Find the energy eigenvalues.

   (b) If k_x,k_y and k_z are equal find the total number of states upto (and including) the four lowest energy levels.

3. Apply the harmonic approximation to the total potential energy function:

$$V(x) = 3x^4 - 2x^2$$

   (a) If ω is the classical frequency of oscillation, for a particle of mass 1 unit, then what is the zero point energy?

   (b) Is the zero point energy the same in both the wells?

   (c) For which quantum number will the overlap be maximum between the wave functions in both the wells?

   (d) How will you calculate the probability of tunnelling from one wellto the other?

4. The particle on a ring is a useful model for the motion of electrons around the porphyrin ring, the conjugated macrocycle that forms the structural basis of the heme group and the chlorophylls. We may treat the group as a circular ring of radius 440 picometers, with 20 electrons in the conjugated system moving along the perimeter of the ring. Assume that in the ground state of the molecule quantized each level is occupied by two electrons.

   (a) Calculate the energy of an electron in the highest occupied level.

   (b) Calculate the frequency of radiation that can induce a transition between the highest occupied and lowest unoccupied levels.

   (c) What is the quantum mechanical probability density for a particle moving on a ring? Compare it with the classical one.

5. Contruct a 4 by 4 matrix representation for the following operators using the Harmonic oscillator wavefunctions as a basis:

   (a) $\hat{x}^2$

   (b) $\hat{a}_+$

   (c) $\hat{a}_+\hat{a}_-$